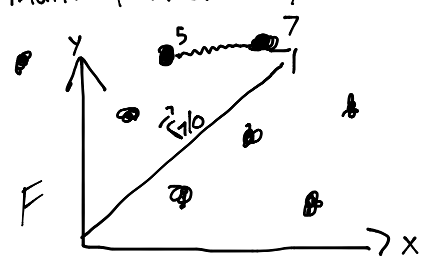
Multi - particle system



$$\vec{F}_F\,tot = m_7 \vec{a}_7$$

$$\vec{L} = \vec{F}_7\left(t_1, \vec{r}_1, \vec{r}_2 \dots \vec{r}_7, \vec{v}_1, \vec{v}_2 \dots \vec{v}_7\right)$$

$$\vec{F}_7 = -m_7 g \hat{j} + K_{75}\left(|\vec{r}_5 - \vec{r}_7| - l_0\right) \frac{\vec{r}_5 - \vec{r}_7}{|\vec{r}_5 - \vec{r}_7|}$$

n particles $\rightarrow$ 4n ode's $\rightarrow$ 1st order

given initial state $\rightarrow$ predict the future:

(Laplace, quantum, Chaos)

<u>Linear Momentum</u> Not microscopically reasonable

one particle $\vec{F}_i = m_i \vec{a}_i$

$\uparrow \vec{F}_i^{tot}$ $\quad \hookrightarrow \vec{a}_{i/e}$

add up the particles: $\sum_i \vec{F}_i = \sum m_i \vec{a}_i$

$$\frac{d}{dt} m_i = 0$$

$$\sum\left(\vec{F}_i^{int} + \vec{F}_i^{ext}\right) = \frac{d}{dt} \sum m_i \vec{v}_i$$

$$\underbrace{\boxed{\sum F_i^{int}}}_{=0} + \sum F_i^{ext} = \frac{d}{dt} \vec{L}$$

for one of 3 reasons: 1. Classical reason: assume central pair-wise forces

a.) $\vec{F}_i^{int} = \sum \vec{F}_{ij}^{int}$

b.) $\vec{F}_{ij} = -\vec{F}_{ji}$

c.) $\vec{F}_{ij} = \dfrac{F_{ij}(\vec{r}_j - \vec{r}_i)}{l_{ji}}$

2. $\vec{F}^{int} = 0$ by assumption

1 implies 2,3

2,3 do not imply 1

3. $\sum \vec{F}^{ext} = \dot{\vec{L}}$ by assumption

Look at $\vec{L} = \sum m_i \vec{v}_i = \sum m_i \dot{\vec{r}}_i = \frac{d}{dt} \sum m_i \vec{r}_i = \frac{d}{dt} m_{tot} \vec{r}_G$

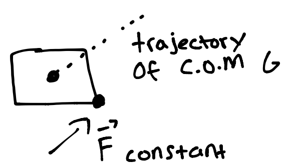
$\vec{r}_G = \dfrac{\sum m_i \vec{r}_i}{m_{tot}}$ $\qquad m_{tot} \vec{r}_{G/O} = \sum m_i \vec{r}_{i/G}$

LMB: $\vec{F}^{ext} = m \ddot{\vec{r}}_G$

$\vec{F} = m\vec{a}$ is correct and precise for any system

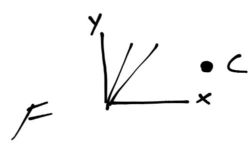
$= m \vec{a}_{G/F}$

Example:



center of mass will go in a straight line

Angular Momentum



one particle $\vec{F}_i = m_i \vec{a}_i$

$\sum \vec{r}_{i/c} \times \vec{F}_i \rightarrow \sum \vec{r}_{i/c} \times (\vec{F}_i^{int} + \vec{F}_i^{ext}) = \sum \vec{r}_{i/c} \times m_i \vec{a}_{i/F}$

Assume $\sum \vec{r}_{i/c} \times \vec{F}_i^{int} = 0$

internal forces have no net torques

$$\sum \vec{r}_{i/c} \times \vec{F}^{ext} = \sum \vec{r}_{i/c} \times (m_i \vec{a}_{i/F})$$