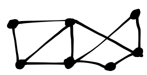
2-d rigid objects $\vec{\omega}, \vec{L}, \vec{H}, + E_K$

Consider lots of point masses with enough massless bars so the object is rigid

$\#\text{bars} \geq (\#\text{masses}) * 2 - 3$ → trusses in Ruina/Pratap

If too many bars: incompatibility or indeterminary

→ still can find motion!

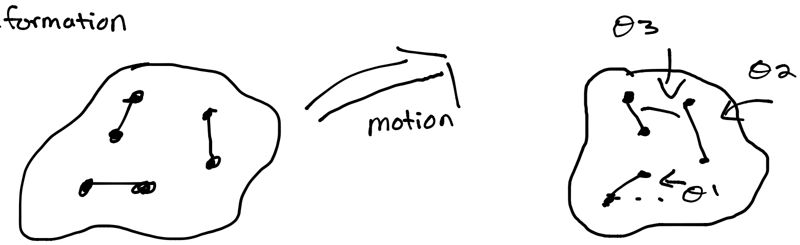
<u>Ex:</u> 

Naive approach: DAE`s

<u>Rigid object</u>: "rigid body"

all lengths between points are constant in time and all marked angles between line segments constant in time

→ no deformation



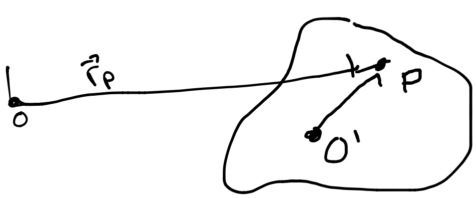
$\theta_1 = \theta_2 = \theta_3 = \theta$ of rotation object

$\boxed{\omega = \text{angular velocity} = \dot{\theta}}$ $\boxed{\vec{\omega} = \omega\hat{k} = \dot{\theta}\hat{k}}$

→ pick a reference point: O', material point on the object

Sometimes use center of mass

or hinge point



$\vec{r}_{p/o'} = \vec{r}_{o'p}$

$\vec{r}_p = \vec{r}_{o'} + \vec{r}_{p/o'}$

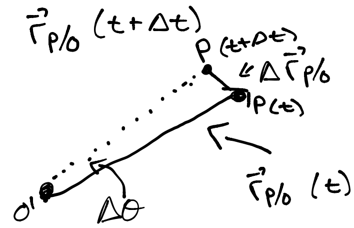
↳ $\vec{r}_{o'/o}$

$$\vec{V}_p = \vec{V}_{o'} + \frac{d}{dt}\vec{r}_{p/o'}$$

<u>relative to points</u>
<u>means subtract</u>

$$\left.\begin{aligned}\vec{a}_p &= \vec{a}_{o'} + \frac{d}{dt}\vec{V}_{p/o'}\end{aligned}\right] \quad \begin{aligned}\vec{V}_{p/o'} &= \vec{V}_p - \vec{V}_{o'} \\ \vec{a}_{p/o'} &= \vec{a}_p - \vec{a}_{o'}\end{aligned}$$

$$\boxed{\vec{V}_{p/o'} = \vec{\omega} \times \vec{r}_{p/o'}}$$



$$\Delta\vec{r}_{p/o'} = |\vec{r}_{p/o}| \cdot \Delta\theta \cdot \hat{n}$$

$\hookrightarrow \perp$ to $\vec{r}_{p/o'}$

$$\Delta\vec{r}_{p/o'} = [\Delta\theta\,\hat{k}] \times \vec{r}_{p/o'}$$

$$\vec{V}_{p/o'} = \frac{\Delta\vec{r}_{p/o'}}{\Delta t} = \frac{\Delta\theta}{\Delta t}\,\hat{k} \times \vec{r}_{p/o'}$$

$$\vec{V}_{p/o'} = \vec{\omega} \times \vec{r}_{p/o'} \quad \checkmark$$

$a_{p/o'} = ?$ $\longrightarrow$ $\vec{a}_{p/o'} = \frac{d}{dt}\vec{V}_{p/o'} = \frac{d}{dt}(\vec{\omega} \times \vec{r}_{p/o'})$

$$= \dot{\vec{\omega}} \times \vec{r}_{p/o'} + \vec{\omega} \times \vec{V}_{p/o'}$$

$$\vec{a}_{p/o} = \dot{\vec{\omega}} \times \vec{r}_{p/o} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{p/o'}) = \ddot{\theta}\,\hat{k} \times \vec{r}_{p/o} - \omega^2\vec{r}_{p/o'}$$

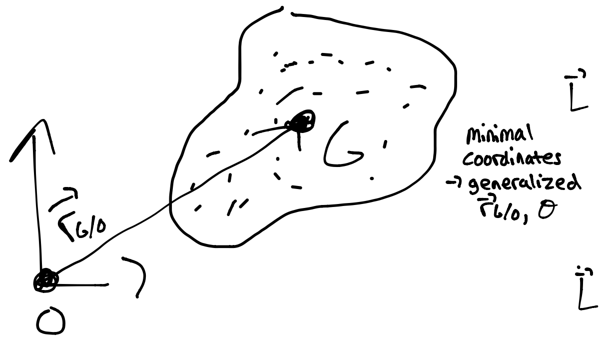
$$\boxed{\vec{a}_{p/o} = \ddot{\theta}\hat{k} \times \vec{r}_{p/o} - \omega^2\vec{r}_{p/o'}}$$

$\ddot{\theta}\,\hat{k} = \dot{\vec{\omega}} = \vec{\alpha}$ angular acceleration of object

a lot like $\vec{a}$ for polar: $\vec{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + \cancel{2\dot{r}\dot{\theta}})\hat{e}_\theta$

$$-r\dot{\theta}^2\,\hat{e}_r = -\omega^2\vec{r}, \quad r\ddot{\theta}\,\hat{e}_\theta = \dot{\vec{\omega}} \times \vec{r}_{p/o'}$$

Look at some object + calculate motion quantities



$$\vec{L} = \underbrace{\sum m_i \vec{v}_i}_{\sum (m_i \vec{v}_G + m_i \vec{v}_{i/G})} = m_{tot} \vec{v}_G$$

$$\dot{\vec{L}} = m_{tot} \vec{a}_G$$

$$\vec{H}_{/i} = \underbrace{\vec{r}_{G/i} \times (m_{tot} \vec{v}_G)}_{\vec{H}_{G/i}} + \underbrace{\sum \vec{r}_{i/G} \times m_i \vec{v}_{i/G}}_{\vec{H}_{/G}}$$

$$\vec{H}_{/G} = \sum \vec{r}_{i/G} \times (m_i \vec{\omega} \times \vec{r}_{i/G}) = \sum r_{i/G}^2 \, \omega m_i \hat{k} = \omega \hat{k} \sum m_i r_i^2$$

I^G = moment of inertia about G = $\begin{bmatrix} \sum m_i r_i^2 \\ \int r^2 dm \end{bmatrix}$

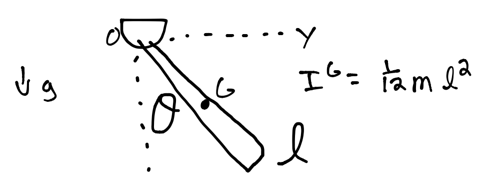
$$\vec{H}_{/i} = \vec{r}_{G/i} \times m_{tot} \vec{v}_G + I^G \dot{\theta} \hat{k}$$

$$\dot{\vec{H}}_{/i} = \vec{r}_{G/i} \times m_{tot} \vec{a}_G + I^G \ddot{\theta} \hat{k}$$

$$E_K = \frac{1}{2} m_{tot} V_G^2 + \frac{1}{2} I^G \dot{\theta}^2$$

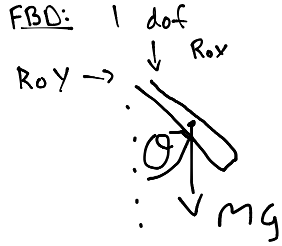
⇒ can find equations of motion for rigid objects!

<u>Example:</u> Pendulum



$I^G = \frac{1}{12} m l^2$

<u>FBD:</u> 1 dof



$$\text{AMB}/o: \quad \sum \vec{M}_{/o} = \dot{\vec{H}}_{/o}$$

$$\vec{r}_{G/o} \times mg\hat{\imath} = \vec{r}_{G/o} \times (m\vec{a}_G) + I^G \ddot{\theta}\hat{k}$$

$$\vec{r}_{G/o} \to \frac{l}{2}\hat{e}_r \qquad \vec{r}_{G/o} \to \frac{l}{2}\hat{e}_r \qquad \vec{a}_G \to \vec{a}_G = \ddot{\theta}\hat{k} \times \vec{r}_{G/o} - \dot{\theta}^2 \vec{r}_{G/o}$$

$$\longrightarrow \boxed{mg\frac{l}{2}\sin\theta = -\Big(\underbrace{I^G + \frac{ml^2}{4}}_{I^o}\Big)\ddot{\theta}}$$