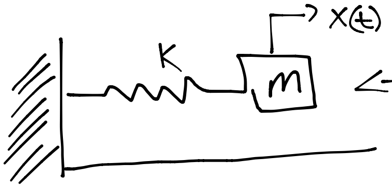
Damping Models (sections 2.10 - .12 inclusive)



$\leftarrow F_d = f_f$ $\qquad m\ddot{x} + Kx = F_d$

What generates damping? $\quad F_d = -c\dot{x}$ $\quad$ Why do we choose this so often?

What else? $\quad F_d = -\mu_s N$ $\quad$ static $\quad N \to mg$

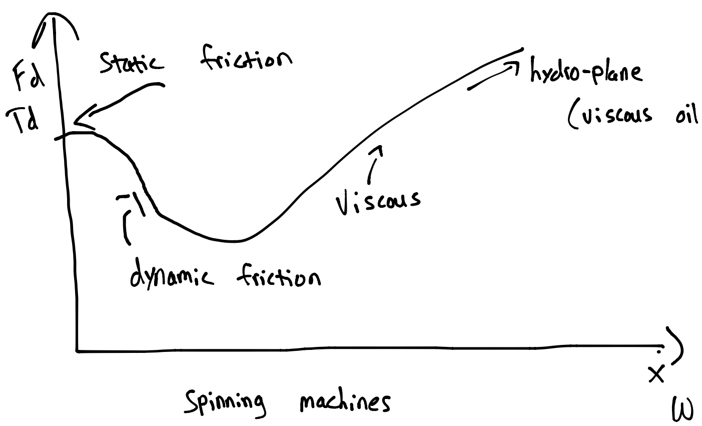
$F_d = -\mu_d N$ $\quad$ dynamic

$F_d = \frac{1}{2} \rho \dot{x}^2 A_{block} C_d$ $\quad$ air damping

$F_d = -K\beta i$ $\quad$ hysteretic damping in the spring

"no perfect spring"

Stribeck Damping



$\to$ Matlab

Section 2.10

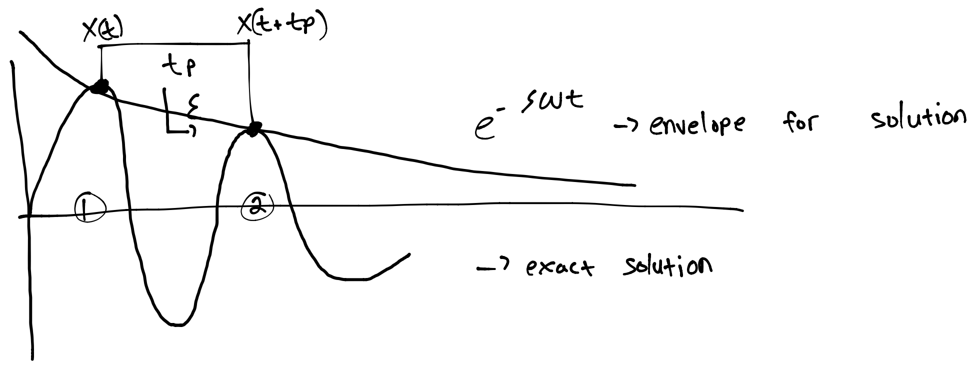
Identification of Damping and natural frequency

- if damping is small $\omega_d \approx \omega_n$ $\qquad \omega_d = \omega_n \sqrt{1-\zeta^2}$

if the system is damped then $\quad t_p = \frac{2\pi}{\omega_d} \to$ period of damped natural frequency

- this is nice, but still don't know what ζ is → let's find it experimentally

define the decrement: $\sigma = \ln \frac{X(t)}{X(t+t_p)}$



① $X(t) = Ae^{-\zeta\omega t}\sin(\omega_d t + \phi)$

② $X(t+t_p) = Ae^{-\zeta\omega(t+t_p)}\sin(\omega_d(t+t_p)+\phi)$

$$\sigma = \ln \frac{Ae^{-\zeta\omega t}\sin(\omega t + \phi)}{Ae^{-\zeta\omega t}\sin(\omega_d(t+t_p)+\phi)}$$

$\omega_d t_p = 2\pi$

$\sin(\omega_d t + \phi) = \sin(\omega_d t + 2\pi + \phi)$

sin's cancel, A's cancel

$\sigma = \ln e^{\zeta\omega_n t_p} = \zeta\omega_n t_p = \zeta\omega_n \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$

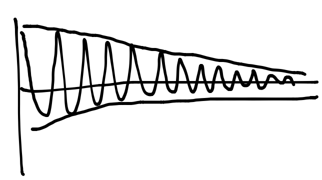
- solving for ζ yields: $\zeta = \frac{\sigma}{\sqrt{4\pi^2+\sigma^2}}$ σ → any two successive peaks

$\zeta = \%$ critical damping $C_{cr} = 2\sqrt{Km}$ $C = \zeta C_{cr}$

- process of identifying ω_n, ζ over many modes is called <u>Modal Analysis</u>

- mode shapes: μ

σ for n peaks: $\sigma = \frac{1}{n}\ln\left(\frac{X(t)}{X(t+nt_p)}\right)$



You get different σ's at different peaks

Peak deflections (read!) and see examples 2.13 - 2.14

$$|\bar{X}| = \frac{\bar{f}}{m}\left(\frac{1}{\sqrt{(\omega_n^2-\omega^2)^2 + (2\zeta\omega_n\omega)^2}}\right) \rightarrow g(\omega)$$

$$g(\omega) = \frac{1}{m\sqrt{(\omega_d^2-\omega^2) + (2\zeta\omega_n\omega)^2}}$$

$$g(\Omega) = \frac{1}{K\sqrt{(1-\Omega)^2 + (2\zeta\Omega)^2}}$$

$$\Omega = \omega/\omega_n$$