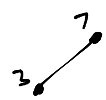
DAE's, Rolling, Questions

Recall $\vec{F} = m\vec{a}$ — easy for 1 particle and collection with Known interaction forces

"Known": $\vec{F} = \vec{F}$(positions and velocities) → not accelerations

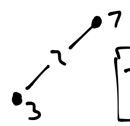
Ex: $G, g, K, -c\vec{v}, -cv\vec{v}$
↳ $\ell = 0$, $\ell \neq 0$

Difficulty in Dynamics: constraints
↳ ex: $|\vec{r}_3 - \vec{r}_7| = \text{constant}$



inextensible bar

FBD:



$\boxed{T_{37} = ?}$

How to deal with this?

Naive solution (DAE)
Advance Solution (Finesse)

Naive Approach: For each particle $\vec{F} = m\vec{a}$ (includes constraint forces)

constraints: differentiate twice, gives restrictions on $\ddot{x}_3, \ddot{y}_3$, etc.

solve set simultaneously at every instant in time

Advanced: (A) Treat collection of rigidly connected particles as a rigid object

$\vec{M}_{/G} = I^G \alpha \hat{k}$ and LMB ✱ internal forces have no net torque

(B) Use judicious dot products and cross products to eliminate constraint forces

What happens if we have a collection of rigid objects?

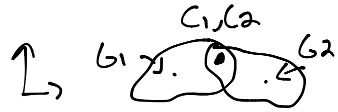
If forces are Known: $\vec{M}_{/G} = I^G \alpha$ In 2-D

$\sum F_x = m\ddot{x}$ 3 equations for each rigid object

$\sum F_y = m\ddot{y}$

I) If forces are "Known": $F_i = F_i$(position, angles, velocities, rates)

II) What about constraints?



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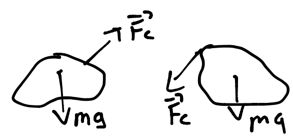


<u>LMB + AMB</u>: 6 scalar equations

unknowns: $\ddot{x}_1, \ddot{y}_1, \ddot{x}_2, \ddot{y}_2, \ddot{\theta}_1, \ddot{\theta}_2, F_{cx}, F_{cy}$ $\longrightarrow$ 8 unknowns

We need 2 more equations: tell the equations the masses are connected

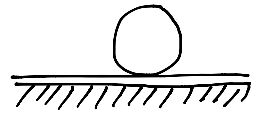
$\vec{a}_{c_1} = \vec{a}_{c_2} \longrightarrow$ 2 equations

$\vec{a}_{c_1} = \vec{a}_{G_1} + \vec{a}_{c_1/G_1} = \ddot{x}_{G_1}\hat{\imath} + \ddot{y}_{G_1}\hat{\jmath} - \dot{\theta}_1^2 \vec{r}_{c_1/G_1} + \ddot{\theta}K \times \vec{r}_{c_1/G_1}$

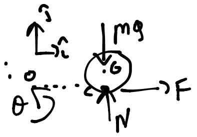
Can now handle any number of rigid objects connected by hinges

<u>Rolling</u>

Disk on a table



FBD:



LMB $\cdot\, \hat{\jmath}$: $N = mg$

AMB/O dt $\rightarrow$ $\vec{H}_{/O}(t) = \vec{H}_{/O}(0) = 0\hat{K}$

$= -mr\dot{x}_G \hat{k} + I^G \dot{\theta} \hat{k} = 0\hat{k}$

at end of experiment: $-mr\dot{x}_G + I^G \dot{\theta} = 0$

rolling constraint applies at end : $\vec{V}_c = \vec{0}$

$$\dot{x}_G + r\dot{\theta} = 0$$

2 equations : linear, homogeneous, independent in $\dot{x}, \dot{\theta} \rightarrow \dot{x} = 0, \dot{\theta} = 0$