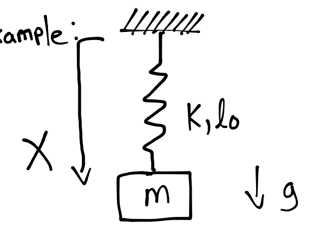
Root Finding

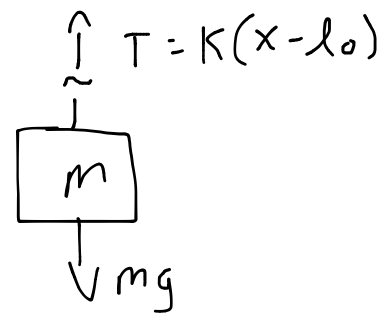
Given $F(x,y)$ Find x_0, y_0 so that $F(x_0,y_0) = 0$

Harmonic Oscillator (continued)

Example:



FDD:



<u>LMB</u>: $m\ddot{x} = mg - K(x - l_0)$

$$m\ddot{x} + Kx = mg + Kl_0$$

$$\ddot{x} + \frac{K}{m}x = g + \frac{K}{m}l_0$$

<u>Method 1</u>

$$x_{general} = x_{homogeneous} + x_{particular}$$

"particular" - any solution to the whole equation

$\ddot{x}_h + \frac{K}{m}x_h = 0$ → homogeneous has 0 on right hand side

$x_h = A\cos\left(\sqrt{\frac{K}{m}}t\right) + B\sin\left(\sqrt{\frac{K}{m}}t\right)$ $\qquad x_p = \frac{mg}{K} + l_0$

$$x = \frac{mg}{K} + l_0 + A\cos\left(\sqrt{\frac{K}{m}}t\right) + B\sin\left(\sqrt{\frac{K}{m}}t\right)$$

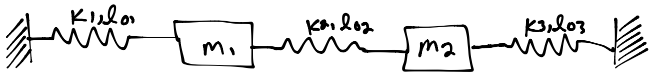
<u>Method 2</u>

Define $z = x - \left(\frac{mg}{K} + l_0\right)$ → $\ddot{z} + \frac{K}{m}z = 0$

solution: $z = A\cos(\omega t) + B\sin(\omega t)$

Summary, in the subject of vibrations, position is generally measured relative to equilibrium

Example:



- takes effort to find equilibrium

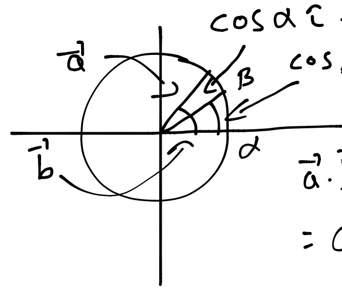
Two forms of solutions

$$x = A\cos\left(\sqrt{\tfrac{K}{m}}\,t\right) + B\sin\left(\sqrt{\tfrac{K}{m}}\,t\right) = C\cos\left(\sqrt{\tfrac{K}{m}}\,t - \delta\right)$$

Why? $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$

$\cos\beta\,\hat{\imath} + \sin\beta\,\hat{\jmath}$

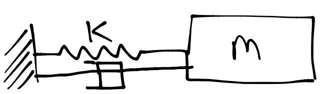


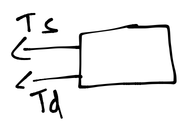
$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos(\alpha - \beta)$$
$$= a_x b_x + a_y b_y$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$C = \sqrt{A^2 + B^2}, \quad \delta = \operatorname{atan2}(A, B)$$

Damped Harmonic Oscillator





$T_s = Kx$, $T_d = c\dot{x}$

LMB: $\sum F_x = m\ddot{x}$ $\qquad m\ddot{x} = -Kx - c\dot{x}$

$$m\ddot{x} + Kx + c\dot{x} = 0$$

Solve by guessing

- get rid of one of the constants