Slight Recap

Sleigh

Relating Coordinates

Frame B accelerates but does not rotate

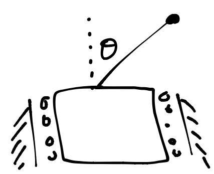
$$\sum \vec{M}_{/c} = \sum \vec{F}_{i/c} \times m_i \vec{a}_i$$

altered gravity field

$$\sum \vec{M}^{other}_{/c} + \sum \vec{r}_{i/c} \times m_i \vec{g} = \sum \vec{r}_{i/c} \times m\left[\vec{g} - \vec{a}_{B/F}\right]$$

$= \sum \vec{r}_{i/c} \times m\vec{a}_{i/B}$ (if we use acceleration relative to moving frame)

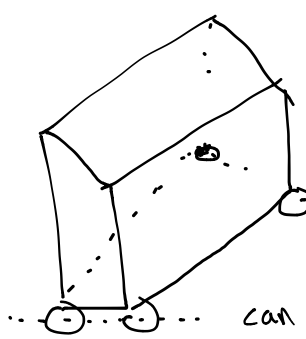
<u>Example:</u>



"altered gravity" gravity field

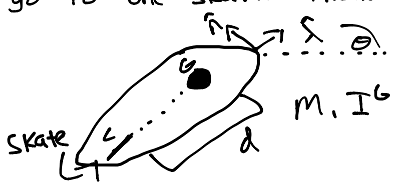
$$\ddot{\theta} + \frac{g+a}{l}\sin\theta = 0$$

Andy's Fav: <u>Chaplygin Sleigh</u>



can only move tangently

two weels go to one skate, frictionless pads



FBD:

minimal coordinates: θ, V_c

$$\vec{V}_G = \vec{V}_c + \vec{V}_{G/c} \qquad \vec{a}_G = \vec{a}_c + \vec{a}_{G/c}$$

$$\dot{\vec{r}}_c = \dot{x}_c \hat{i} + \dot{y}_c \hat{j} = V_c \hat{\lambda} = V_c(\cos\theta \hat{i} + \sin\theta \hat{j})$$

$$\vec{a}_c = \vec{a}_c + \vec{a}_{G/c} \qquad \vec{a}_c = \frac{d}{dt}\vec{V}_c = \frac{d}{dt} V_c \hat{\lambda} = \dot{V}_c \hat{\lambda} + V_c \dot{\hat{\lambda}}$$

$$\dot{\hat{\lambda}} = \vec{\omega} \times \hat{\lambda} = \dot{\theta} \hat{k} \times \hat{\lambda} = \dot{\theta} \hat{n}$$

$$\vec{\omega} = \dot{\theta} \hat{k}$$

Aside: $|\vec{A}| = \text{constant}$

$$\dot{\vec{A}} = \dot{\theta}\hat{k} + \vec{A} \qquad \vec{a}_{G/c} = -d\omega^2 \hat{\lambda} + d\dot{\omega}\hat{n}$$

We don't want N: LMB $\cdot\, \hat{\lambda}$ and AMB/c

LMB $\cdot\, \hat{\lambda}$: $\{\sum \vec{F} = m\vec{a}\} \cdot \hat{\lambda}$ $\qquad$ $\vec{F} = 0$, cross off m

$$0 = [(\dot{V}_c \hat{\lambda} + V_c \dot{\theta} \hat{n}) + (-d\omega^2 \hat{\lambda} + d\dot{\omega}\hat{n})] \cdot \lambda$$

$$\boxed{\dot{V}_c = d\omega^2} \; (1)$$

$$AMB_{/c} = \sum \vec{M}_{/c} = \dot{\vec{H}}_{/c}$$

$$0 = \vec{r}_{G/c} \times m\vec{a}_c + I^G \dot{\omega} \hat{k}$$

$$= (d\hat{\lambda}) \times m(\dot{V}_c \hat{\lambda} + V_c \dot{\theta} \hat{n} - d\omega^2 \lambda + d\dot{\omega}\hat{n}) + I^G \dot{\omega} \hat{k}$$

$\hat{\lambda} \times \hat{\lambda} = 0 \qquad \hat{\lambda} \times \hat{n} = \hat{k}$

$$= d\, m V_c \dot{\theta}\, \hat{k} + d^2 m \dot{\omega}\, \hat{k} + I^G \dot{\omega}\, \hat{k}$$

Dot both sides with $\hat{k}$: $\boxed{\dot{\omega} = - \dfrac{d m V_c \omega}{d^2 m + I^G}}$ ②

$\dot{x}_c = V_c \cos\theta$ ③ $\qquad \dot{y}_c = V_c \sin\theta$ ④

Equations ①-④ : ODE's for motion

ω diminishes in time if $V_c > 0$

Unstable motion if $V_c < 0$