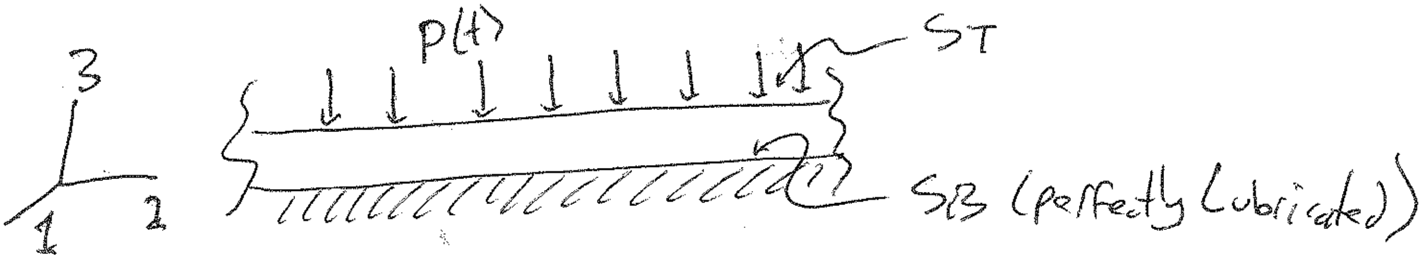
Solid Mechanics.

(2) You have a isotropic elastic material confined at infinity so $U_1 = U_2 = 0$ there (see picture). Imagine that a pressure is applied on the top $P = P(t)$. Derive the equation of motion for u_3 what is the wave speed.



B.C.s:

ST : $\sigma_{33} = P(t)$, all others 0.

S_B : $U_3 = 0$

we'll do it later.

~~From before we saw that [σ] had only~~

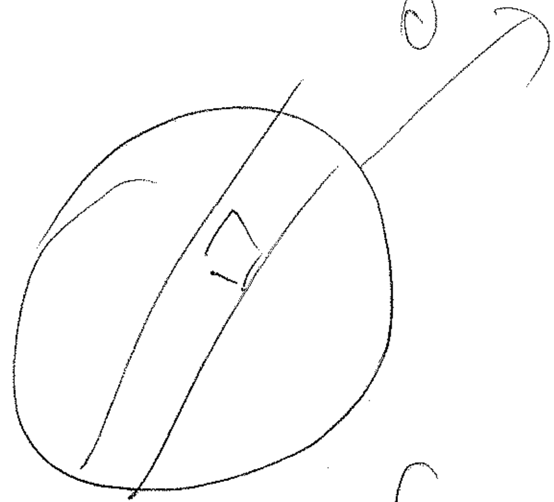
from before, we saw that $U_3 = \dfrac{P x_3}{E}$. Start with

$$\sigma_{ij,j} + \cancel{b_i}^{\,0} = \rho a_i \quad ; \quad \sigma_{33,3} = \rho \ddot{x}_3$$

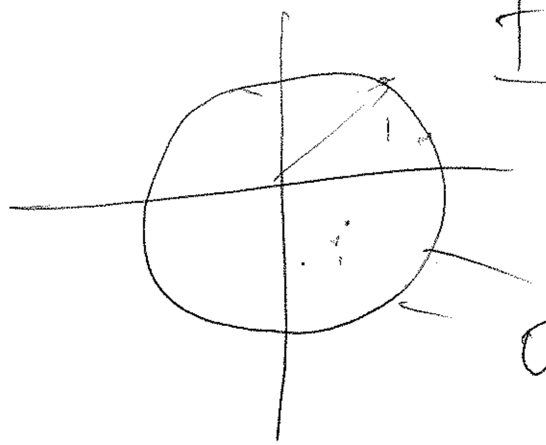
$$\gamma_{33} = U_{3,3} = \frac{P(t)}{E}, \quad \sigma_{33} = 2\mu\gamma_{33} + \lambda\gamma_{kk} \; ; \; \gamma_{kk} = \gamma_{33}$$

$$\sigma_{33} = (2\mu + \lambda)\frac{P(t)}{E}$$

$e^{e^{i\theta}}$



$f(z)$

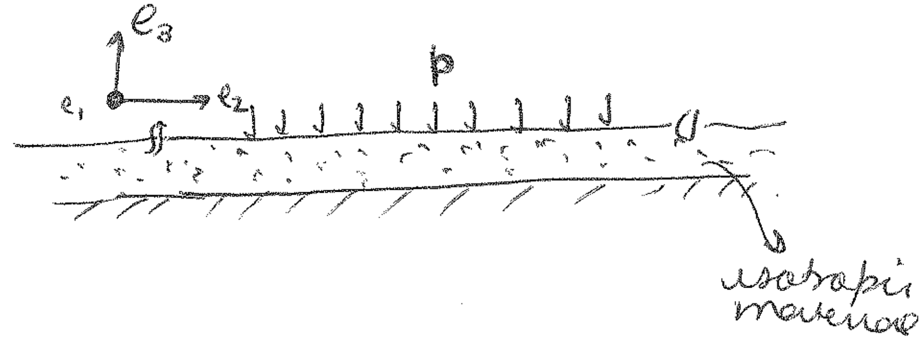


$\frac{1}{2\pi i}\int_z \frac{f(z)}{z-t}\,dz =$

$\frac{P+3}{E}$

solid dynamics 2008

Q1)

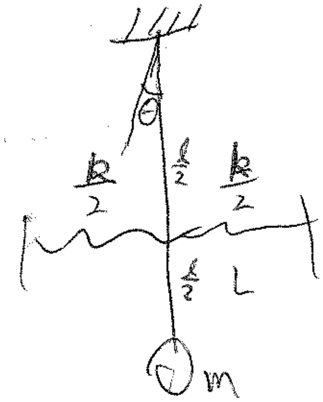


Dynamics 2008

Chooo + Nooks

consider a pendulum (massless rod (length l) with a mass 'm' at end).
midway it has two springs attached to it, each of sp. constant $\frac{k}{2}$

→ using Lagrangian method, to find equation of motion
→ find frequency for small amplitude motion



(1) Kinetic Energy of the system

$$T = \frac{1}{2}m(\dot{\theta}L)^2 = \frac{1}{2}m\dot{\theta}^2L^2$$

$$V = 2\left(\frac{1}{2}\frac{k}{2}\left(\frac{L}{2}\theta\right)^2\right) + mgL(1-\cos\theta)$$

$$= \frac{k}{8}L\theta^2 + mgL(1-\cos\theta)$$

Lagrangian Equations

$$L = T - V = \frac{1}{2}m\dot{\theta}^2L^2 - \frac{k}{8}L\theta^2 - mgL(1-\cos\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = 0$$

$$m\ddot{\theta}L^2 + \frac{k}{4}L\theta + \sin\theta\, mgL = 0$$

$$m\ddot{\theta} + \frac{\frac{k}{4}\theta + \theta mg}{L} = 0$$

$$\boxed{m\ddot{\theta} + \frac{1}{L}\left(\frac{k}{4} + mg\right)\theta = 0}$$

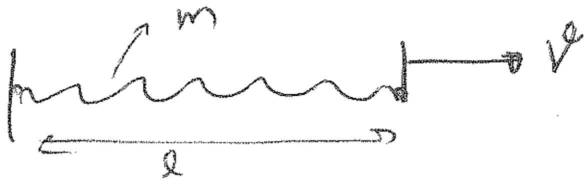
The frequency is

$$\omega = \sqrt{\frac{1}{Lm}\left(\frac{k}{4} + mg\right)}$$

② if spring has mass what happens?

→ there would be an extra K.E energy term in Lagrangian. due to spring. P.E is same

How to get K.E of spring



- assume mass is uniformly distributed
- if one end is at velocity v, other end is at 0

assume linear distribution of velocity along length of spring

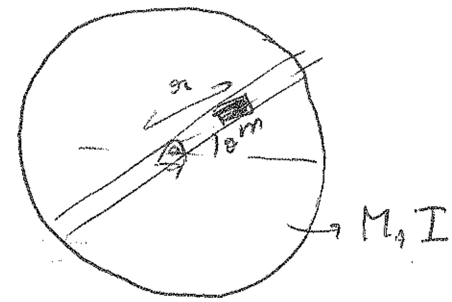
$$K.E = \frac{1}{2}\int dm\, v^2$$

$$= \frac{1}{2}\int_{x=0}^{x=l}\left(\frac{x}{l}v\right)^2 \underbrace{\frac{m}{l}\,dx}_{dm}$$

= ? do that yourself

Dynamics 2008 Noofuy

Q2 There is a disc hinged frictionlessly at centre. (I, M)
it has a frictionless groove.
On which a mass m can freely slide



→ find e.o.m

gen coordinate r, θ

$$\mathcal{L} = K.E = \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m (r^2 \dot{\theta}^2 + \dot{r}^2)$$

$\frac{\partial \mathcal{L}}{\partial q} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right)$

e.o.m finally come out

for $q = r$: $\ddot{r} - r\dot{\theta}^2 = 0$] no force on 'm' in $\hat{e}_r$ direc — ①

for $q = \theta$ $\xrightarrow{\text{integrating}}$ $I\dot{\theta} + m r^2 \dot{\theta} = \text{constant}$ ②] this is same as total angular momentum conservation

also there is energy conservation

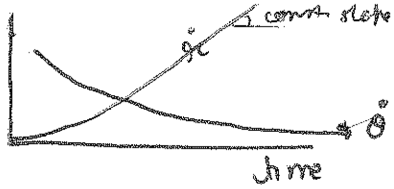
$$\frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m (r^2 \dot{\theta}^2 + \dot{r}^2) = \text{Constant} \longrightarrow ③$$

→ does the θ goes to ∞ as $t \to \infty$ for any of the initial conditions.
(Typical Prof Ruina style)
(also in Dynamic 570 HW's).

from ① we see $\ddot{r} = r\dot{\theta}^2$ always posit $\therefore$ r always increase as $t \to \infty$ $r \to \infty$

from ②
$$\dot{\theta} = \frac{\text{const}}{I + mr^2} \quad \therefore \text{ as } t \to \infty \quad r \to \infty$$
$$\therefore \dot{\theta} \to 0$$

from ③ $\dot{\theta} \to 0$ this mean total energy (which is conserved.) goes to $\dot{r}$ motion of particle (disc kinda stops).

$\therefore$ @ $t \to \infty$ $\dot{r} \to$ constant with this info → 

from ②

$$\dot\theta = \frac{\text{const}}{I + mr^2} = \frac{k}{I+mr^2}$$

$$\therefore \int_0^\infty \dot\theta\, dt = k\int_0^\infty \frac{dt}{I+mr^2}$$

$$\theta_\infty = k\int_0^a \frac{dt}{I+mr^2} + k\int_a^\infty \frac{dt}{I+mr^2}$$

•) if a is some large value of time but still finite.

then first integral is finite.

•) in second integral. we can use fact that after large time $\dot r$ = const r = const + const t

$$r = C_1 + C_2 t$$

$$\therefore \text{second integral} = k\int_a^\infty \frac{dt}{(I + mC_1^2 + mC_2^2t^2 + 2mC_1C_2t)}$$

now this integral also converges. because it is comparable with $\int_a^\infty \frac{dt}{t^2}$ which converges to $\frac{1}{a}$.
(remember maths 191).

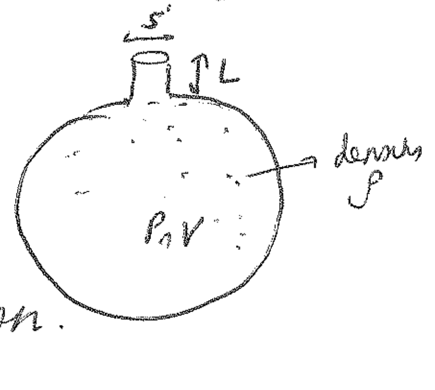
$\therefore$ both integrals converge

$\boxed{\text{hence } \theta_\infty \text{ is finite}}$

gosh!!

Dynamics 2008 Noopy

Q3 Consider a big flask with volume V, filled with air at pressure P. It has a small neck of length l, and crossection area S. Air follows the law $PV^{\gamma} = \text{constant}$ when you blow over the neck, a sound comes. find frequency of that sound (after modelling it as a spring mass system).



→ STATE ASSUMPTIONS

→) big flask's volume and pressure is almost P & V, it fluctuates a little, because of up and down motion of air in the neck.

→) wavelength of the motion is small (I don't what exactly it helps in ask Sachse)

→ model the system, what is 'moving mass', and what is the 'spring constant'.

:) so mass which is moving is the air in neck.

$$m = \rho L S$$

:) spring effect comes from fluctions in large flask. due to movement of air colum.

<u>if air colum moves down by x</u>

- volume decreases to $V - Sx$
- now $PV^{\gamma} = \text{const}$

$$\rightarrow \frac{\Delta P}{P} + \gamma \frac{\Delta V}{V} = 0$$

$$\therefore \Delta P = -\frac{\gamma P \Delta V}{V} = -\gamma \frac{P}{V}(-Sx)$$

. <u>this generates a force upwards</u>

$$F = S\Delta P = \underbrace{\frac{S^2 \gamma P}{V}}_{\text{spring constant}} x$$

so model is

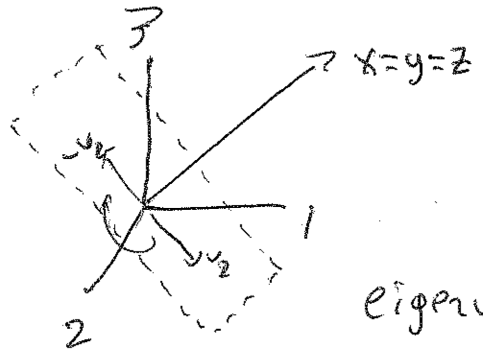
$m = \rho L S$

$k = \dfrac{\gamma S^2 p}{V}$

$\therefore$ frequency of motion $= \sqrt{\dfrac{k}{m}}$

Math 2008 David

1) The Linear Transform $\underline{L}$ rotates vectors in 3space $\overset{180°}{\wedge}$ about the line $x=y=z$. Find the eigenvectors and eigenvalues of $\underline{L}$. Write $\underline{L}$ as a matrix



a) Note that vectors on the line $x=y=z$ get mapped onto themselves.

thus $\vec{V}_1 = \frac{1}{\sqrt{3}}(\hat{e}_1 + \hat{e}_2 + \hat{e}_3)$ is an eigenvector with eigenvalue of 1.

Next, note that vectors in the plane $\perp$ to $x=y=z$ & passing through the origin get mapped to their opposite. Thus the e-value is -1, and we need 2 evectors to span the plane.

let $\vec{V}_2' = \hat{e}_1$, $\vec{V}_3' = \hat{e}_3$, use gram-schmidt to find $\vec{V}_1$ & $\vec{V}_3$

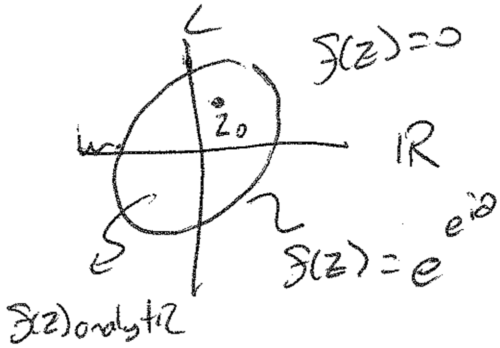
. . . . . .

b) in the basis V, $\underline{L} = \begin{bmatrix} 1 & & \\ & -1 & \\ & & -1 \end{bmatrix} \rightarrow [L]^e = [R]^T [L]^V [R]$

with $R^T = \left[[\vec{V}_1]^e \; [\vec{V}_2]^e \; [\vec{V}_3]^e \right]$

2008 David

(2) Can you find a function s.t. $f(z) = e^{e^{i\theta}}$ on the unit circle, $f(z) = 0$ outside the unit circle circle and $f(z)$ is analytic inside the unit circle?



we know that for any point inside the unit circle,

$$f(z_0) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-z_0}$$

if $f(z)$ is analytic on the domain enclosed by the simple closed curve, positively oriented C. Take C = unit circle.

$z = e^{i\theta}$
$dz = ie^{i\theta}d\theta$

$$\longrightarrow \quad f(z) = \frac{1}{2\pi i}\int_0^{2\pi} \frac{e^{e^{i\theta}}\, ie^{i\theta}d\theta}{e^{i\theta} - z}$$

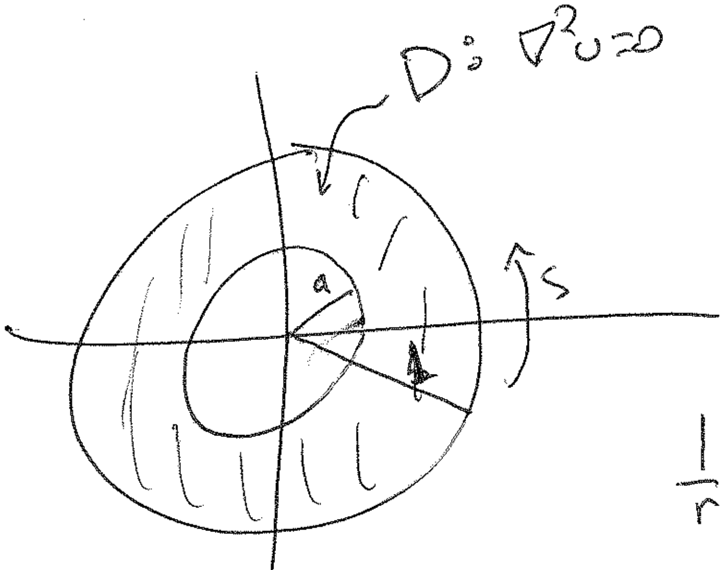
Note that if z_0 is outside the unit circle, $g(z) = \frac{f(z)}{z-z_0}$ is analytic, thus $f(z) = \frac{1}{2\pi i}\oint g(z)\,dz = 0.$

2008 David

(3) let $\nabla^2(u) = 0$ on the annulus bound by concentric circles of radius $b > a > 0$. let

$U(b,\theta) = 0$, $\frac{\partial u}{\partial s}(b,\theta) = 0$ where s is the curvilinear coordinate giving distance around the circle. Solve for u.

Note $u = u(\theta, r) \rightarrow \nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$



$U(b,\theta) = 0$

$\frac{\partial u}{\partial s}(b,\theta) = 0.$

$U(r,\theta) = U(r)$ by symmetry.

$\frac{1}{r}\frac{d}{dr}\left(r\,U'(r)\right) + \frac{1}{r^2}\cdot 0 = 0$

$\frac{1}{r}\left(r\,U''(r) + U'(r)\right) = 0$

$U''(r) + \frac{1}{r}U'(r) = 0. \quad (1)$

note general solution to (1) is $C_1 - C_2 \ln(r)$

$U = C_1 - C_2 \ln(r)$

B.C.s: $u(b,\theta) = 0 \rightarrow C_1 = 0$

$\frac{\partial u}{\partial s}(b,\theta) = \frac{\partial U}{\partial \theta} = 0 \rightarrow$ not unique sol. $U = C_2 \ln(r)$ for any C_2

Q-Exams Dynamics 1/19/07 (Jan 07) Pranav & Krishna

1) A body is acted on by a drag force. ~~which~~ ~~is~~ $F_D \propto v^n$ $n = 0, 1, 2$ & v is the velocity

(i) Discuss examples for various values of n

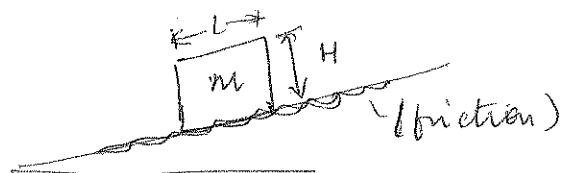
(ii) Is energy conserved?

(iii) Is momentum conserved?

(iv) Find time and distance covered by the body ~~before~~ as it comes to rest.

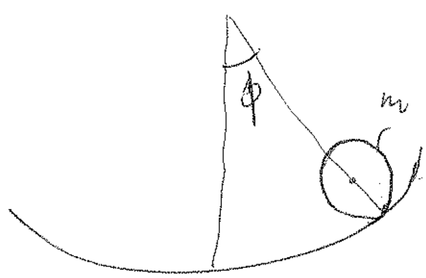
(Cady)

2)



Discuss dynamics (Rand)

3)



Find equations of motion for any one case

(i) The arc is frictionless

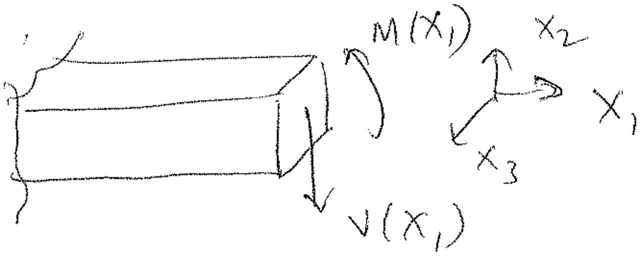
(ii) The body rolls without slipping

(Healey)

Q-Exams Solid/Continuum Mechanics 1/19/07 (Jan 07)

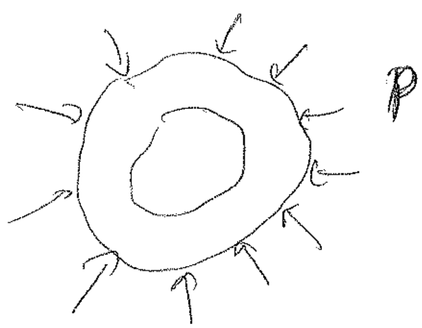
Krishna & Pranav

(1)



Find $\sigma_{12}(X_1, X_2, X_3)$ using continuum mechanics approach (Zehnder)

(2)



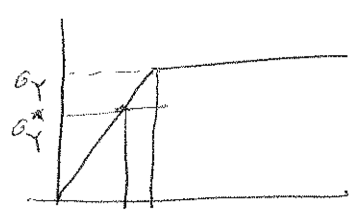
(Stress distribution known for this case)

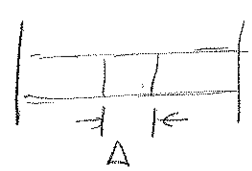
Find stress distribution for

?

(Hint: Superposition) (Jenkins)

(3)





A rod is stressed to σ_Y (yield point) and strain is $\varepsilon_0 = \sigma_Y / E$, ~~Now~~ & clamped as shown. Now in a small part (Δ) the stress drops to σ_Y^*. Find increase in length of this part.

(Phoenix)

Q-Exams Mathematics 1/19/07 (Jan 07)

Krishna & Pranav

1. Solve

$$x y'' + y' + xy = 0$$

(Bessel's eq^{n} of first kind!)
(Rand)

2. Solve $$\int_{-\infty}^{\infty} \frac{\cos x}{(x-4)}\, dx$$ (Rand)

3. Write any PDE you know and solve it analytically. Give different methods. (Weng)

4. $z = x + iy$

$W(z) = u + iv$

$u = u(x, y)$

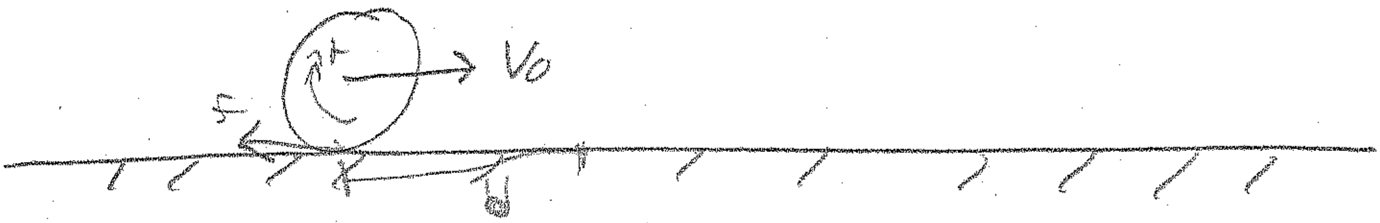
$v = v(x, y)$

given $\nabla^2 \phi = 0 = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$

Prove $$\frac{\partial^2 \phi}{\partial v^2} + \frac{\partial^2 \phi}{\partial u^2} = 0$$

(Mukherjee)

A ball is released on a flat floor



How far does it slide before it is only rolling?

Make any reasonable assumptions.

$$a = -\frac{\mu \cancel{m} g}{\cancel{m}}$$

$$\alpha = \frac{\mu m g R}{\frac{2}{5} m R^2} = \frac{5\mu g}{2R}$$

$$\omega(t) R = v(t)$$

$$[\omega(0) + \alpha t] R = [v(0) + at]$$

$$\frac{5\mu g t}{2} = v_0 - \mu g t$$

$$\frac{7}{2}\mu g t = v_0 \qquad t = \frac{2v_0}{7\mu g}$$

$$d = v(0)t + \frac{1}{2}at^2$$

$$= v_0 \frac{2v_0}{7\mu g} - \frac{\mu g}{2}\frac{4}{49}\frac{v_0^2}{\mu^2 g^2}$$

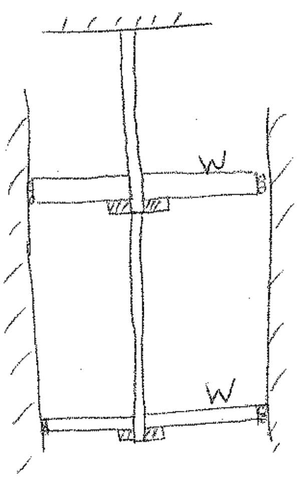
$$= \frac{2v_0^2}{7\mu g}\left[1 - \frac{1}{7}\right] = \frac{12}{49}\frac{v_0^2}{\mu g}$$

SOLID MECHANICS

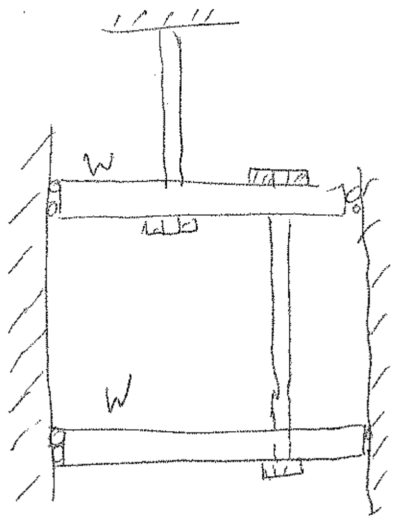
Q summer '06

Manish
~~Bryin~~ Brian
Joshua

Q)



What is the practical problem in this design



Make FBD and argue what might happen?

Q) Why is a plane required to define traction. What is stress tensor and how it is related to traction. How do you get it (derivation)?

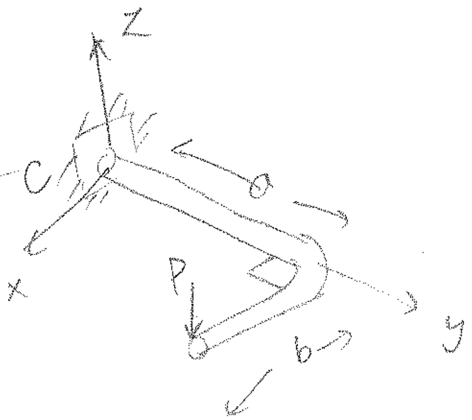
Q) What is modulous?
What is E (Young's mod.) how do you find it?
What is G (shear mod.) how do you find it?
What is ν (Poisson ratio) how do you find it?
What is strain?
the relation between ϵ & σ
definition of ϵ, ν, bulk mod.

Solid Mechanics

Jan 2006

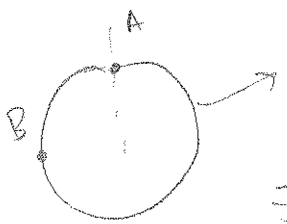
Carlos
Erik
Sanjay

1)



a) Find reactions at the support

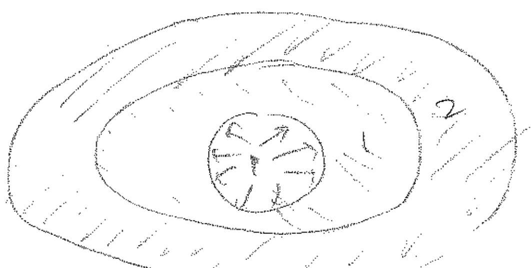
b)



crossection @ the support 'C'

Find stress (linear) at A and B

2)



Two elliptical planes joined as shown.

plate ① has a circular hole
plate (2) elliptical (obviously)

describe boundary and interface conditions (No need to solve)

Applied Math — Jan 2006

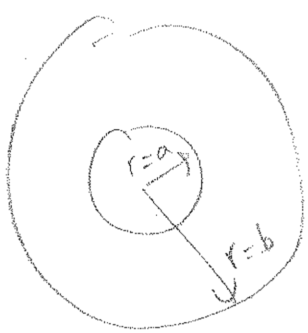
Carlos
Erik
Sanjay

1) Define (any of) the following terms.

1) Cauchy-Riemann eqns
2) Christoffel Symbols
3) Nonlinearity
4) Eigenvectors
5) Infinitesimals

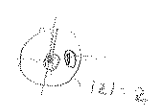
2) Solve

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} - \frac{A}{r^2}u = 0 \qquad \left.\frac{du}{dr}\right|_{r=a} = -1, \qquad u(r=b)=0$$



3) Evaluate

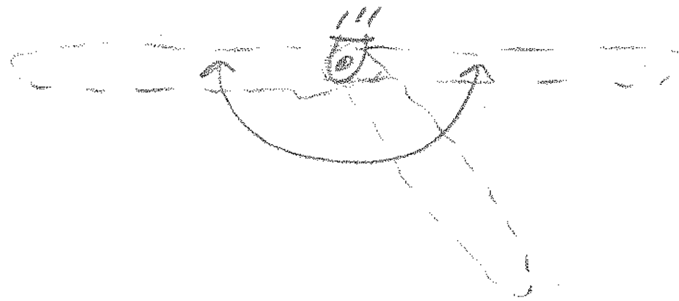
$$\oint_{|z|=2} \frac{e^{\frac{1}{z-1}}}{z}\,dz$$



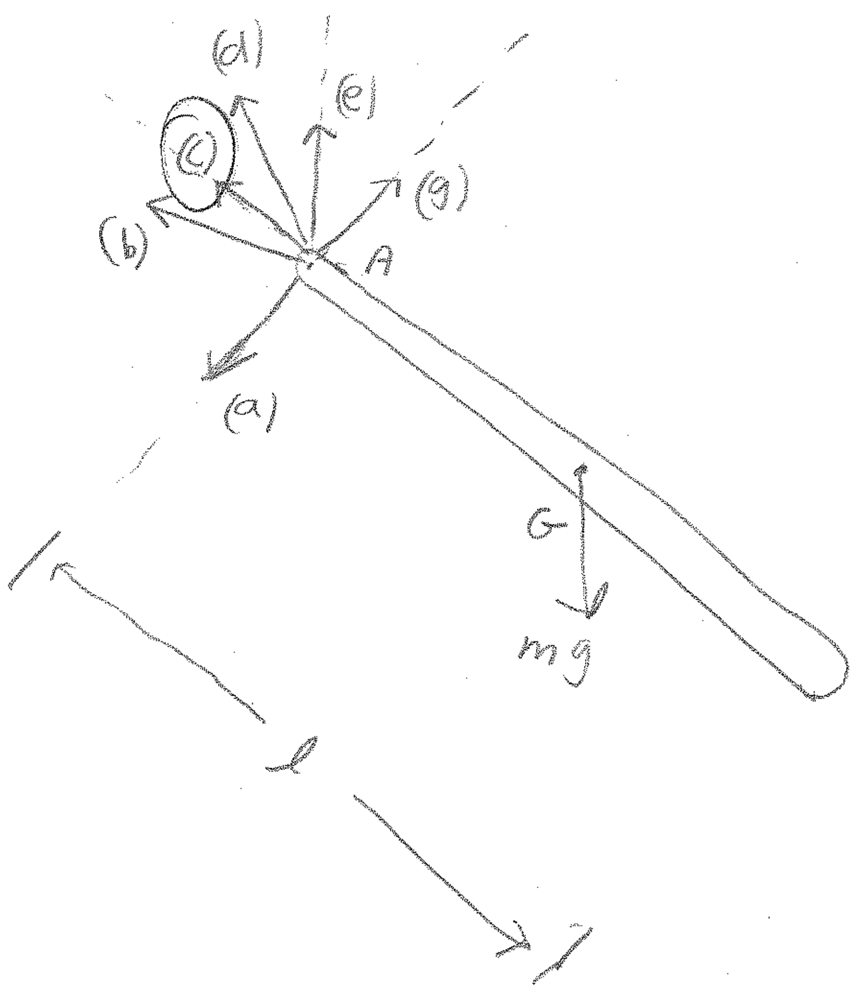
A swinging rod

✓ given l, m, g & release angle $= \theta_0 = \pi/2$

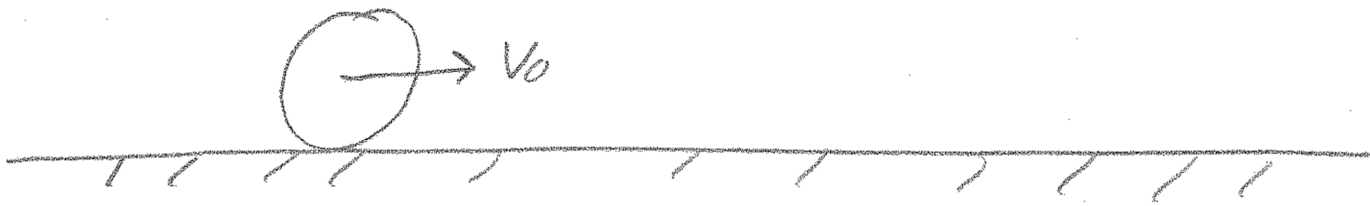
(Uniform mass distribution)



At $\theta = \pi/4$ which is correct reaction force at A : a, b, c, g, e or g?



✓ A ball is released on a flat floor



How far does it slide before it is only rolling?

Make any reasonable assumptions.

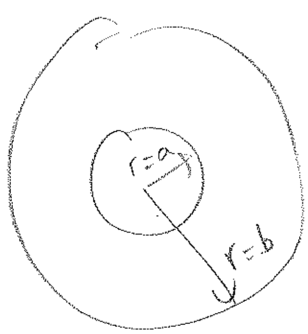
Applied Math — Jan 2006

Carlos
Erik
Sanjay

1) Define (any of) the following terms.

1) Cauchy-Riemann eq^ns
2) Christoffel Symbols
3) Nonlinearity
4) Eigenvectors
5) Infinite series

2) Solve $\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} - \frac{A}{r^2}u = 0$ $\quad \left.\frac{du}{dr}\right|_{r=a} = -1;$ $\quad u(r=b) = 0$



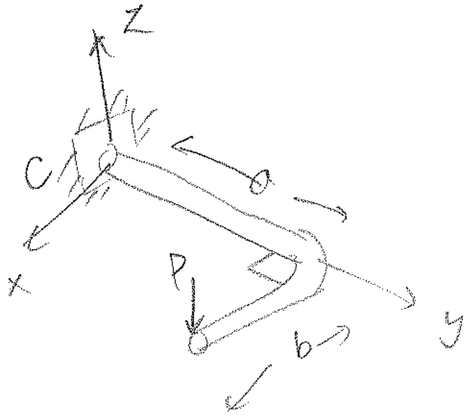
3) Evaluate $\oint_{|z|=2} \frac{e^{\frac{1}{z-1}}}{z}\,dz$

Solid Mechanics

Jan 2006

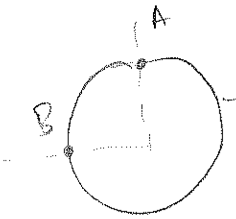
Carlos
Erik
Sanjay

1)



a) Find reactions at the support

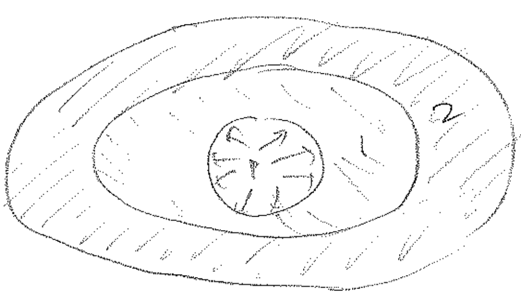
b)



crossection @ the support 'C'.

Find stress (tensor) at A and B.

2)



Two elliptical planes joined as shown.

plate 1 has a circular hole

plate (2) elliptical (obviously)

describe boundary and interface conditions (No need to solve)

(3) Anti plane shear.

displacements

Given $u_1 = 0$; $u_2 = 0$; $u_3 = u_3(x_1, x_2)$

<u>Body forces</u>

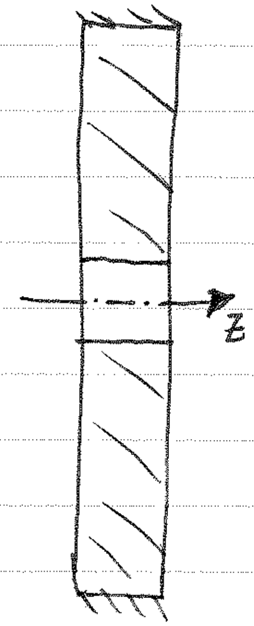
$b_1 = 0$; $b_2 = 0$; $b_3 = b_3(x_1, x_2)$

(a) Set up the equations governing u_3

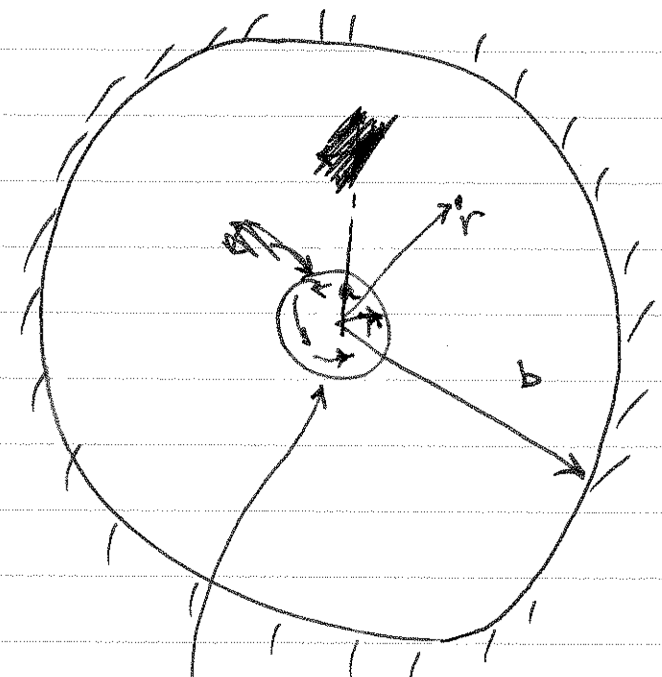
(Ans $G\nabla^2 u_3 = -\rho b_3$) — {use constitutive & equilibrium eqns}

(3)

side view

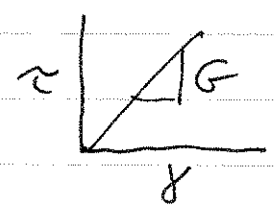


front view (or top view)

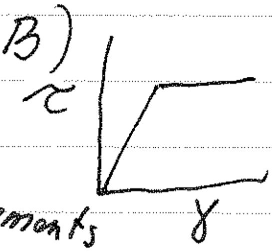


shear, ~~stress~~ τ, applied on inner radius, $r = a$

A)



B)



Assume small displacements

Torque applied to inner rigid disk or radius a, causing shear ~~stress~~ traction, τ

a) Find shear stress as function of r

b) Find shear strain " " " r

c) Find rotation of inner disk (of radius a)

Hints : $\gamma_{r\theta} = r\dfrac{\partial u_r}{\partial \theta} + \dfrac{\partial u_\theta}{\partial r} - u_\theta / r$

$\cancel{\gamma_{r\theta} = r\dfrac{\partial u}{\partial \theta} + \dfrac{\partial v}{\partial r} - \dfrac{v}{r}}$

u_r is r-component u_θ is θ-component

3

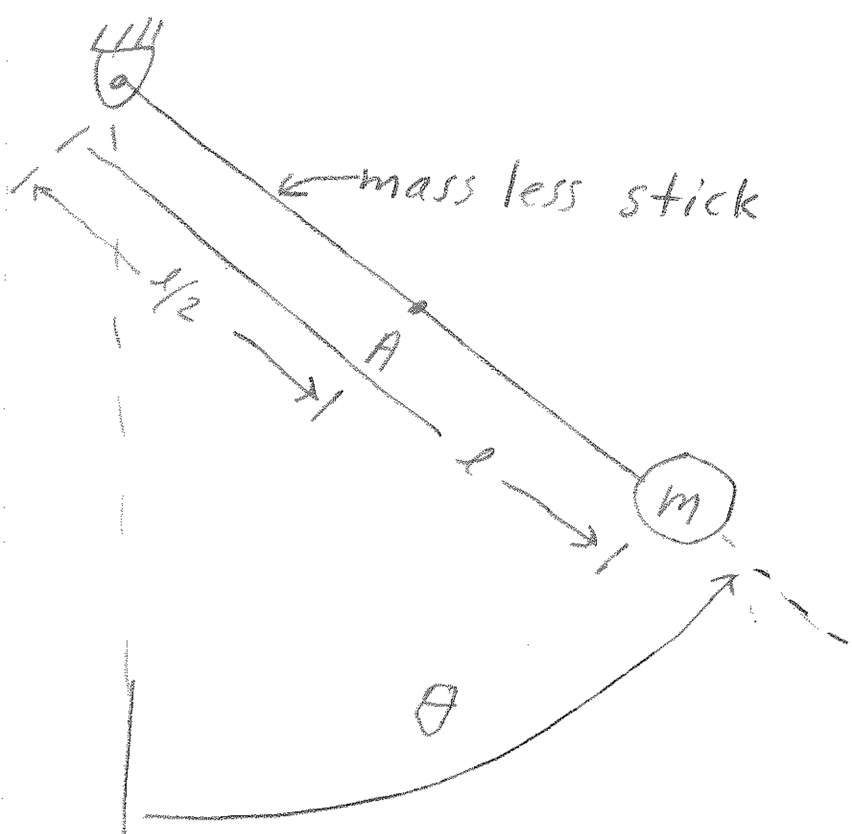
Suppose an $n \times n$ matrix A has real entries, and satisfies

$$A^T = -A.$$

Discuss the relationship between $A\underline{v}$ and $\underline{v}$, for an arbitrary vector $\underline{v}$.

Simple pendulum; given m, g, ℓ

Released from rest at $\theta_0 = \pi/2$.



a) Just at release $(t = 0^+)$ what is acceleration of pt. A?

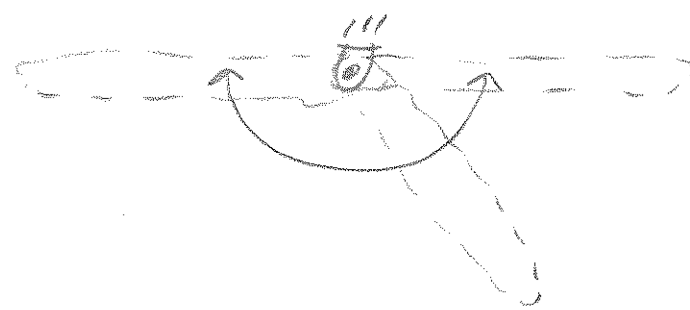
b) At $\theta = 0$ what is tension in rod?

c) What is the period of oscillation?

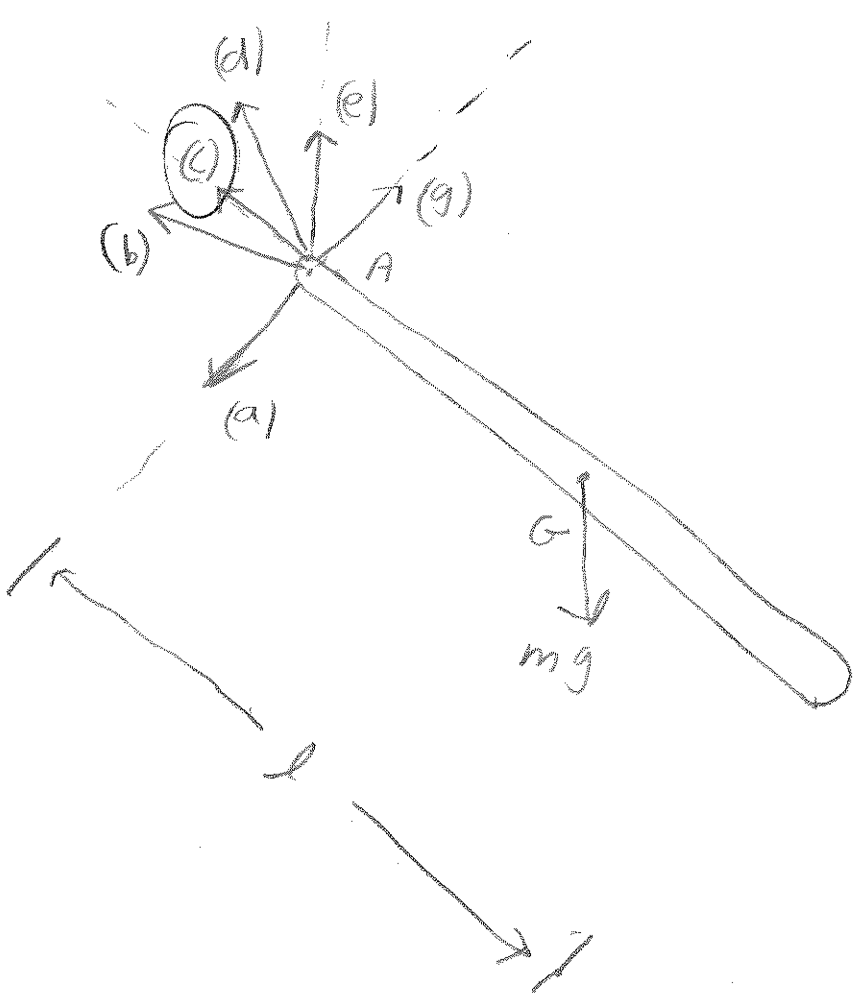
A swinging rod

given l, m, g & release angle $= \theta_0 = \pi/2$

(Uniform mass distribution)



At $\theta = \pi/4$ which is correct reaction force at A: a, b, c, g, e or g?



A ball is released on a flat floor

How far does it slide before it is only rolling?

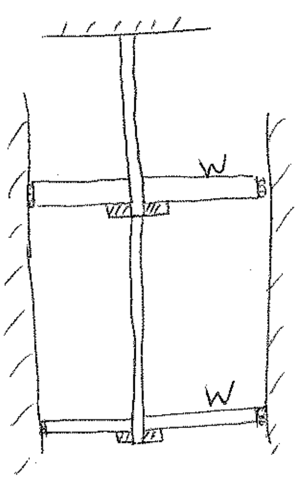
Make any reasonable assumptions.

SOLID MECHANICS

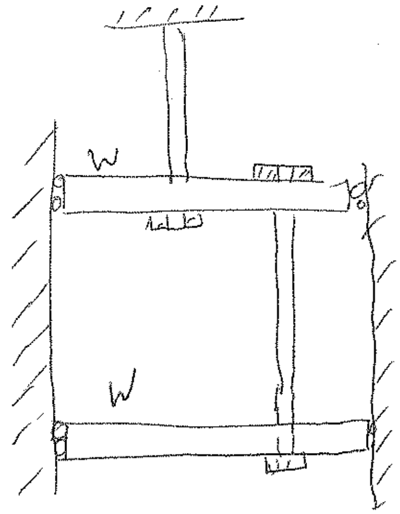
Q summer '06

Manish
~~Bryin~~ Brian
Joshua

Q)



what is the practical problem in this design



Make FBD and argue what might happen?

Q) Why is a plane required to define traction. What is stress tensor and how it is related to traction. How do you get it (derivation)?

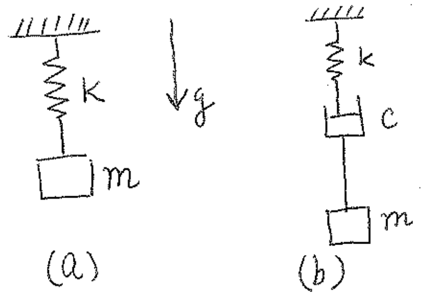
Q) What is modulous?
What is E (Young's mod.) how do you find it?
what is G (shear mod.) how do you find it?
what is ν (poisson ratio) how do you find it?
What is strain?
the relation between ϵ & σ
definition of ϵ, ν, bulk mod.

DYNAMICS

Q Summer '06

Manish
Brian ~~Bryan~~
joshua

Q)



(a) Find the equ^n of motion of the mass for 1) $kx = F$ and 2) $F = kx(1-x)$

compare the two situations will 2) be a periodic motion

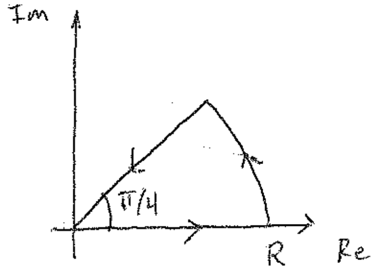
(b) what effect does the damper have on the motion of m. derive the equation

Q) A stone is dropped from the top of empire state bld. ($h = 1250$ ft) find the distance it falls away from the base (due to rotation of earth) $\boxed{\text{Ans } 4''}$

Q) derive the equ^n of motion for a rocket with initial mass m_0, emmition rate $= r$ (constant) and the discharge has relative velocity u wrt rocket.

1) Use e^{ix^2} and contour in fig. A to evaluate $\int_0^\infty \cos(x^2)\,dx$

$$\int_0^\infty \sin(x^2)\,dx$$



2) $$\frac{x^{100}}{(x-1)(x-2)\cdots(x-101)} = \frac{A_1}{x-1} + \frac{A_2}{x-2} + \cdots + \frac{A_{101}}{x-101}$$

Find A_n

3) $L(A) = A + A^T$ A is $n \times n$ real matrix

a) find eigenvalues of L

4) a) Find talor's expansion of $f(x)$ about $x = a$

b) give t. expansion of two functions (say $\sin x$, e^x)

c) $f_{b^+}(x) - f_{b^-}(x) = h$ $b \in [a, c]$

find expansion of f about a.

Moment-free motion of a rigid body

Let the moment of inertia relative to body fixed xyz axes be

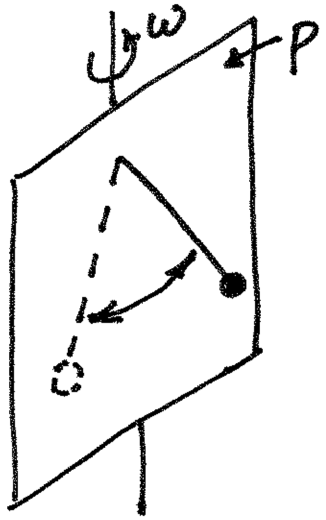
$$\bar{\bar{I}} = I_1 \hat{i}\hat{i} + I_2 \hat{j}\hat{j} + I_3 \hat{k}\hat{k}$$

$$= \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$$

Let $\bar{\omega} = \omega_1 \hat{i} + \omega_2 \hat{j} + \omega_3 \hat{k}$ be the angular velocity of the body relative to an inertial frame.

1. Obtain the eqs of motion governing $\omega_i(t)$.

2. Use these to explain why uniform rotation about the middle moment of inertia is unstable.



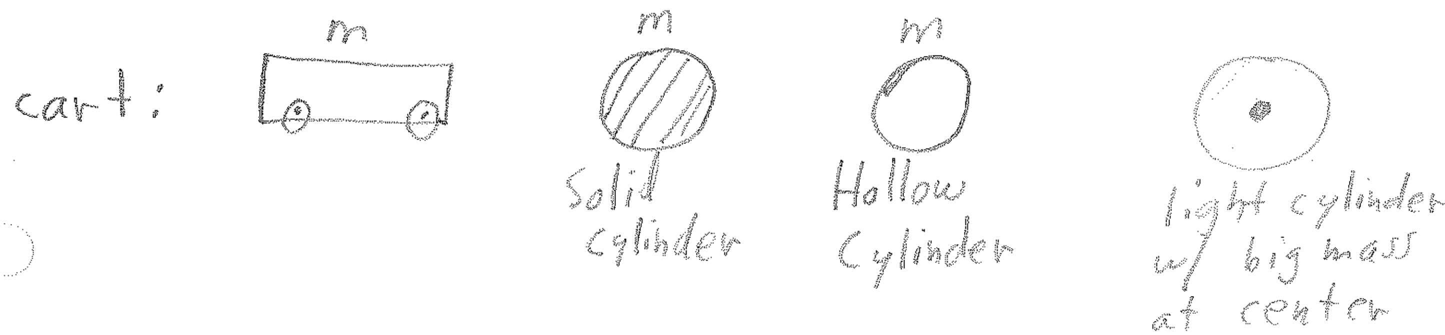
A plane pendulum is restricted to move in plane P. Plane P is forced to rotate about a vertical axis at fixed angular speed ω.

1. How many degrees of freedom does this problem have?
2. Use Lagrange's equations to derive the equation(s) of motion for this system.

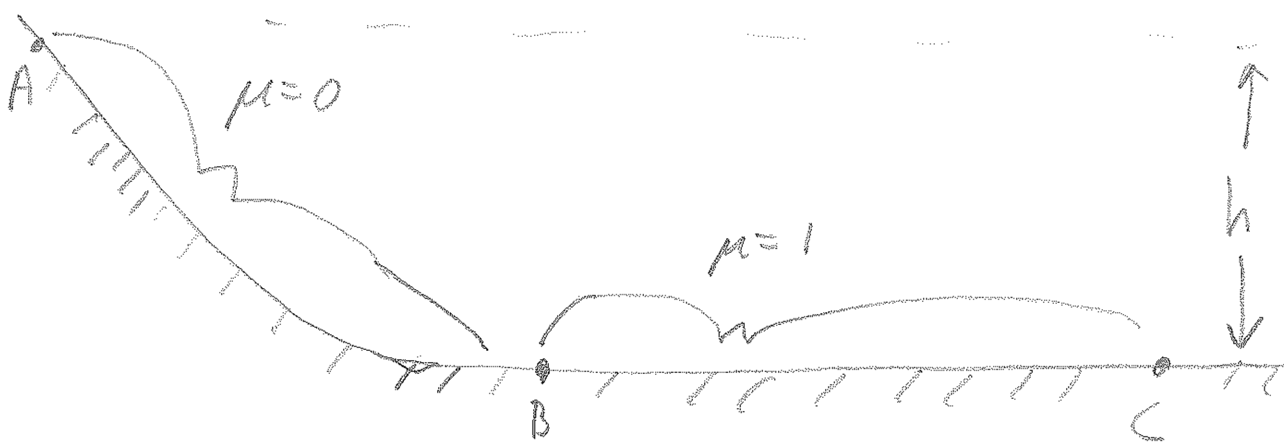
Which object will travel from B to C fastest? slowest?

Objects released from rest at A.

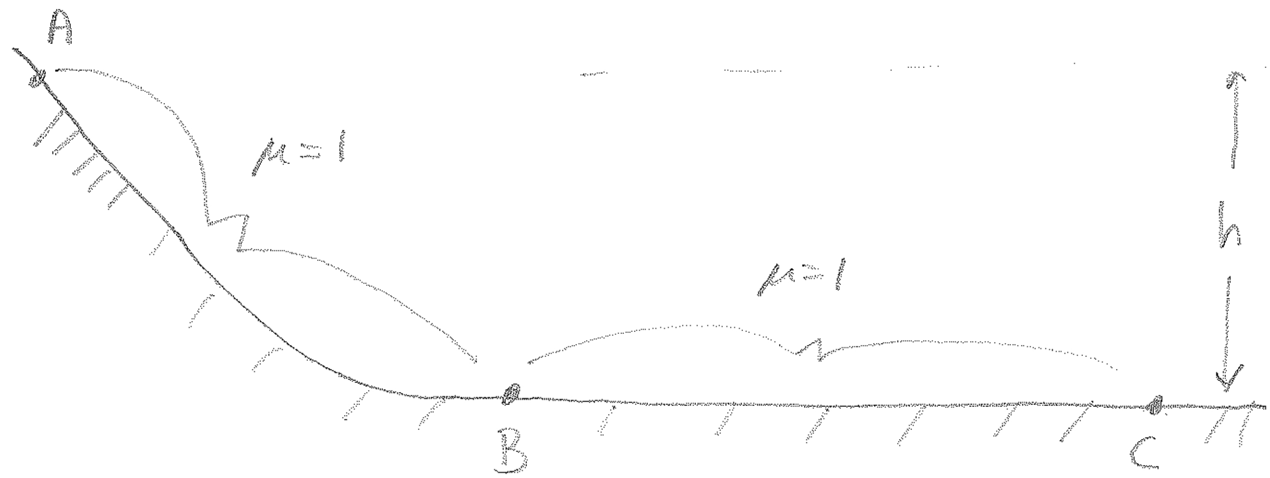
Answer for ramp I & ramp II.



Ramp I



Ramp II



Define any four of the terms below and illustrate your definition with an example:

Centripetal acceleration

Inertia

Harmonic oscillator

Linear momentum

Nutation

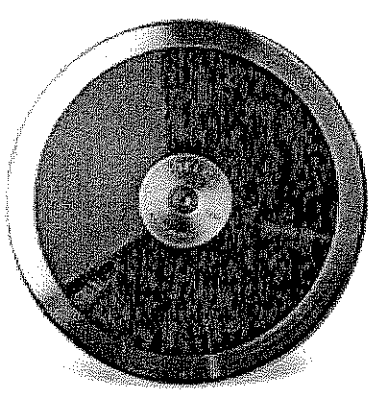
Free body diagram

An Olympic Question:

Write out the equations that describe the motion of a discus (a heavy spinning disk) after it leaves the competitor's hand. Start with the simplest possible assumptions. What is the motion?

Discuss how this model may be made more realistic. What new features will appear in the motion? How do these features arise in the solution.

Orbit Hi-Spin Discus 1k



~~Retail Price: $193.70~~

Our Price: $149.00

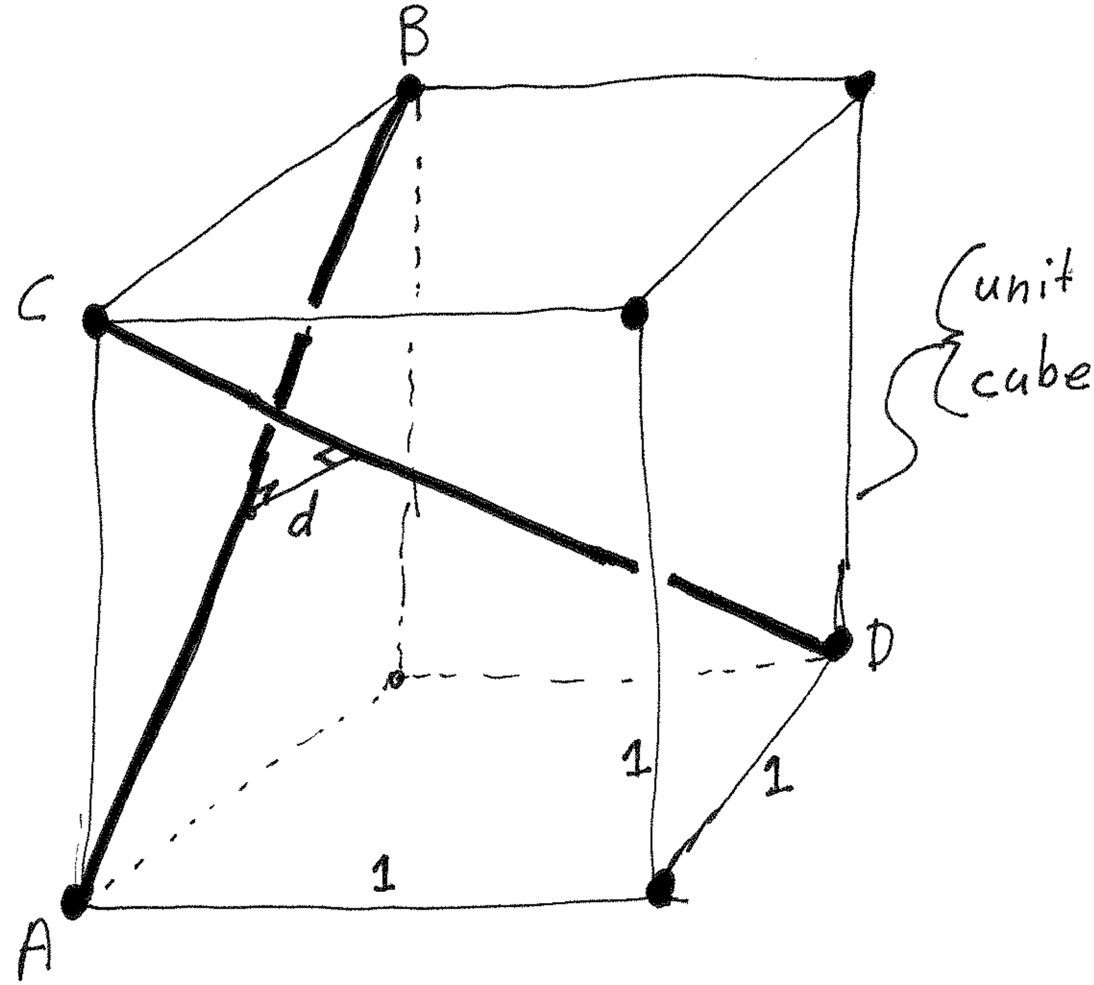
Designed for championship performance, this discus has a rim weight of 80-85% and is the preferred discus of top class throwers. High-impact resistant black ABS side plates, combined with a stainless steel rim, make this one of the best discus in the world.

Model Number: T82

Discus Videos

(not used)

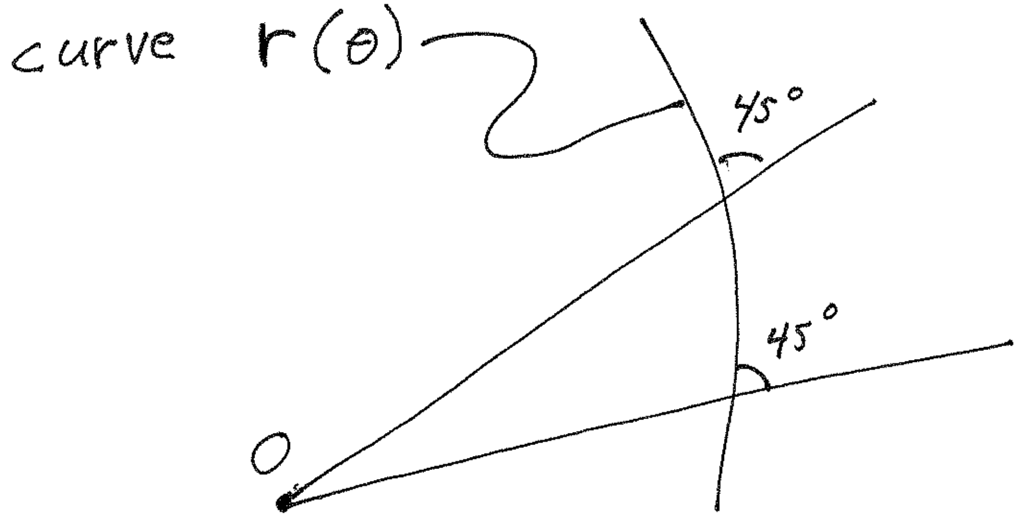
What is the distance between line segment AB and line segment CD?



$d = ?$

(2)

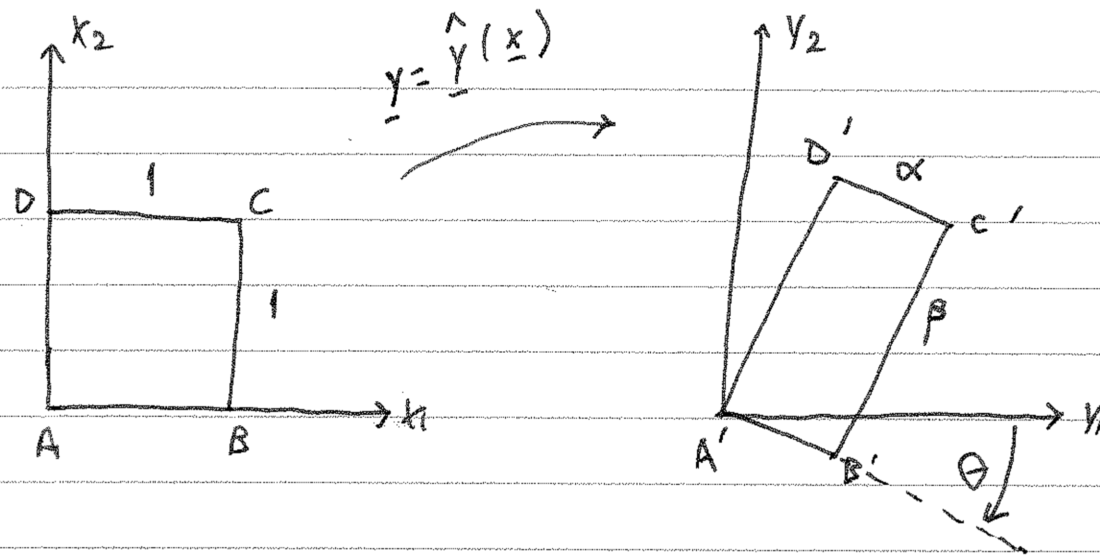
Find the curve $r(\theta)$ that makes a 45° angle with every radial line from the origin.



a) Any such $r(\theta)$.

b) Most general $r(\theta)$.

(1)



Find F, Q, U

What are principal stretches?

principal directions?

3) Alternate form

Using balance of linear momentum, derive the equations of motion for a solid body B with surface S.

~~Describe quantities~~

What, exactly, are traction, stress, and how are they related?

~~Write down the~~

(2) ~~Starting from N~~

Consider a body B of volume V and surface ~~area~~ S. Based on Newton's second law, Write down the equations of motion in integral form over the whole body. Now find a local form of this equation valid at any point in the body.

Do you know how to write the virtual work principle?

(4)

Show

$$\int_0^{\infty} \frac{\cos x}{\sqrt{x}}\,dx = \frac{1}{\sqrt{2}} \int_0^{\infty} \frac{e^{-t}}{\sqrt{t}}\,dt$$

(not used)

Consider the equation

$$2f(x) + \frac{d}{dx} f(x) + \int_0^x f(x')\,dx' = 0 .$$

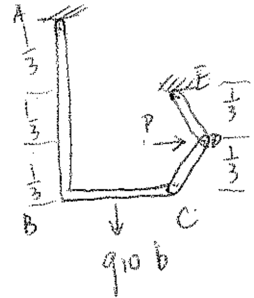
a) Find a solution.

b) Find the most general solution.

Q - Exam Jan. 2005

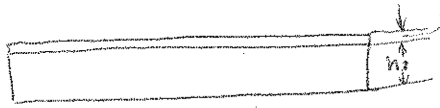
Solid Mechanics: Committee: Zender, Baker, Papoulia, Sachse

Question 1:



Find the force P which makes $^{A}_{B}L_{C}$ stay that way.

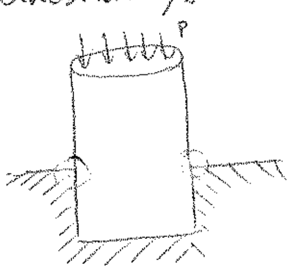
Question 2:



① A large substrate with a thin layer (film) ~~a~~ on the top. These 2 materials have different Thermal expansion Coefficients. When temperature changes, find the relationship of the curvature and the stress in the top layer

② what if $h_1 \approx h_2$

Question 3:



① find the stress field

② which part will yield first, the top part or the bottom part.

Q Exam Jan 2005

Mathematics Committee: Hui, Rand*, Mukherjee, Cady

Question 1: $F(z) = u(x,y) + i v(x,y)$ is analytic on domain Ω. There are two curve families $u(x,y)=c$ & $v(x,y)=c$. They interacted ~~on~~ at pt $z_0(x_0,y_0)$. Show the two ~~fama~~ curve families are $\perp$ to each other.

Question 2: ① $A=\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ Find R s.t. RAR^{-1} is a diagonal ~~tensor~~ Matrix

② what's wrong with $\begin{pmatrix} 7 & -1 \\ 9 & 1 \end{pmatrix}$, why?

Question 3: f is defined as $f = \underline{A}\underline{V} \cdot \underline{V}$ $\underline{A}$ is a symmetric tensor. $\underline{V} \in \mathbb{R}^n$

Show that $\underset{\|V\|=1}{\mathrm{Min}}(f) = \min(\lambda_1, \lambda_2, \cdots \lambda_i)$ λ_i are the eigenvalues of A.

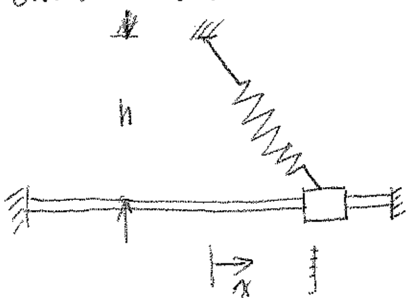
Question 4: $y = f(x)$.

please find $\frac{d}{dy}\frac{dy}{dx}$ in terms of ~~diver~~ derivative of ~~x~~ y

Q Exam Jan. 2005.

DYNAMICS Committee : Ruina, Burns, Healey* Phoenix

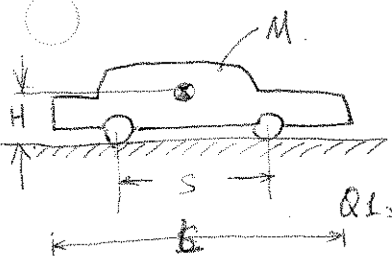
Question 1:



The stress-free length of the spring is l

1. if $h > l$, write the motion equation(s).
2. if x is small, describe the motion
3. if $h < l$, what happen?

Question 2:

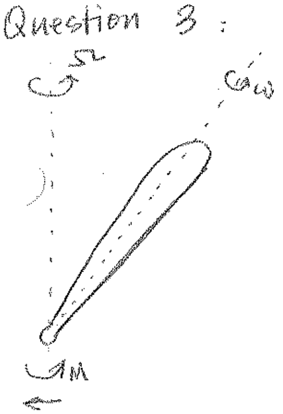


The wheel roll without friction on their axises.
The friction parameter of the ground is μ. You can chose Front or Rear wheel to be the driven wheel.

Q1: what is the maximum ~~minimum~~ acceleration of the car.
(Note: assume pure-rolling ~~for~~ on the non-driven wheel)

Q2: Which dimensions might affect the maximum acceleration: H, L, S, OR M?

Question 3:



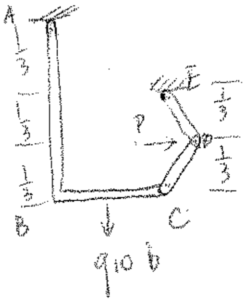
a baseball bat rotates with constant Ω & ω as shown on the left. ① Find the moment on the bottom. (magnitude and direction)

② " " Force needed.

Q - Exam Jan. 2005

Solid Mechanics: Committee: Zender, Baker, Papoulia, Sachse

Question 1:



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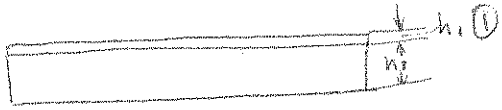

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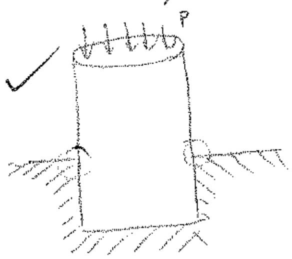

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Q Exam Jan 2005

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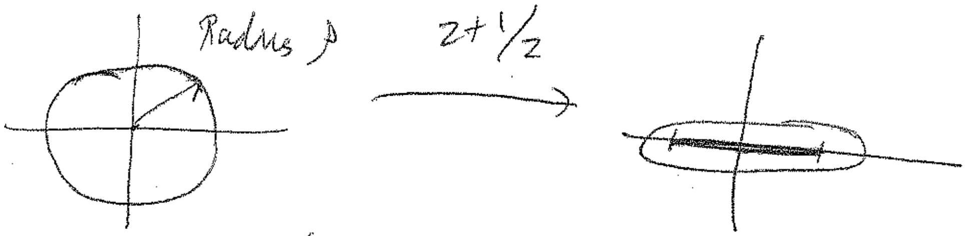
Question 4: $y = f(x)$

please find $\dfrac{d}{dy}\dfrac{dy}{dx}$ in terms of ~~diver~~ derivative of [illegible] y

Qualifying Exam – Aug 23, 2002 – MATHEMATICS

1. $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{x}\, dx$

(or)



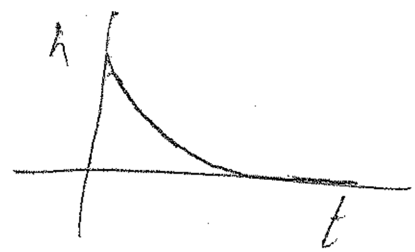
2. $h(t) - h_0 = -a \displaystyle\int_0^t d\tau \sqrt{h(\tau)}$

a) Is the eqn. Linear or Non-Linear?

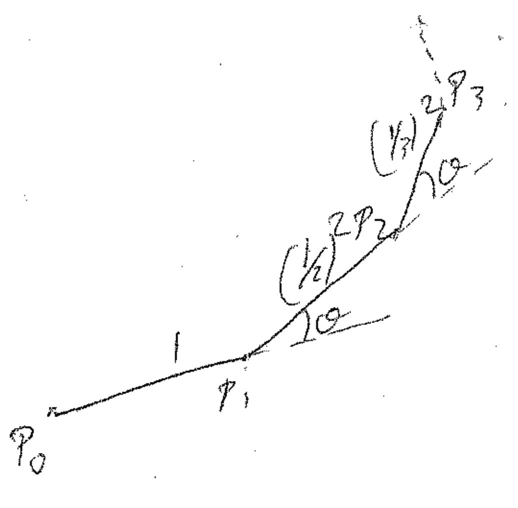
b) Equivalent Differential eqn.?

c) Sketch $h(t)$ without solving

d) Numerical Scheme



3.



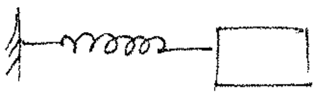
$P_n \to ?$ as $n \to \infty$

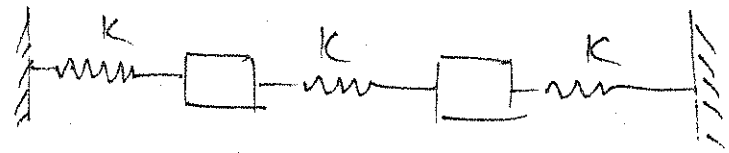
$\sum \frac{e^{in\theta}}{n^2}$

Dynamics

08/23/2002

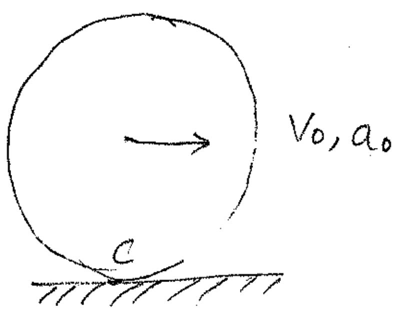
1)





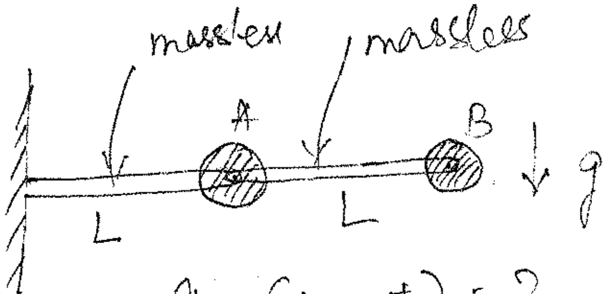
what is resonance, natural frequency, normal modes ...

2)



find ω, a_c, V_c etc

3)

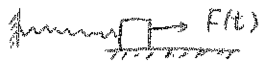


$a_A\ (t=0^+) = ?$

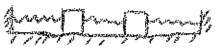
08/23/2002

Dynamics

1\.



(a)

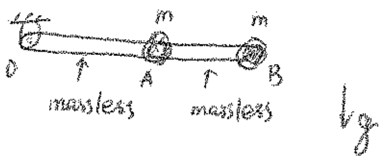


(b)

① Explain what is natural frequency? what is resonance?

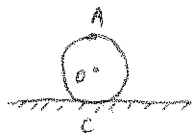
② what is natural modes? where are the natural modes of (b)? what are the frequencies of the natural modes?

2\.



Double Pendulum, released from the current position. what are the accelerations of A and B?

3\.



Given $\underline{V_o}$ $\underline{a_o}$.

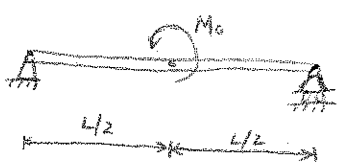
① Find $\underline{V_c}$, $\underline{a_c}$?

② Find $\underline{V_A}$ a_A ?

③ If there is wind coming from right, all the masses concentrates in A. what are the forces acting on A? what is total force?

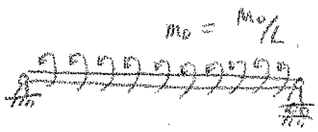
Solid Mechanics

1\.

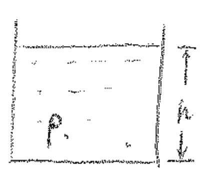


① Find the distribution of shear force and moment in the beam, and draw them.

② what if there is a distributed moment $m_0 = \frac{M_0}{L}$?

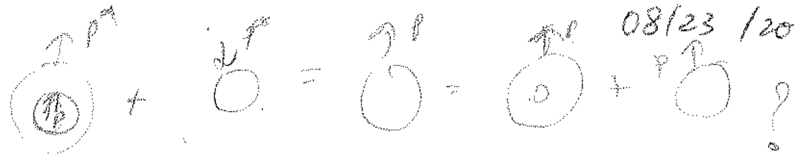


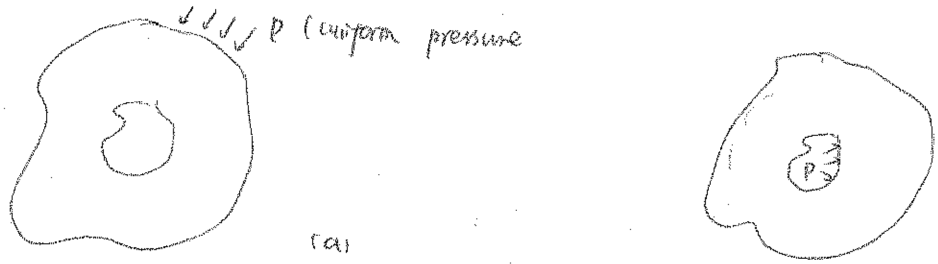
2\. what is stress? what is traction? what is the difference between stress in solid and stress in fluid? Find the stresses in the fluid.



08/23/2002.

Solid Mechanics (Cont'd)





(a) (b)

① Give stresses in (a). Can you find stresses in (b)? (how many solutions?)

② what if the material is not homogeneous. isotropic?

Math ?

1. Conformal Mapping $\xi(z) = z + \frac{1}{z}$, where z is a circle with radius ρ.

① Find $\xi(z)$ and draw it. what if $\rho = 1$? $\rho \neq 1$?

② At which points $\xi(z)$ is not conformal?

---

2. Integral equation $h(t) = h_0 - a_0 \int_0^t \sqrt{h(\tau)}\, d\tau$ $(h_0, a_0 > 0)$.

① Can you write down the D.E formulation?

② draw a sketch of $h(t)$.

③ Any numerical method to solve this problem?

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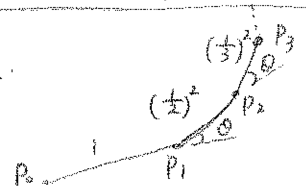

① Will P_n converge or not?

② If it converges, what's the limite?

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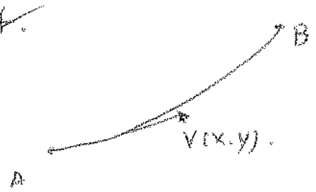
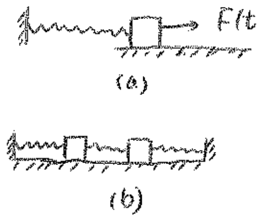


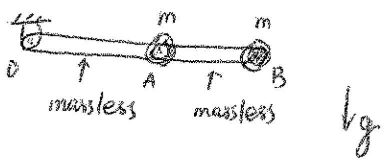
Given $V(x, y)$. Find out the path between A and B which minimize the time needed to travel from A to B.

08/23/2002

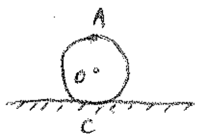
Dynamics

1. 

① Explain what is natural frequency? what is resonance?
② what is natural modes? where are the natural modes of (b)? what are the frequencies of the natural modes?

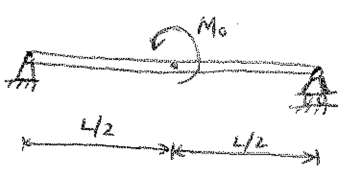
2. 

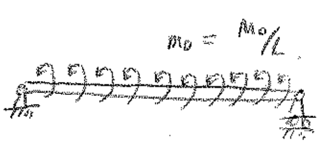
Double Pendulum, released from the current position. what are the accelerations of A and B?

3. 

Given $\underline{v}_0$, $\underline{a}_0$.
① Find $\underline{v}_C$, $\underline{a}_C$?
② Find $\underline{v}_A$, a_A?
③ If there is wind coming from right, all the masses concentrates in A. what are the forces acting on A? What is total force?

Solid Mechanics

1. 



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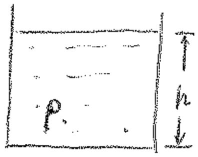

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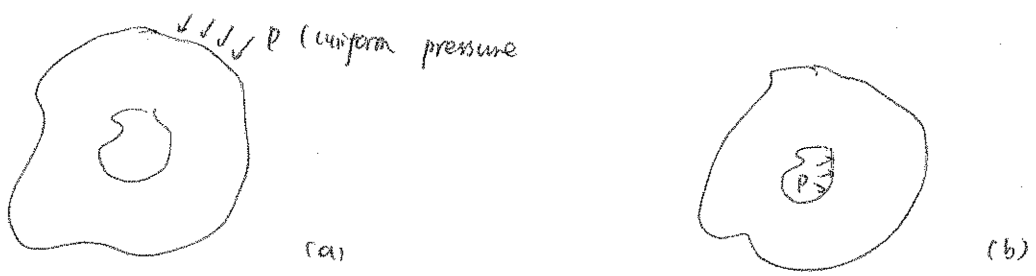

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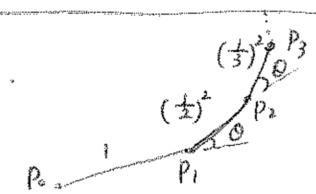

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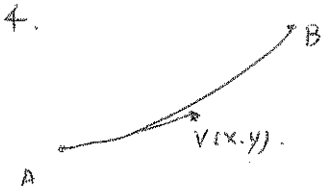


Given $V(x, y)$. Find out the path between A and B which minimize the time needed to travel from A to B.

8/24/01

Applied Math

(Rand)

(5) State the Fredholm Alternative.

Applied Math. 8/24/01

(1) $\sin A = A - \frac{A^3}{3!} \cdots$ A is a matrix.

Given $A = \frac{\pi}{4}\begin{pmatrix} 7 & -3 \\ -3 & 10 \end{pmatrix}$ (Carlos Castillo-Chavez)

Find $\sin A$ in closed form.

(2) P.DE. $\frac{\partial u(x,t)}{\partial x} = c\frac{\partial u}{\partial t}(x,t)$. defined (Wang)

on $x \in [0,1]$. given periodic boundary conditions so that $u(x+1,t) = u(x,t)$, initial condition $u(x,0) = \sin 2\pi x$. Solve for $u(x,t)$.

What is the nature of the solution?

(3) Given $f(z) = u(x,y) + i v(x,y)$ is analytic (Wang)

Consider the level curves $u(x,y) = C_1$
$V(x,y) = C_2$

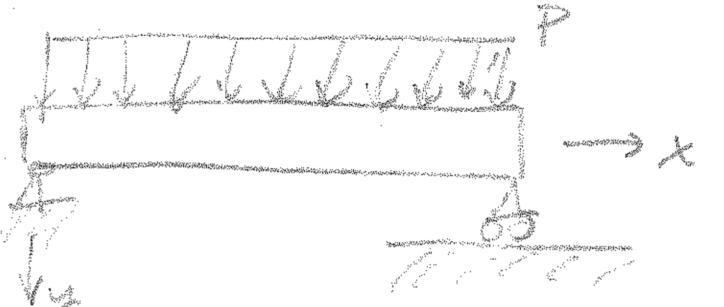
Show that the level curves are orthogonal.

(4) $\frac{\partial \phi}{\partial t} + p(t)\phi = s(t)$ $p(t)$, $s(t)$ given (Cady)

What else, if anything, is necessary to solve?
What is conserved?
Can you set this up as an integral eq?
How to solve numerically?

Cont. Mech. 8/24/01 (Jenkins)

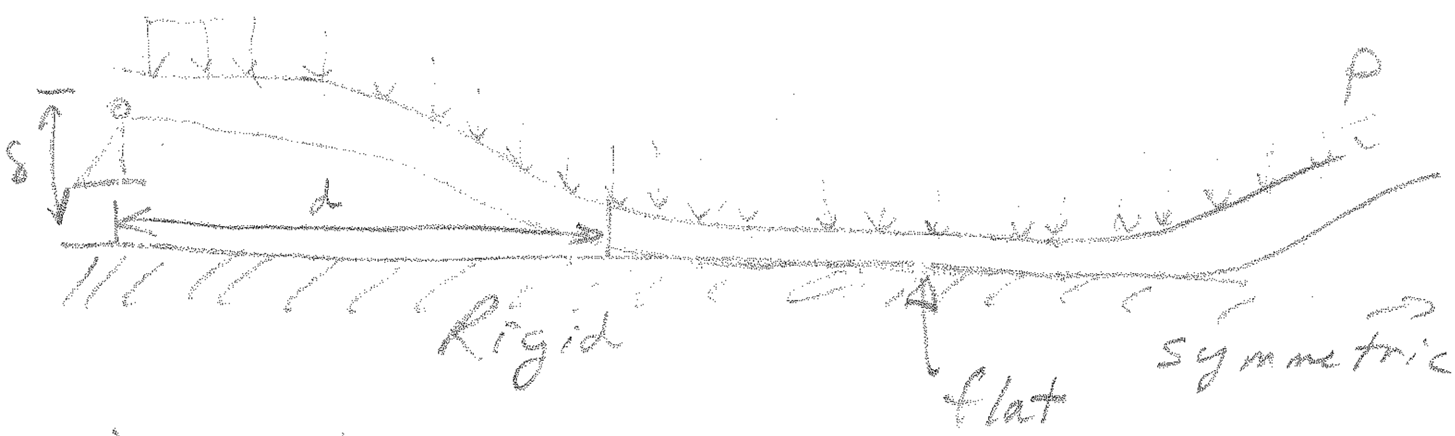
1. What is stress? What is shear stress? What is normal stress?
Write stress/strain constitutive law for lin. elast. isotropic solid. Do the same for an incompressible fluid. How do you define strain for small displacements

2. 

(Healey)

✓ Solve for y(x).

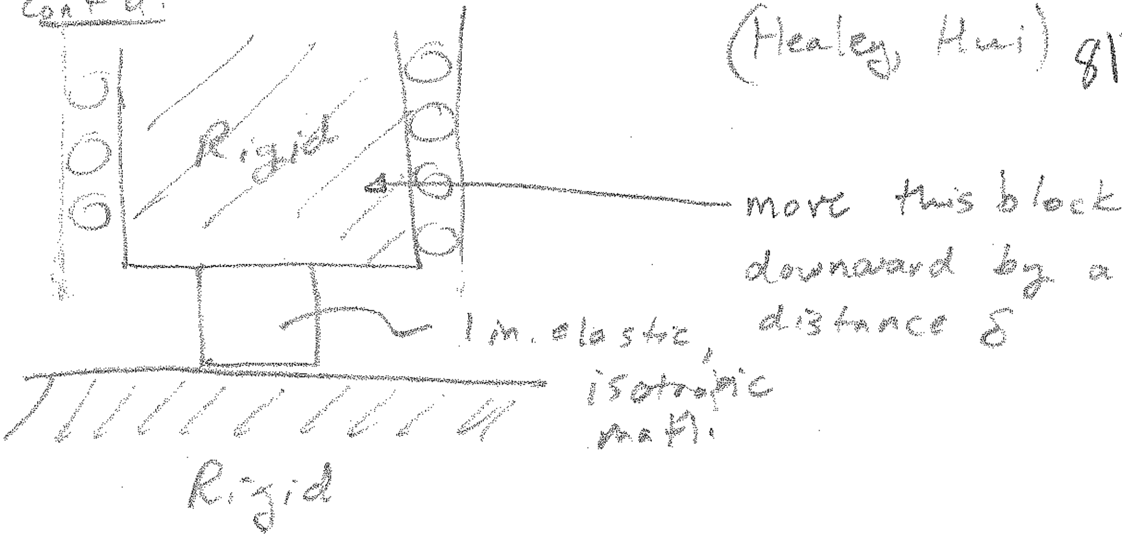
Now also consider p large enough such that



Now state B.C.'s such that we can solve the problem for y(x) (and d).

Cont. Mech. Cont'd. (Healey, Hui) 8/24/10

3.



Set up everything you need to solve the problem (possibly stress - strain, strain-disp., equil, compatibility, boundary conditions)

for two cases: i) top and bottom surfaces perfectly lubricated

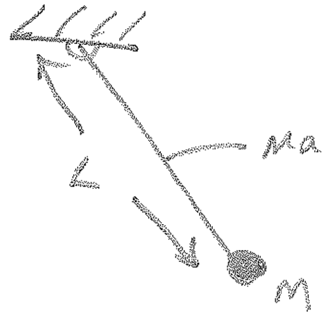
ii) top and bottom surfaces glued

What will the deformed block look like in each case. What if $\nu = 0$? (poisson ratio)

Dynamics

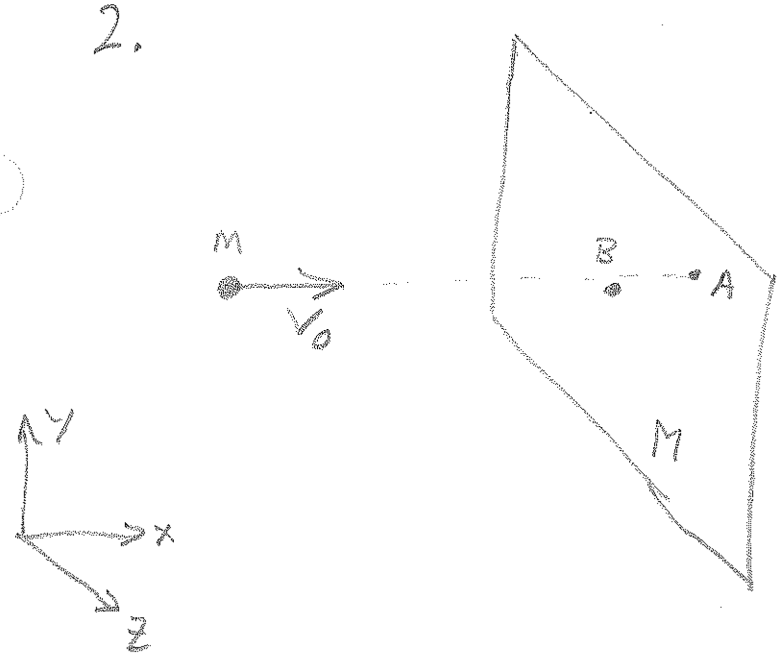
8/24/01

1. Derive eq(s). w/ Lagrange : (Burns)



Describe 2 other ways to derive eq (s) of motion.

2. (Ruina)



Particle collides and sticks at A.

Is $\underset{\sim}{L}$ conserved?,

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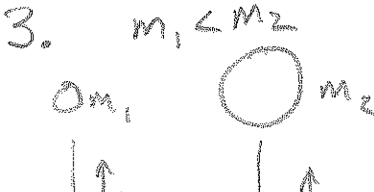
About what points? for what systems? (particle, plate, or particle + plate)

Setup the problem to solve for the subsequent motion.

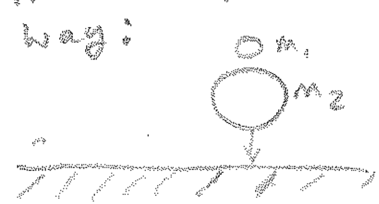
3. $m_1 < m_2$ (Zehnder)



When m_1 and m_2 bounce they return to heights h_1, h_2 respectively. (Approximately the same e.)

Explain ⟶

When dropped this way:



m_1 returns to a much higher height
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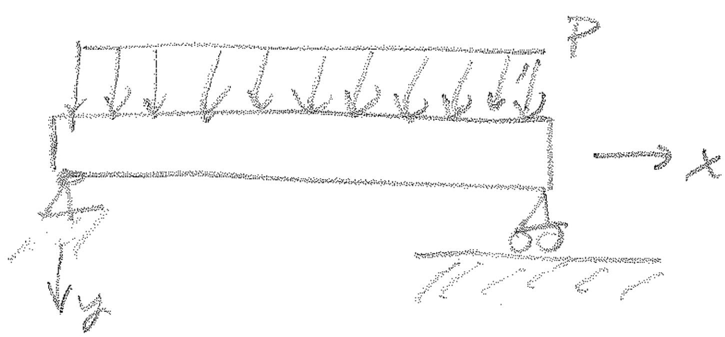
2. 

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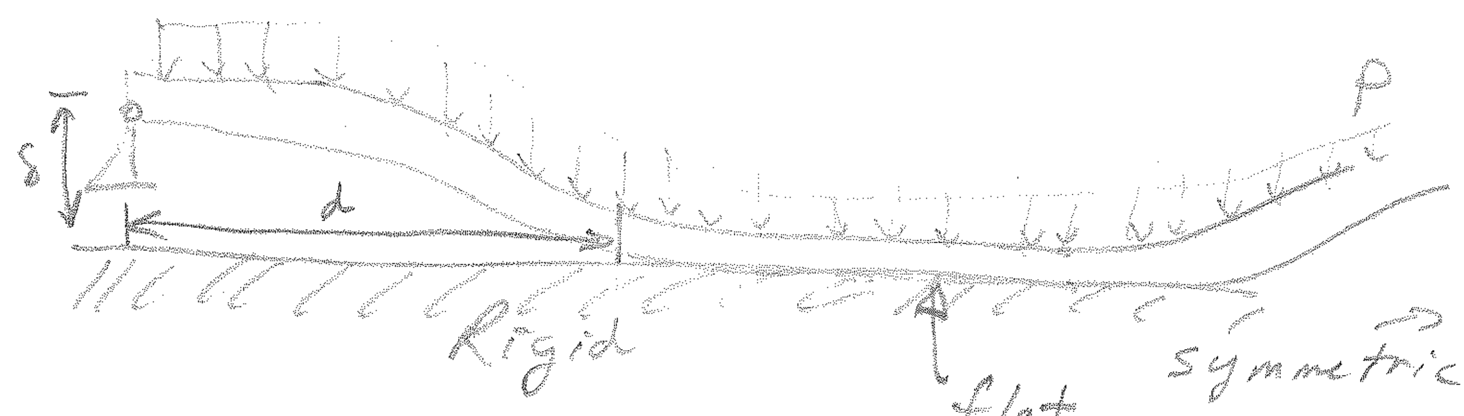

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(Healey, Hui) 8/24/01

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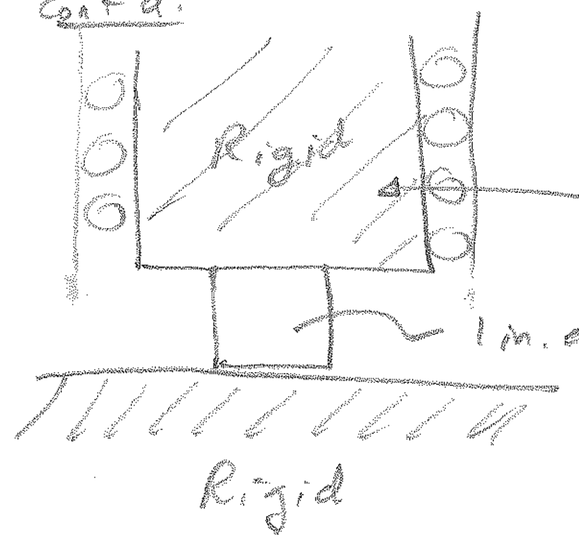

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8/24/01

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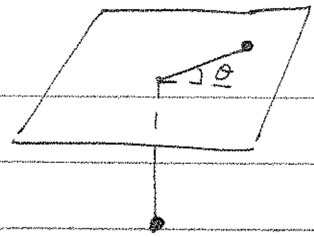
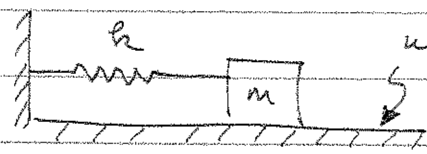
2000 Q-Exam
08/18/00
Dynamics - AFTERNOON

Problem 1.



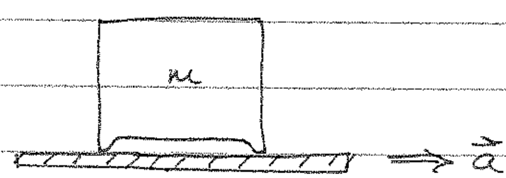
a) How many degrees of freedom?
b) Are there any conserved quantities?
c) Find the equations of motion.

Problem 2.



a) What is the natural frequency?
b) Now, suppose the spring has mass M, what is the new natural frequency?
c) Some other stupid questions...

Problem 3.



floor has constant acceleration $\vec{a}$, block is at rest at $t=0$

a) What happens?

2000 Q-Exam
08/18/00
Dynamics - AFTERNOON

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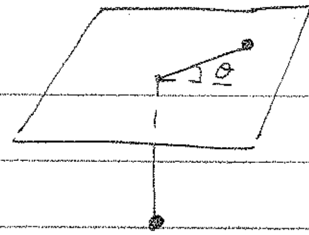

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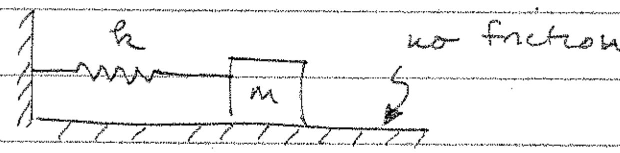

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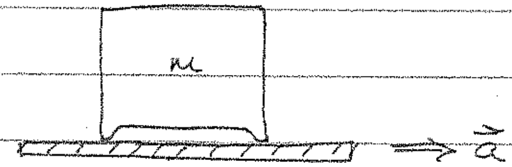

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8/24/01

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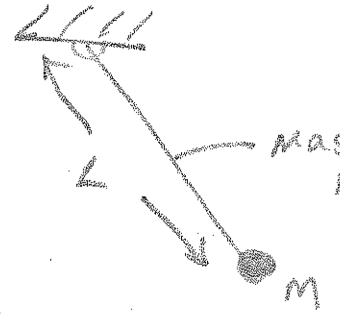

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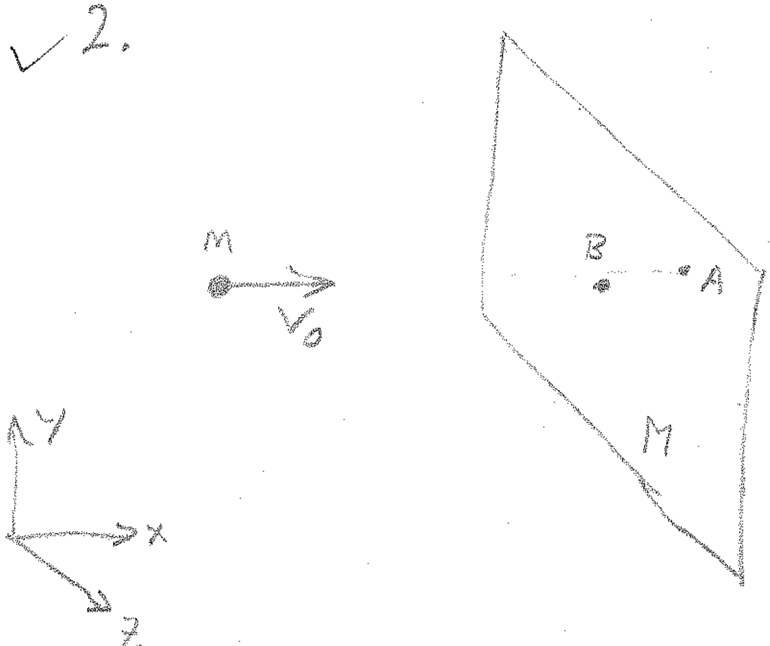

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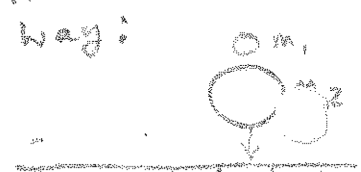

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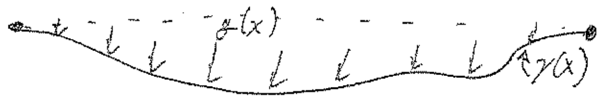
2000 - Q EXAM
06/18/00
APPLIED MATH

1) $\frac{d^2y}{dx^2} = g(x)$ ON $0 \le x \le 1$ $\quad y(0) = y(1) = 0$



a) WHAT CAN YOU SAY ABOUT THE EXISTANCE OF A SOLUTION?

b) NOW SUPPOSE THE BOUNDARY CONDITIONS DID NOT EXIST, WHEN DOES A SOLUTION EXIST?

c) HOW DOES THESE RESTRICTIONS ON $g(x)$ RELATE TO MECHANICS?

---

2) GIVEN SQUARE MATRIX $\underline{\underline{A}}$ AND $\underline{\underline{B}}$

BOTH HAVE DISTINCT (BUT NOT EQUAL) EIGENVALUES AND THEY SHARE THE SAME EIGENBASIS.

a) SHOW $\underline{\underline{A}}\,\underline{\underline{B}} = \underline{\underline{B}}\,\underline{\underline{A}}$

b) SHOW THAT $\underline{\underline{A}}\,\underline{\underline{B}} = \underline{\underline{B}}\,\underline{\underline{A}}$ IMPLIES STATEMENTS ABOVE.

---

3) HEAT DIFFUSION IN A RING

$u_{xx} = \alpha u_t \qquad T(\theta, \underset{t=0}{0}) = \sin(2\theta)$

AT WHAT TIME IS $\max|T(\theta,t)| - \min|T(\theta,t)| = 1/2$?

---

4) GIVEN $I = \int_0^1 \left[\left((y')^2 - 1 \right)^2 + y^2 \right] dx \qquad y(0) = y(1) = 0$

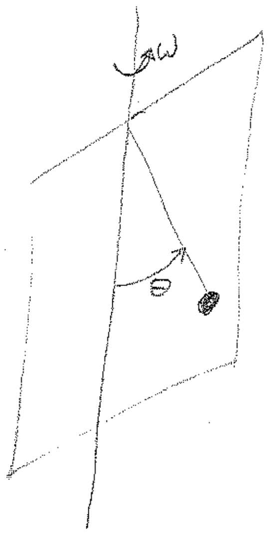
a) FIND EULER EQUATION FOR THIS FUNCTIONAL

b) DOES THIS MINIMIZE I? MAXIMIZE I?

c) CAN YOU FIND A $y(x)$ THAT GIVES AN I LESS THAN YOUR FIRST SOLUTION?

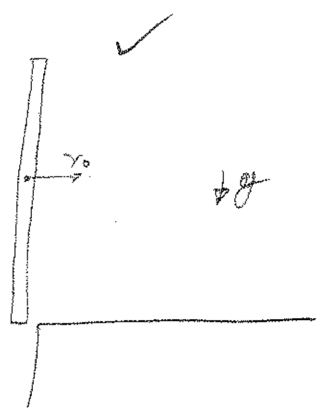
HINTS C1 CONTINUOUS...

2000 Q-EXAM
08/18/00
DYNAMICS – MORNING

① 

PENDULUM CONFINED TO A PLANE ROTATING AT ANGULAR VELOCITY ω.

FIND EQUATIONS OF MOTION USING LAGRANGE.

② 

A THIN ROD MOVING HORIZONTALLY WITH VELOCITY v_0 BARELY STRIKES A TABLE IN AN INELASTIC COLLISION – THEN ROTATES ABOUT THE IMPACT POINT UNDER INFLUENCE OF GRAVITY.

WHAT IS THE FINAL MOTION OF THE ROD?

③ ✓WRITE OUT EQUATIONS OF MOTION FOR A RIGID BODY ROTATING ABOUT ITS CENTER OF MASS.

2R200 - QEXAM11
08/18/00
CONTINUUM MECHANICS

① a) WHAT IS THE DIFFERENCE BETWEEN A SOLID AND A LIQUID?
b) WRITE DOWN A CONSTITUTIVE MODEL FOR A SOLID.
c) WRITE DOWN A CONSTITUTIVE MODEL FOR A LIQUID (STATIC).
d) WRITE DOWN A CONSTITUTIVE MODEL FOR A LIQUID (DYNAMIC).
e) WRITE DOWN THE MOST GENERAL CONSTITUTIVE MODEL YOU CAN THINK OF.

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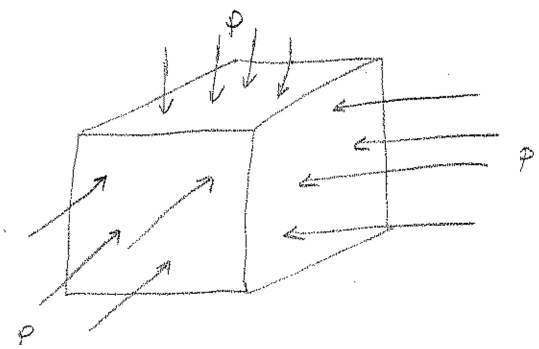
② 

SAME PRESSURE ON ALL SIX SIDES OF CUBE

3D ELASTICITY

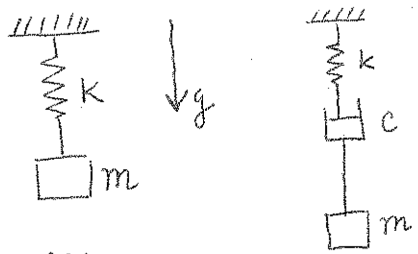
a) FIND THE DISPLACEMENT FIELD
b) IS THE ANSWER UNIQUE?
c) WHAT ASSUMPTIONS DID YOU MAKE IN ADDITION TO 3D ELASTICITY?
d) IS THE ANSWER THE SAME IF THE BODY HAD A DIFFERENT SHAPE?
e) WHAT IF THERE WAS A HOLE IN IT?
f) SUPPOSE THERE ARE TWO MATERIALS - ONE INSIDE THE OTHER - CAN YOU COME UP WITH A RESTRICTION ON E, ν SUCH THAT THE DISPLACEMENT FIELD REMAINS THE SAME?
g) HOW WOULD YOU GET RID OF THE NON-UNIQUE TERMS IN THE DISPLACEMENT FIELD TO SOLVE IT ON THE COMPUTER?

DYNAMICS

Q Summer 'C

Manish
Brian ~~Bryan~~
joshua

Q)



(a) (b)

(a) Find the equ^n of motion of the mass for 1) $kx = F$ and 2) $F = kx(1-x)$

compare the two situations will 2) be a periodic motion

(b) what effect does the damper have on the motion of m. derive the equation

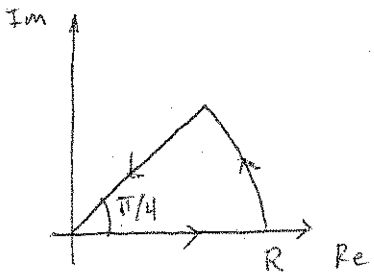
Q) A stone is dropped from the top of empire state bld. ($h = 1250$ ft) find the distance it falls away from the base (due to rotation of earth) $\boxed{\text{Ans } 4''}$

Q) derive the equ^n of motion for a rocket with initial mass m_o, emmition rate $= r$ (constant) and the discharge has relative velocity u wrt rocket.

Q Summer '06

1) Use e^{ix^2} and contour in fig. A to evaluate $\int_0^\infty \cos(x^2)\,dx$

$\int_0^\infty \sin(x^2)\,dx$



2) $$\frac{x^{100}}{(x-1)(x-2)\cdots(x-101)} = \frac{A_1}{x-1} + \frac{A_2}{x-2} + \cdots + \frac{A_{101}}{x-101}$$

Find A_n

3) $L(A) = A + A^T$ A is $n \times n$ real matrix

a) find eigenvalues of L

4) a) Find talor's expansion of $f(x)$ about $x = a$

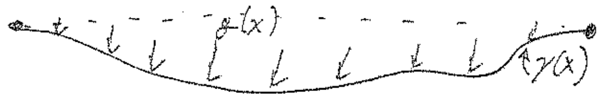
b) give t. expansion of two functions (say $\sin x$, e^x)

c) $f_{b^+}(x) - f_{b^-}(x) = h$ $b \in [a, c]$

find expansion of f about a.

2000 - Q EXAM
08/18/00
APPLIED MATH

1) $\dfrac{d^2y}{dx^2} = g(x)$ ON $0 \le x \le 1$ $\quad y(0) = y(1) = 0$



a) WHAT CAN YOU SAY ABOUT THE EXISTANCE OF A SOLUTION?

b) NOW SUPPOSE THE BOUNDARY CONDITIONS DID NOT EXIST, WHEN DOES A SOLUTION EXIST?

c) HOW DOES THESE RESTRICTIONS ON $g(x)$ RELATE TO MECHANICS?

---

2) GIVEN SQUARE MATRIX $\underline{\underline{A}}$ AND $\underline{\underline{B}}$

BOTH HAVE DISTINCT (BUT NOT EQUAL) EIGENVALUES AND THEY SHARE THE SAME EIGENBASIS.

a) SHOW $\underline{\underline{A}}\,\underline{\underline{B}} = \underline{\underline{B}}\,\underline{\underline{A}}$

b) SHOW THAT $\underline{\underline{A}}\,\underline{\underline{B}} = \underline{\underline{B}}\,\underline{\underline{A}}$ IMPLIES STATEMENTS ABOVE.

HEAT DIFFUSION IN A RING

$u_{xx} = \alpha u_t \qquad T(\theta, \underset{\uparrow\; t=0}{0}) = \sin(2\theta)$

AT WHAT TIME IS $\max|T(\theta,t)| - \min|T(\theta,t)| = \tfrac{1}{2}$?

---

4) GIVEN $I = \int_0^1 \left[\left((y')^2 - 1 \right)^2 + y^2 \right] dx \qquad y(0) = y(1) = 0$

a) FIND EULER EQUATION FOR THIS FUNCTIONAL

b) DOES THIS MINIMIZE I? MAXIMIZE I?

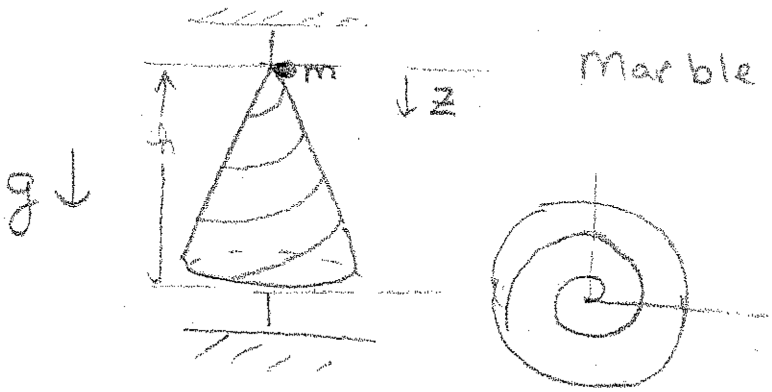
c) CAN YOU FIND A $y(x)$ THAT GIVES AN I LESS THAN YOUR FIRST SOLUTION?

HINT: C1 CONTINUOUS...

DYNAMICS

January, 2000

(1)



Marble mass m

$r = z = \theta$

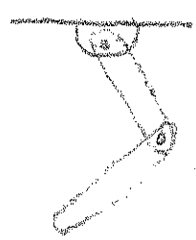
marble travels through a frictionless tube

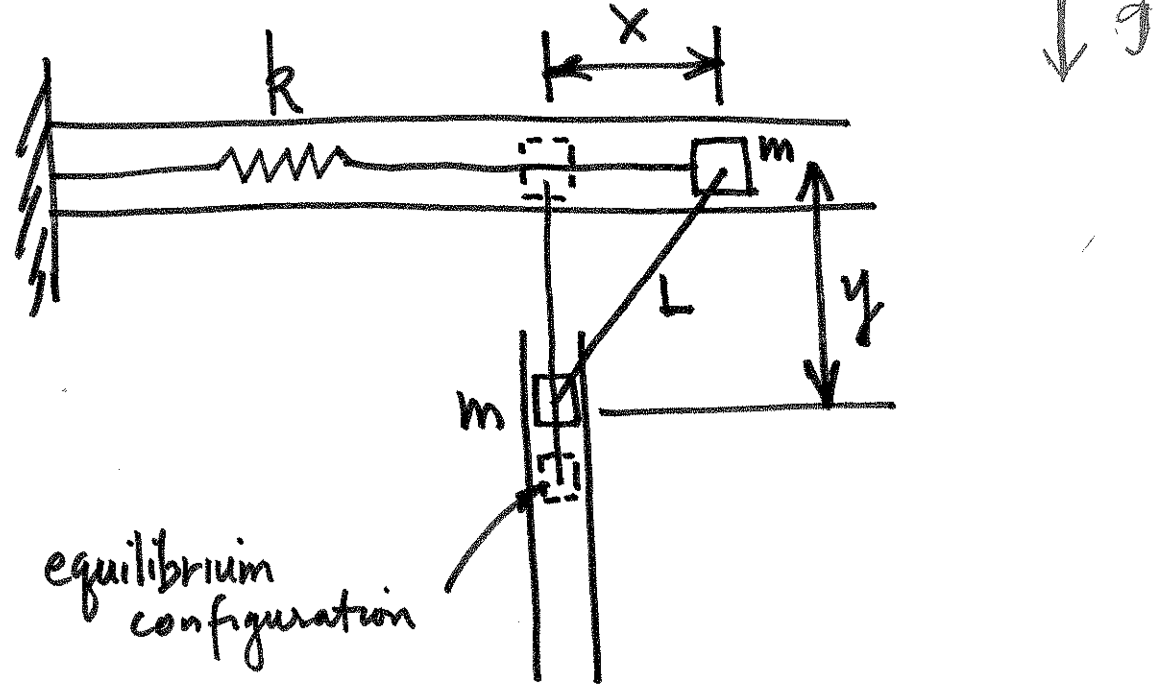
a) find velocity when marble reaches bottom of cone

b) the cone can spin do (a) again

(2) double pendulum

derive equations of motion





Derive the equation of motion.

<u>Assumptions</u>

L = rigid rod

neglect friction

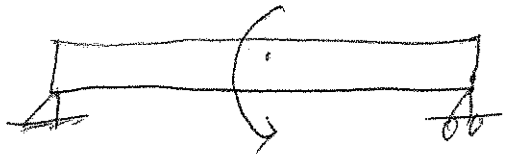
include gravity

Use any method you like.

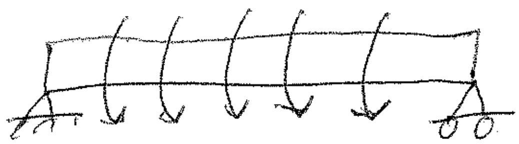
SOLID MECHANICS

1. a)

Shear & Moment Diagram



b)

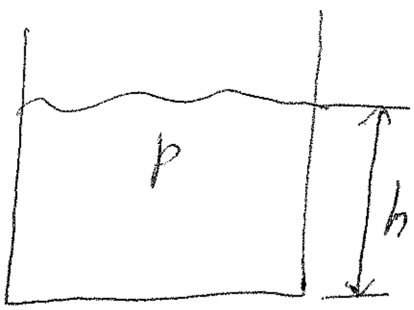


Distributed Moment

2. a) What is Stress (Traction etc.)

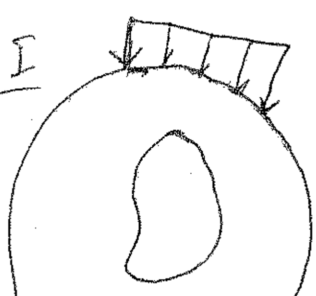
b) Compare stress state in a fluid vs. Solid

c)



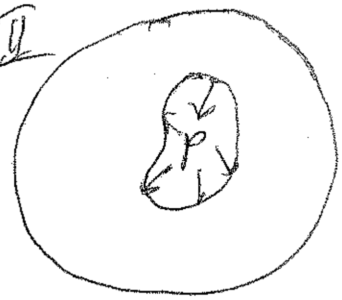
Find stress tensor ~~of~~ at every point.

3. Prob I

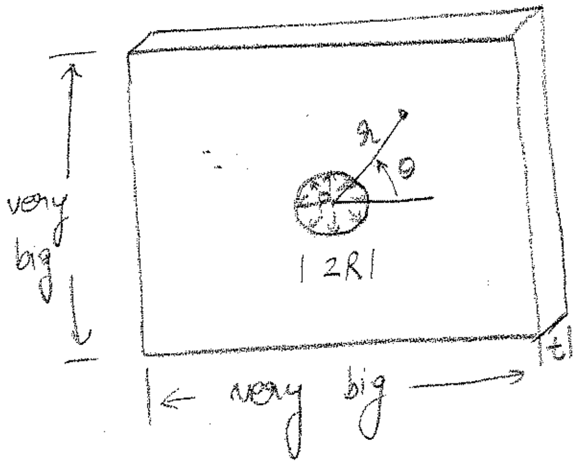


p constant

Prob. II



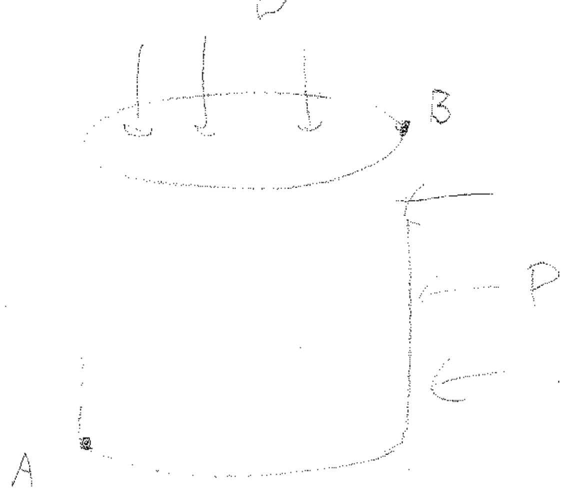
1. Number of solutions for Problem I
2. I Known, find soln. to II
3. What happens if matl. is inhomogeneous?



Pressure 'p' in hole with radius 'R' in a thin linear-isotropic elastic plate

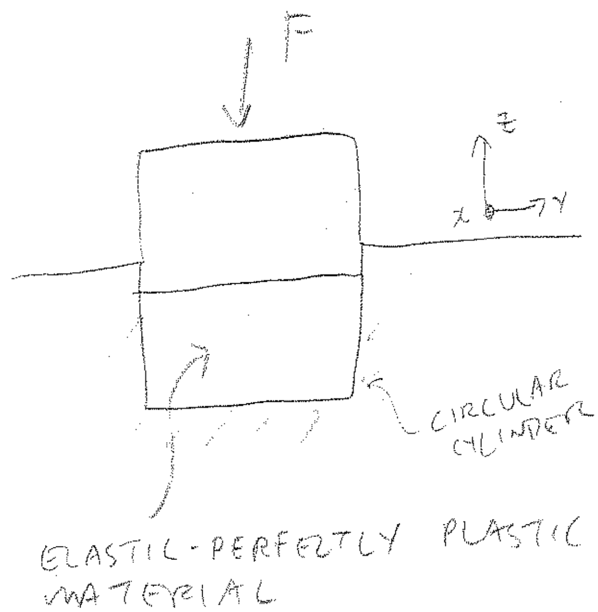
Q-EXAM - AUG. 20, 1999

SOLIDS (RUINA - MODERATOR, JENKINS, ZEHNDER, BAKER, CONWAY)

1) 

SOLID CYLINDER UNDER UNIFORM PRESSURE P.

FIND CHANGE (IN LENGTH) BETWEEN POINTS A & B.

2) 

a) MAKE A PLOT OF σ_{zz} VS ?

SOMETHING ABOUT TRESCA'S YIELD CRITERION

c) WE DIDN'T GET VERY FAR ON THIS

3) SHOW $\oint_{\Gamma} (W n_j - t_i u_{i,j}) \, d\Gamma = 0$

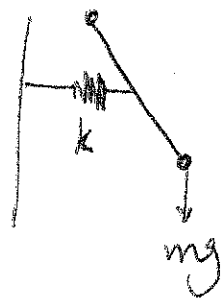
FOR ANY CLOSED CURVE Γ IN x, y PLANE

W = strain energy

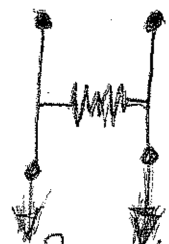
Jan 98.

<u>Dynamics:</u>

(1)



spring + pendulum.



2 pendula + spring

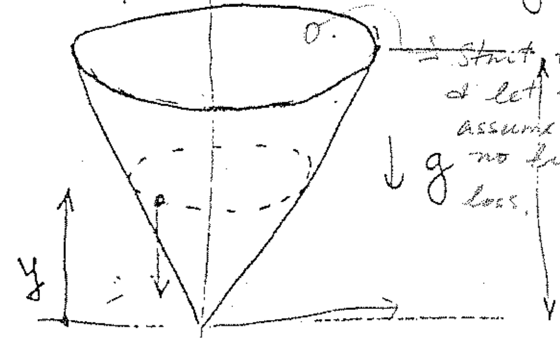
(2) $\dot{x} = x^2 - xy$?
$\dot{y} = xy + y^2$

Discuss – f.p. linearize, stability, phase plane diagram
integrable?

(3) pure elastic collision of 2 balls (no gravity)

use conservation of momentum etc...?

(9) A particle travels in a circular path on the inside of a cone. Find its speed v as a function of y.



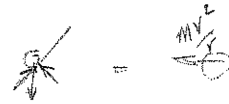
Start from here & let go

assume no frictional loss

Conservation of energy?

(1)

(Newton)



(10) A particle travels along a helical spring, until it reaches the end of the spring and falls off. What is its trajectory?

(1)

$$\tilde{r} = \varphi \cos t\,\hat{i} + \varphi \sin t\,\hat{j} + a t\,\hat{k}$$

$$\dot{\tilde{r}} = -\varphi \sin t\,\hat{i} + \varphi \cos t\,\hat{j} + a\,\hat{k}$$

$$\ddot{\tilde{r}} = -\varphi \cos t\,\hat{i} - \varphi \sin t\,\hat{j}$$

$$\Rightarrow \ddot{r} = -\varphi(\cos t\,\hat{i} + \sin t\,\hat{j})$$

$$\ddot{r} = -\varphi(\cos t_0\,\hat{i} + \sin t_0\,\hat{j}) - g\,\hat{k}$$

$$= \cancel{\varphi \cos t_0}\ \ddot{x}_0\,\hat{i} + \ddot{y}_0\,\hat{j} - g\,\tilde{\hat{k}} \quad \text{constant}$$

(1)

(11) a) State Newtons 3 laws

b) D'Alambert's Principle (Notes)

1st law = particle @ rest remain @ rest unless acted upon by external forces.

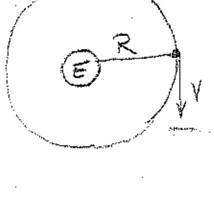
2nd law = change in particle when acted upon by external force is $F = \frac{dp}{dt}$

3rd law = equal & opposite forces.

D'Alamberth principle = reaction forces or force of constraints does no work under virtual displ.

(12) Find the altitude of a geosynchronous orbit.

(2)



geosynchronous

$$\boxed{m\frac{v^2}{R} = G\frac{mM_e}{R^2}}$$

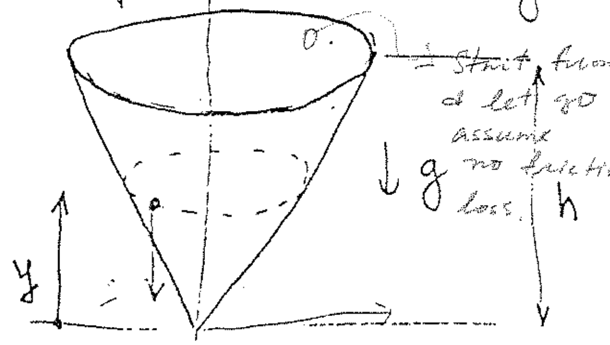
$$R\dot{\theta} = V$$

$$\dot{\theta}\Delta t = 360 \quad \text{1 sidereal da}$$

Note: Geosynchronous orbit is a circular orbit that completes one full revolution about the earth in one sidereal day.

(9) A particle travels in a circular path on the inside of a cone. Find its speed. v as a function of y.

1



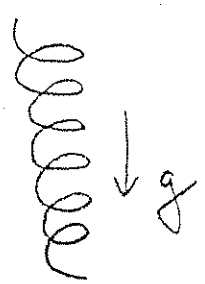
Conservation of energy?

(Newton)

$\frac{mv^2}{r}$

(10) A particle travels along a helical spring, until it reaches the end of the spring and falls off. What is its trajectory?

1



$$\tilde{r} = \varphi\cos t\,\hat{i} + \varphi\sin t\,\hat{j} + at\,\hat{k}$$

$$\dot{\tilde{r}} = -\varphi\sin t\,\hat{i} + \varphi\cos t\,\hat{j} + a\,\hat{k}$$

$$\ddot{\tilde{r}} = -\varphi\cos t\,\hat{i} - \varphi\sin t\,\hat{j}$$

$$\Rightarrow \ddot{\tilde{r}} = -\varphi(\cos t\,\hat{i} + \sin t\,\hat{j})$$

$$\ddot{\tilde{r}} = -\varphi(\cos t_0\hat{i} + \sin t_0\hat{j}) - g\hat{k}$$

$$= \cancel{-\varphi\cos t_0}\ \ddot{x}_0\hat{i} + \ddot{y}_0\hat{j} - g\tilde{\hat{k}} \quad \text{constant}$$

1 (11) a) State Newtons 3 laws

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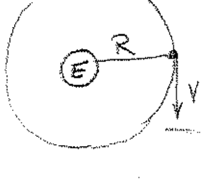
(12) Find the altitude of a geosynchronous orbit.

2



geosynchronous

$$\boxed{m\frac{v^2}{R} = G\frac{mM_e}{R^2}}$$

$R\dot{\theta} = V$

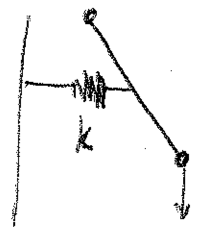
$\dot{\theta}\Delta t = 360$ 1 sidereal day

Note: geosynchronous orbit is a circular orbit that completes one full revolution about the earth in one sidereal day.

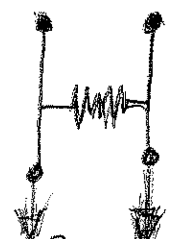
Jan 98.

<u>Dynamics:</u>

(1)



spring + pendulum.



2 pendula + spring

(2) $$\dot{x} = x^2 - xy \qquad ?$$
$$\dot{y} = xy + y^2$$

Discuss – !!. linearize, stability, phase plane diagram
integrable?

(3) pure elastic collision of 2 balls (no gravity)

Use conservation of momentum etc...?

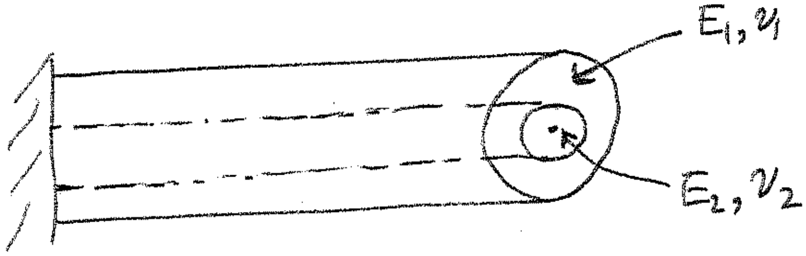
19th Jan' 96.

1) (Sachse) What is modulus of Elasticity? What is stress? What is Strain?

What about a material like wood?

Can you say something about the constitutive equations of a transversely Isotropic material?

2) (Phoenix)



Teflon material between the two cylinders.

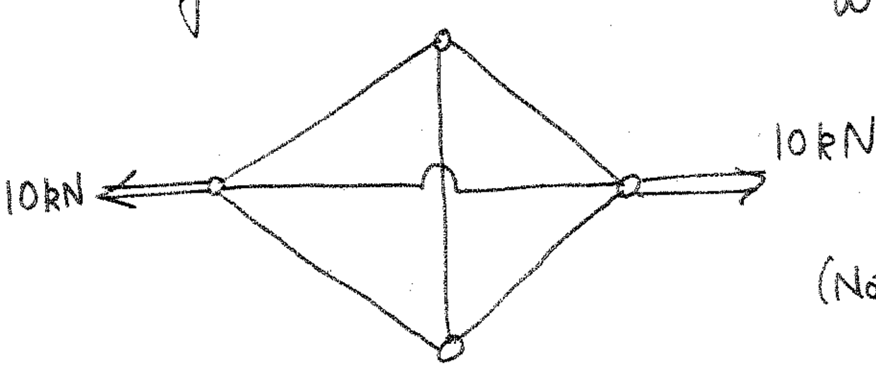
Discuss the Torsion Problem.

Torque applied at the end.

Also, talked about the extension problem i.e Axial loading.

1) $\nu_2 < \nu_1$ 2) $\nu_2 > \nu_1$

3) (Conway)



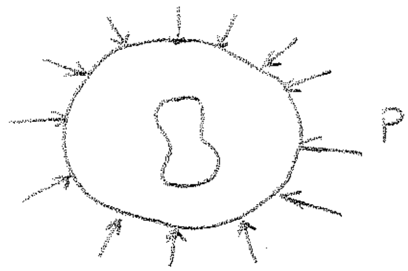
What are the forces in each of the members?

(Note: The structure is statically indeterminate).

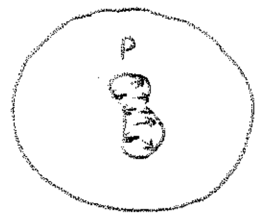
Compiled by
Mandar K. Chati

Define Stress. Describe how balance of momentum works for the case of a two-Dimensional medium (either fluid or solid). Tal about other balance laws, and tensor nature of stress.

2.)



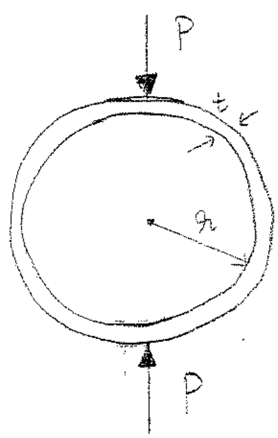
Sphere A



Sphere B

Spheres A and B are identical 3D bodies both having an arbitrary shaped cavity. Sphere A is subjected to a hydrostatic pressure of magnitude 'p', & it is assumed that the resulting stress distribution is known. The problem is in the context of linear elasticity (small strains, homogeneous isotropic material etc.). find the stress distribution for sphere B (hydrostatic pressure loading of magnitude 'p' inside the cavity). Using the result of Sphere A. (Hint: Use Superposition).

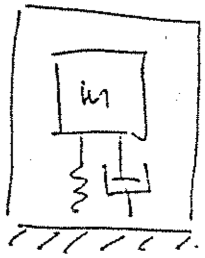
3.)



Set up the free body diagram for a ring loaded by identical forces P. Use body symmetries to eliminate certain terms.

What are the required boundary conditions ?

(53) Puls hammer ? . Discuss !
How to measure the force ?
Equation of the mechanism inside the hammer ?

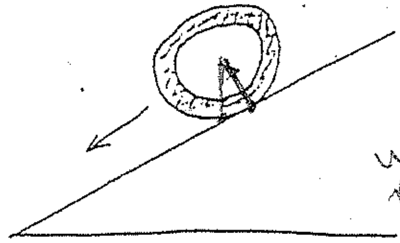


2

(54) What is constraint ? limitation placed on the degrees of freedom a system can have. usually, it has the form of an equation relating the different degrees of freedom. Hence reducing the # of independent degrees of freedom.

1

(55)



use momentum principle is better.

1

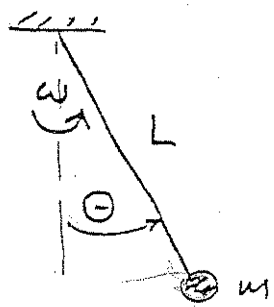
a) Solve the problem :
   (i) with slippage
   (ii) without slippage

b) What are restrictions on $\underset{\sim}{T} = \frac{d\underset{\sim}{L}}{dt}$

fixed pt in space or CM.

c) Where is the above equation derived from? $\longrightarrow$ $\vec{F} = m\vec{a}$

(50) Spherical pendulum ($\dot{\theta} = 0$)
Given an I.C. on v, find the period.
If the mass is given a slight disturbance ($\dot{\theta}$ no longer $= 0$), find the period!

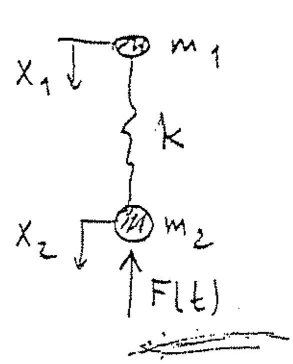


$T = \sqrt{\frac{g}{R}}\,2\pi$

1

EOM: $\ddot{\theta} + \frac{g}{R}\sin\theta = 0$

(51)



Set up eq.'s of motion.
Given $(\ddot{x}_1 - \ddot{x}_2)$ how would you find $\underline{F(t)}$?

1

$m_1\ddot{x}_1 - k(x_2 - x_1) = 0$
$m_2\ddot{x}_2 + k(x_2 - x_1) = F(t)$

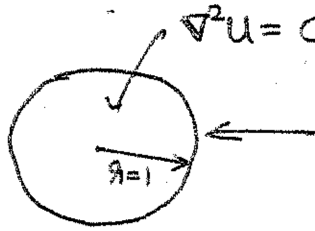
(52) What is the difference between LE' and NE's?

1

Applied Mathematics.

19th Jan '96

1) (Andy Ruina) Laplace's Equation on a unit disk.



$\left.\dfrac{\partial u}{\partial n}\right|_{r=1} = \cos\theta$

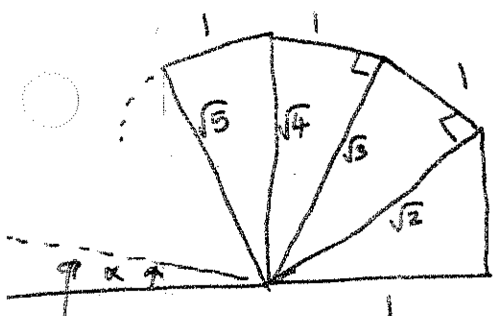
What is $u(r,\theta)$ or $u(x,y) = ?$

2) (Hui) Solve the following equation:

$$\frac{dy}{dx} = \int_0^x y(t)\,dt \qquad y(0) = 1$$

What is $y(x) = ?$

3) (Strogatz)



Does this Converge?

compiled by
Mandar K. Chati

1> What is Euler's constant, γ?

Show $\lim_{n\to\infty}\left[1+\frac{1}{2}+\frac{1}{3}+ \cdots\cdots +\frac{1}{n}-\log(n)\right]$ converges. give a bound on this limit.

2> How do you solve the Cauchy-Euler equation.

$$x^2y''+xy'-y=0?$$

How do you solve the in-homogeneous case:

$$x^2y''+xy'-y=f(x)?$$

3> What is $\operatorname{curl}(f\underline{u})$? $\quad f: \mathbb{R}^3 \to \mathbb{R}$

4> State Residue Theorem.

5> Define the derivative of a complex function.

6> Give a brief lecture on Fourier Series.

7> What can you say about the following:

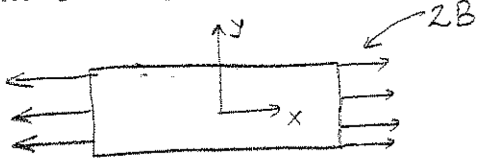
$$L(u)+\alpha u=0 \qquad u(0)=u_0$$
$$u(a)=u_1$$

'L' is a linear operator.

8> Evaluate: $\displaystyle\int_C \frac{dz}{z^{2n}+1}$ where C is: $|z|=3$

13> Derivation of the governing equations:

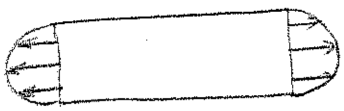
a) Solutions to:



$$\phi = By^2$$

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} = 2B$$

b) Solution to:



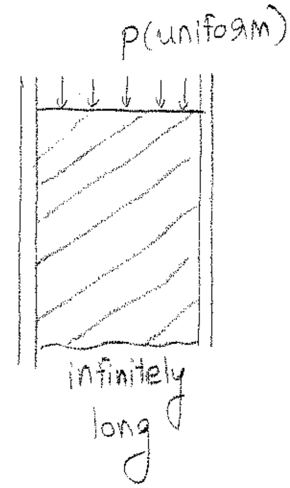
$$\phi = S\left(\frac{y^2}{2} - \frac{y^4}{12b^2}\right) + \cdots\cdots$$

$$\sigma_x = S\left(1 - \frac{y^2}{b^2}\right)$$

need additional terms. So, that $\nabla^2\nabla^2\phi = 0$ is satisfied.

14> Given a tube of paper which is then subjected to torsion. At what angle will the paper buckle & why?

15> Find the stress?



a) No friction on the walls?

b) Now with friction. State the Shearing stress distribution on the walls.

16> Why does buckling not come under the classical theory of Elasticity? — (Ans.: by the assumption of smallness).

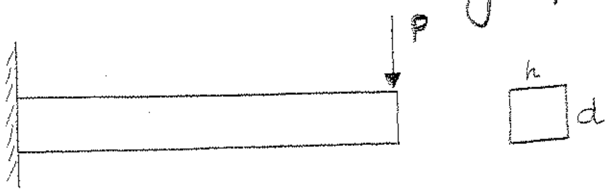
17> What are compatibility equations? What do they mean?

18> Given the displacement field $w = f(x,y)$? $u = v = 0$.

What are the governing equations.
Which of σ ; ε are zero!

How do you measure E & G experimentally?

19.> Consider the following problem :



a> Draw the shape of the transverse cross-section of the beam after it has been loaded.

b> What assumptions are being made? In which region of the beam is the kinematic constraint approximation of beam theory good. Are there regions where the cross-section warps?

c> What is the relation between the anti-elastic radius of curvature & the bending radius of curvature? Is there a value of ν (Poisson's ratio that immediately tells you what the relation should be!)

20.> A piece of chalk (a brittle material) fails when subjected to loading:

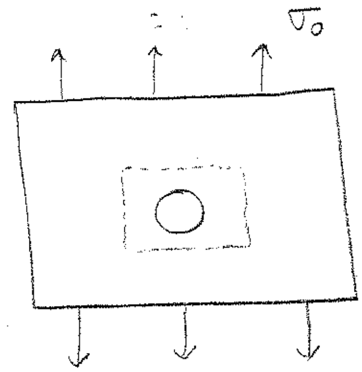
a>

b>

Failure criterion : Max. normal stress.

Explain different failure modes.

Consider a small hole in a large plate subjected to tensile loads.



a) How would you formulate the problem as a problem in linear elasticity?

b) Wants to know if one uses stress function to solve it.

c) What are the B.C's?

d) Is the solution for the displacement unique?

e) How would you use this solution to check the accuracy of a code that solves the problem of a large hole in a plate (dotted rectangle)?

f) Is this solution good if the radius of the hole is of the order of the thickness of the plate?

2) The well known formula for beam stress is:

$$\sigma = -\frac{My}{I} \quad *$$

a) Very briefly describe the meaning of this equation.

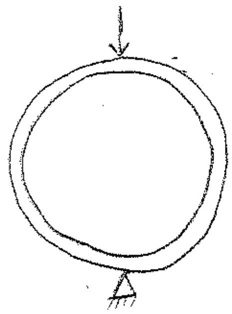
b) What are the conditions under which * is accurate?

What materials?

What shapes?

What loads?

23> Draw FBD for left half of the ring. What is the stress distribution on the surface of cut.



24> Define ν.

How could you measure ν. More than one way if you can think of them?

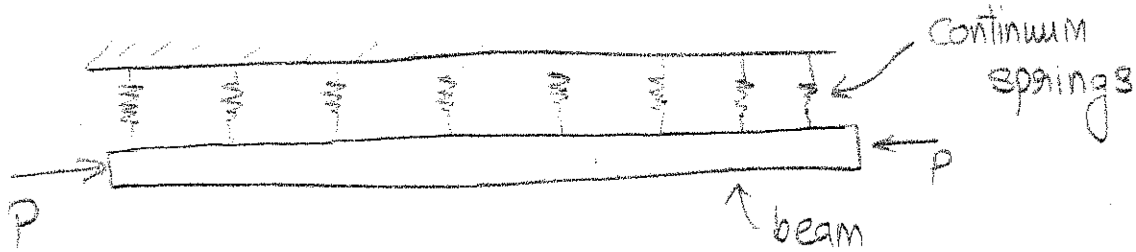
What are the biggest & smallest values of ν! and why

What are the biggest & smallest values of ν for materials you know about?

When $\nu = 0.5$ a material is "incompressible". How would would you describe a material with $\nu = -1$?

25> Beam on an elastic foundation:

$P_{buckle} = ?$



Erich + Deepak's Q exam! 1996. August.

Q Applied Math! Cauchy Euler eqn

1) $x^2 y'' + xy' + \lambda y = 0$

$X'(0) = 0$
$X'(0) = 0$

(sub $y = x^k$
- discuss solution for various values of λ - unique?)

2) $\underline{A}$ linear symmetric pd. $A: \mathbb{R}^n \to \mathbb{R}^n$.

Use Gibbs decomposition of $\underline{A}$

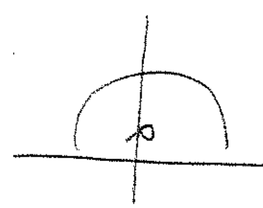
$\langle v, w \rangle$ = inner product

min $\langle Av, v \rangle$ subject to $\langle v, v \rangle = 1$

?3) $\displaystyle\int_{-\infty}^{\infty} \frac{a^{iz}}{z^2+1}\, dt$ $\quad a \in \underline{\mathbb{R}}$ $\quad 1 > a > 0.$ $\quad$ Use Jordan's lemma.

$= \displaystyle\int_{-\infty}^{\infty} \frac{e^{i \ln a\, z}}{z^2+1}$ $\qquad \int_{C_R} \to 0$ as $\lambda \to \infty$

(hint! consider different contours?



$a > 1$)

C. Applied Math

1. i^i

2. Prove if $\lim_{z\to 0} f(z)=0$ & $|G(z)|\le M$ in some neighborhood of $z=0$

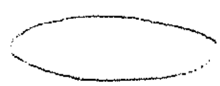
   then $\lim_{z\to 0} f(z)\,G(z)=0$

3. $\underline{\underline{A}}_{n\times n}$ has eigenvalues $\lambda_1,\cdots,\lambda_n$

   3.1 What are the eigenvalues of $\underline{\underline{A}}^{-1}$?

   3.2 Prove it.

4. 4.1 Write down wave equation (1-Dimensional)

   4.2. Given initial conditions, If the system is a circular string  . What are the boundary conditions

   4.3 solve it

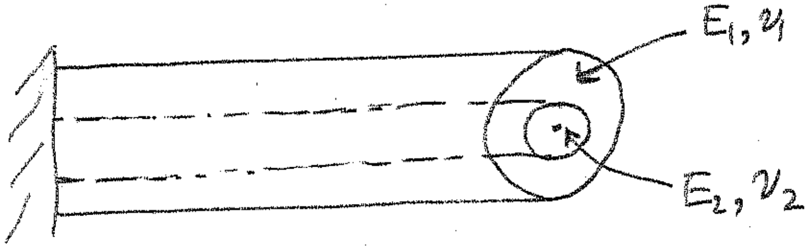
   4.4 Any other way to solve the equation?

   4.5 How can a traveling wave represent a standing wave?

19th Jan '96.

1.) (Sachse) What is modulus of Elasticity?
What is stress? What is Strain?
What about a material like wood?
Can you say something about the constitutive equations of a transversely Isotropic material?

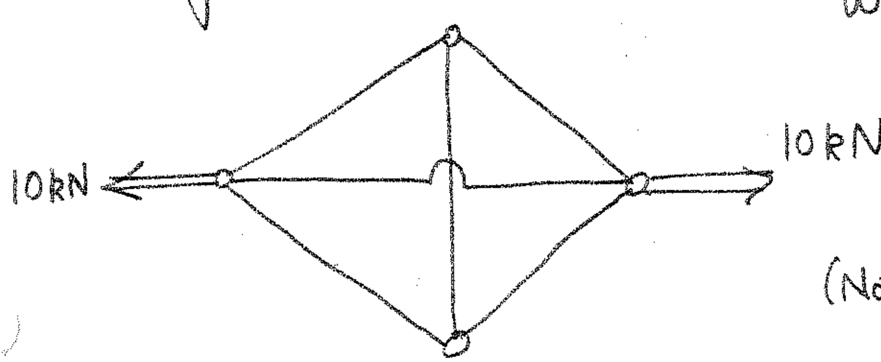
2.) (Phoenix)



Teflon material between the two cylinders.

Discuss the Torsion Problem.
Torque applied at the end.
Also, talked about the extension problem i.e Axial loading.
1.) $\nu_2 < \nu_1$ 2.) $\nu_2 > \nu_1$

3.) (Conway)

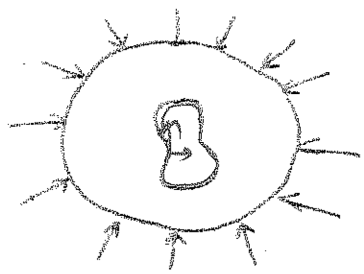


What are the forces in each of the members?
(Note: The structure is statically indeterminate).

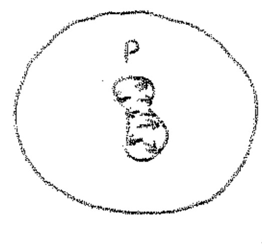
Compiled by
Mandar K. Chati

Define Stress. Describe how balance of momentum works for the case of a two-Dimensional medium (either fluid or solid). Tal about other balance laws, and tensor nature of stress.

~~2)~~



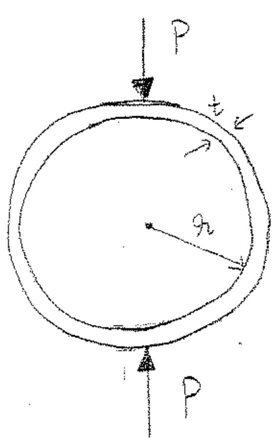
Sphere A



Sphere B

Spheres A and B are identical 3D bodies both having an arbitrary shaped cavity. Sphere A is subjected to a hydrostatic pressure of magnitude 'p', & it is assumed that the resulting stress distribution is known. The problem is in the context of linear elasticity (small strains, homogeneous isotropic material etc.). find the stress distribution for sphere B (hydrostatic pressure loading of magnitude 'p' inside the cavity). Using the result of Sphere A. (Hint: Use Superposition).

3)



Set up the free body diagram for a ring loaded by identical forces P. Use body symmetries to eliminate certain terms.

What are the required boundary conditions ?