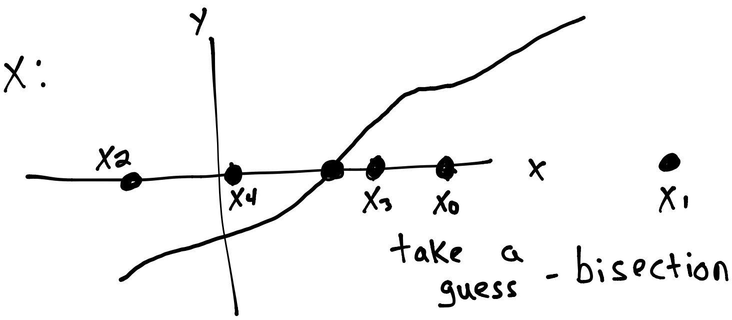
Root Finding and Periodic Orbits

puzzle: find x so that $f(x) = 0$ $\longrightarrow$ root finding

Example:

$x \in \mathbb{R}^1$
$f(x) \in \mathbb{R}^1$

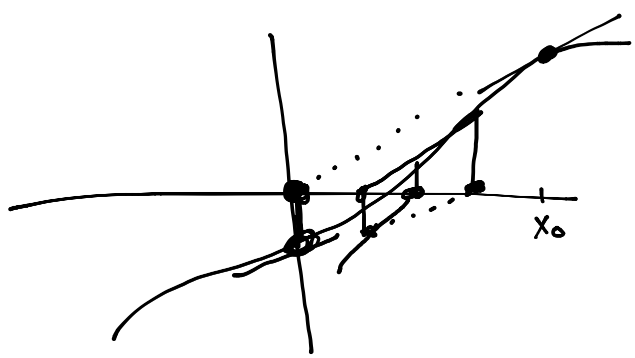
Find x:



2 approaches:

bisection approach: 2^{nd} guess is wildly different

Newton's Method:



can go bad

- need a good first guess

Root Finding with multi-variables

1.) Bisection doesn't work

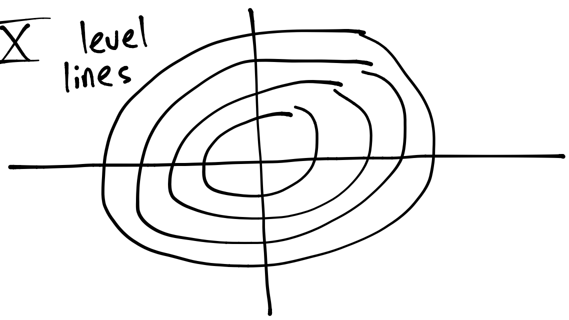
2.) Newton's method still works

Example:

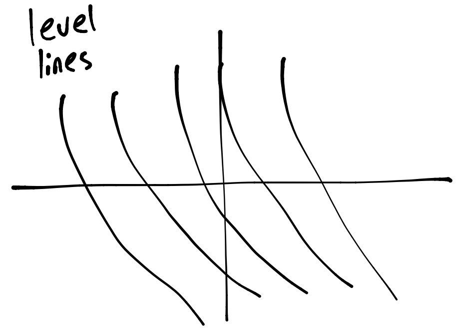
"X" = $\mathbb{R}^2$ $\qquad f = \mathbb{R}^2$

$$\left.\begin{aligned} \mathbb{X} &= \mathbb{X}(x,y) \\ \mathbb{Y} &= \mathbb{Y}(x,y) \end{aligned}\right\} \text{ given, components of } f$$

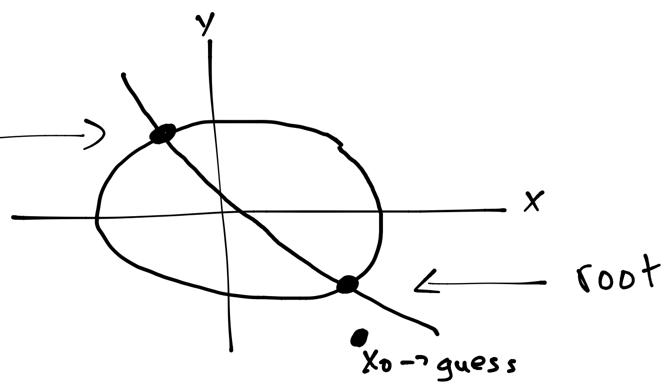
$\mathbb{X}$ level lines



$\mathbb{Y}$ level lines



plot together:



- Initial guess (x_0, y_0)
- gives us $\mathbb{X}_0, \mathbb{Y}_0$
- approximation
  - linear

$$\mathbb{X} = \mathbb{X}_0 + (x-x_0)\left.\frac{\partial \mathbb{X}}{\partial x}\right|_{x_0,y_0} + (y-y_0)\frac{\partial \mathbb{X}}{\partial y}$$

$$\mathbb{Y} = \mathbb{Y}_0 + (x-x_0)\frac{\partial \mathbb{Y}}{\partial x} + (y-y_0)\frac{\partial \mathbb{Y}}{\partial y}$$

- tangent planes at location of first guess
- We would like the second guess to be zero

To find x_1, y_1 solve for x, y:

$$0 = X_0 + \frac{\partial X}{\partial x}(x_1 - x_0) + \frac{\partial X}{\partial y}(y_1 - y_0)$$
$$0 = Y_0 + \frac{\partial Y}{\partial x}(x_1 - x_0) + \frac{\partial Y}{\partial y}(y_1 - y_0)$$

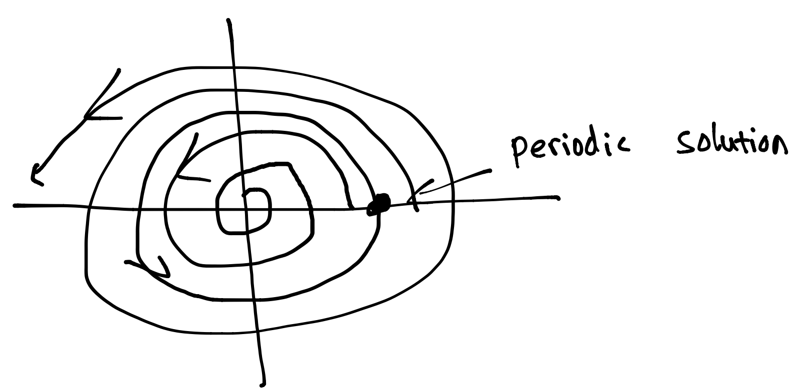
$$\underbrace{\begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} - \begin{bmatrix} \frac{\partial X}{\partial x} & \frac{\partial X}{\partial y} \\ \frac{\partial Y}{\partial x} & \frac{\partial Y}{\partial y} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}}_{b} = - \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

J = Jacobian

Solve $Ax = b$ for x

- gives x_1 and repeat

Find periodic solutions of O.D.E's



fix one variable

- "picking a Poincare Solution"

guess X_0, t_f input to f

solve equations, get X_1, Y_1

evaluate $f = \begin{bmatrix} X_1 - X_0 \\ Y_1 - 0 \end{bmatrix}$ Want $f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

- root finding problem

Matlab:

```
funcrion name
xo= 1;
x = fsolve(@myfun,x0)
end

function fval = myfun(x)
fval = 5-x^2
end


options =optionset(display,iter,tolfun, 1e, tolx, 1e);
[x,fval,exitflag] = fsolve(myfun,xo,options);
```

```
u=x(1)   y=(x(2)
x = sin(u) + v y= tan
```