Generalized forces Q_i associated with q_i

Solution: $-\frac{\partial \mathcal{L}}{\partial q} + \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = Q_i$

Not all forces are conservative. ex: applied, viscous, constraint

How to find Q_i?

Key idea: $dW = dW$ increments in work

$$\sum_i \vec{F}^i_{applied} \cdot d\vec{r}_i = \sum Q_i \, dq_i$$

$d\vec{r}_i$ associated with the dq_i

Simplify: $\sum_i \vec{F}^i \cdot d\vec{r}_i = Q_7 \, dq_7$

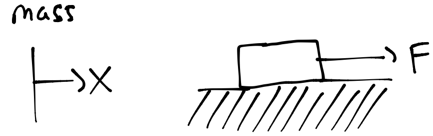
↳ motion associated with small change in only q_7

$$Q_i \, dq_i = \sum_j \vec{F}_j \frac{\partial \overset{x's,\,y's}{X_j}}{\partial q_i} dq_i$$

all X_i where forces applied

$$\frac{dX_j}{dq_i} = J \text{ (the jacobian)}$$

<u>Example:</u> 1D mass



$q = x$

$$\mathcal{L} = E_K - E_P = T - V = \frac{1}{2} m \dot{q}^2$$

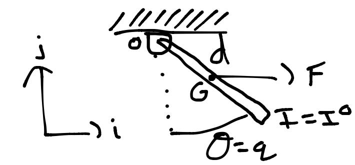
L.E. $-\frac{\partial \mathcal{L}}{\partial q} + \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = Q \longrightarrow m\ddot{q} = Q$

$dw = dw$

$Q \, dq = F \, dx$

$Q = F \longrightarrow \boxed{m\ddot{x} = F}$

Example: pendulum, no gravity



$$\mathcal{L} = \frac{1}{2} I \dot{\theta}^2$$

L.E. $-\dfrac{\partial \mathcal{L}}{\partial \theta} + \dfrac{d}{dt}\left(\dfrac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = Q_\theta$

$I\ddot{\theta} = Q_\theta$ $\qquad$ $Q_\theta = ?$ $\qquad$ $Q_\theta \, d\theta = \vec{F} \cdot d\vec{r}_G$

$$= \vec{F} \cdot \underbrace{\frac{\partial \vec{r}_G}{\partial \theta}}_{J} d\theta$$

$$Q_\theta \, d\theta = \vec{F} \cdot \frac{d}{d\theta}\left(-d\cos\theta\,\hat{j} - d\sin\theta\,\hat{i}\right) d\theta = \hat{k}\, d\theta \times \vec{r}_{G/O}$$

$$= \vec{F} \cdot (\hat{k}\, d\theta) \times \vec{r}_{G/O} = \underbrace{\left(\vec{r}_{G/O} \times \vec{F}\right)}_{\vec{M}_{/O}} \cdot \hat{k}\, d\theta$$

$\longrightarrow \; I\ddot{\theta} = \vec{M}_{/O}$

$\quad \hookrightarrow \; M_O = -mgd\sin\theta$ if the force is a gravity force

$I = I^G$ $\qquad$ $q = [x \;\; y \;\; \theta] = [q_1 \;\; q_2 \;\; q_3]$

$$\mathcal{L} = \frac{1}{2} m \dot{q}_1^2 + \frac{1}{2} m \dot{q}_2^2 + \frac{1}{2} I \dot{q}_3^2$$

x: $-\dfrac{\partial \mathcal{L}}{\partial x} + \dfrac{d}{dt}\dfrac{\partial \mathcal{L}}{\partial \dot{x}} = Q_x$ $\qquad \longrightarrow \; m\ddot{x} = Q_x$

$$m\ddot{y} = Q_y$$

$$I\ddot{\theta} = Q_\theta$$

$Q_x\,dx = \vec{F}\cdot d\vec{r}_G$ when only $dx \neq 0$

$= F_x\,dx$

$Q_x = F_x$, $Q_y = F_y$

$Q_\theta\,d\theta = \vec{F}\cdot \underline{d\vec{r}_c}$ when only $d\theta \neq 0$

$= \vec{F}\cdot \dfrac{\partial \vec{r}_c}{\partial \theta}\,d\theta$

$Q_\theta = \vec{F}\cdot \dfrac{\partial \vec{r}_c}{\partial \theta}$ 3rd column of $J_{2\times3}$ dotted with $\vec{F}$ $\rightarrow Q_\theta = (\vec{r}_{G/O} \times \vec{F})\cdot \hat{K}$

Can supplement equations with Kinematic constraint equations:

$\ddot{\vec{r}}_c = \vec{0}$ solve constraint equations

Set of DAE's: