

U-grad or M-Eng or PhD? & 4730 or 5730?

"SOLUTIONS"

Your name:

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ME 470/5730

Prelim 1

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No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes extra if you want it)

## How to get the highest score?

Please do these things:

- ↖ • ↗ Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- ↖ • Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- TIDILY REDUCE and box in your answers (Don't leave simplifiable algebraic expressions).
- >> Make appropriate Matlab code clear and correct.  
You can use shortcut notation like " $T_7 = 2\pi$ " instead of, say, " $T(7) = 2*\text{pi}$ ";.  
Small syntax errors will have small penalties.
- ↑ ↘ Clearly **define** any needed dimensions ( $\ell, h, d, \dots$ ), coordinates ( $x, y, r, \theta \dots$ ), variables ( $v, m, t, \dots$ ), base vectors ( $\hat{i}, \hat{j}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \dots$ ) and signs ( $\pm$ ) with sketches, equations or words.
- | **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Problem 1:        /25

Problem 2:        /25

Problem 3:        /25

1) A particle  $m$  moves near a fixed central mass  $M$  to which it is attracted with a force with magnitude  $|\vec{F}_g| = GmM/r^2$ . A quadratic drag also acts with magnitude  $|\vec{F}_d| = cv^2$ .

- a) Write a vector expression for the total force  $\vec{F}$  acting on the particle in terms of its position  $\vec{r}$  and velocity  $\vec{v}$ .
- b) The particle is launched from the  $x$  axis at  $x = x_0$  with a velocity  $\dot{y}_0$  in the  $y$  direction.

Write the Matlab code to find the distance from the origin at given time  $t_{end}$ .

Some lines of working code are given on the next page, fill in the missing lines.

[In the 'official' solution, which is readable and not minimal, this takes 12 lines total.

It can be done with 5 impenetrable lines.]

- c) For what values of the parameters  $G$  and  $c$ , is linear momentum conserved? Angular momentum? Total energy?

$\vec{r} = x\hat{i} + y\hat{j}$   
 $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$   
 $v = |\vec{v}| = \sqrt{\dot{x}^2 + \dot{y}^2}$

$\vec{F}_g = \frac{mMG}{r^2} (-\hat{e}_r) = -\frac{mMG}{r^2} \frac{\vec{r}}{r} = \boxed{-\frac{mMG}{r^3} \vec{r}}$

$r = |\vec{r}| = \sqrt{x^2 + y^2}$

$\vec{F}_d = cv^2 (-\hat{e}_t)$   
 $= cv^2 \left( \frac{-\vec{v}}{v} \right)$   
 $= \boxed{-cv\vec{v}}$

$\vec{F} = \vec{F}_g + \vec{F}_d = -\frac{mMG}{r^3} \vec{r} - cv\vec{v} \quad (a)$

c) Energy conserved iff no dissipation  $\boxed{c=0}$   
 If and only if

Ang. Mom (about 0) conserved iff only central forces  $\Leftrightarrow \boxed{c=0}$  (unless there are special init. conditions so that  $\vec{r} \times \vec{v}$  always = 0)

L in. Mom. only conserved if  $\vec{F} = \vec{0} \Leftrightarrow \boxed{G=0 \ \& \ c=0}$

1) (cont'd)

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%All of this goes in one text file.
```

```
%The lines already written need no fixing
```

```
%Find the motion of a particle  
function orbit
```

```
p.c=0.1; p.m=1; p.M=1; p.G=1;
```

```
x_0=1; ydot0=4;
```

```
t0=0; tend=10;
```

```
tspan=[t0 tend]';
```

```
r0=[x_0 0]'; v0=[0 ydot0]';
```

```
z0=[r0; v0]';
```

```
[tarray zarray]=ode45(@myrhs,tspan,z0,[],p);
```

```
d=norm(zarray(end,1:2));
```

```
disp(['The distance is' num2str(d)]);
```

```
end
```

```
function zdot=myrhs(t,z,p)
```

```
r=z(1:2); v=z(3:4);
```

```
Fg=-p.G * p.m * p.M * r/norm(r)^3;
```

```
Fd=-p.c * v * norm(v);
```

```
rdot=v;
```

```
vdot=(Fg + Fd)/p.m;
```

```
zdot=[rdot; vdot];
```

```
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

2) A mass  $m$  hangs a variable distance  $x(t)$  down from the ceiling from a spring ( $k, \ell_0$ ) and dashpot  $c$ , pulled by gravity. It is forced by a downwards force  $F = F_0 \sin(\omega t)$  that is transmitted by another spring and dashpot ( $k, \ell_0, c$ ) that are connected by a bar with negligible mass ( $m_1 = 0$ ). You can neglect the physical dimensions of the masses.

- Find the equations of motion for the big block (ie, the ODE governing  $x(t)$ ). That is, find a formula for  $\ddot{x}$  given  $k, \ell_0, c, g, m, F_0, \omega, x, \dot{x}$  and  $t$ .
- For given initial conditions  $(x_0, \dot{x}_0)$ ,  $x$  can be thought of as the sum of three terms: A constant term, a transient term, and a steady state term.
  - Find the constant term.
  - What is the general form of the transient term, and what simpler equation does it satisfy?
  - What is the general form of the steady state term, and what simpler equation does it satisfy?
- Assume the same system is released from rest with  $F_0 = 0$ . For what value of  $c$  does the mass most rapidly return to it's stationary equilibrium (holding all other variables constant)?
- Holding all other values constant, for what value of  $\omega$  is the steady state vibration amplitude of the mass the largest? If you don't know the exact value, give an approximate value (answers in terms of all variables, but for  $\omega$ ).

$$T_k = k(x - \ell_0)$$

$$T_c = c\dot{x}$$

LMB:

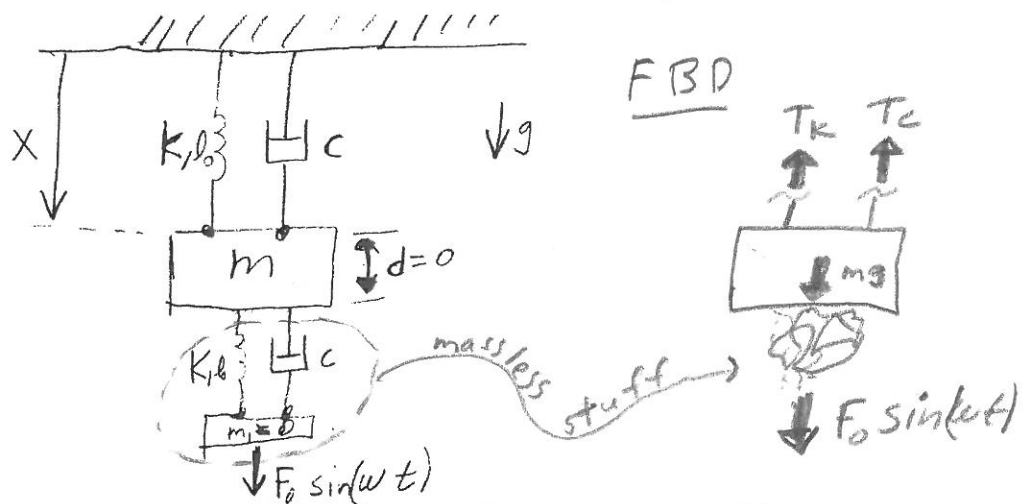
$$m\ddot{x} = \sum F_x$$

$$= F_0 \sin \omega t +$$

$$+ mg$$

$$- k(x - \ell_0)$$

$$- c\dot{x}$$



(a) Num. Soln. Form

$$\ddot{x} = \frac{c}{m} \dot{x} - \frac{k}{m} x + g + \frac{k\ell_0}{m} + \frac{F_0 \sin \omega t}{m}$$

$$\text{or } m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t + mg + k\ell_0$$

Standard Form

b)  $x(t) = x_{\text{const}} + x_{\text{trans}} + x_{\text{SS}}$

$$x_{\text{const}} = \ell_0 + \frac{mg}{k}$$

$$x_t = e^{-\lambda t} (A \cos(\omega_d t) + B \sin(\omega_d t))$$

⊙ i.k. to find  $\lambda, \omega_d$ . But not needed!

Assume underdamped

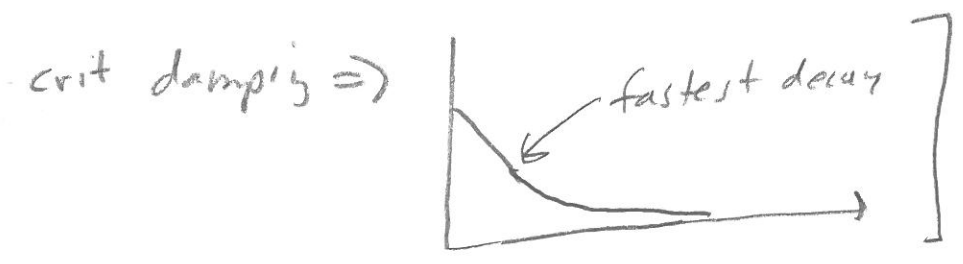
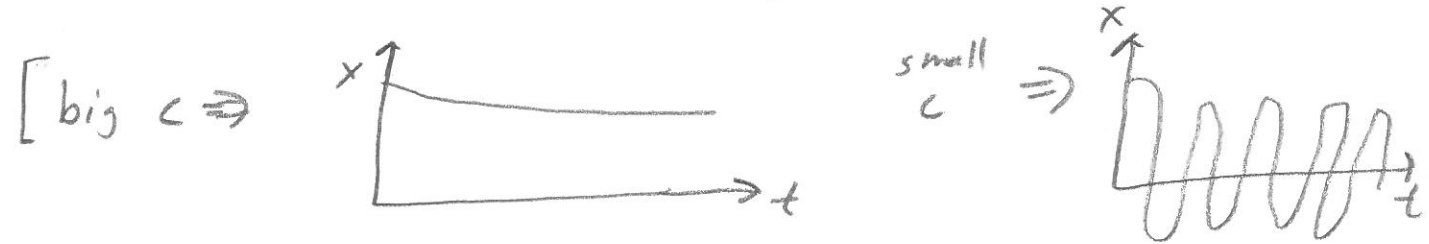
2) cont'd

$$x_{ss} = C \cos(\omega t) + D \sin(\omega t)$$

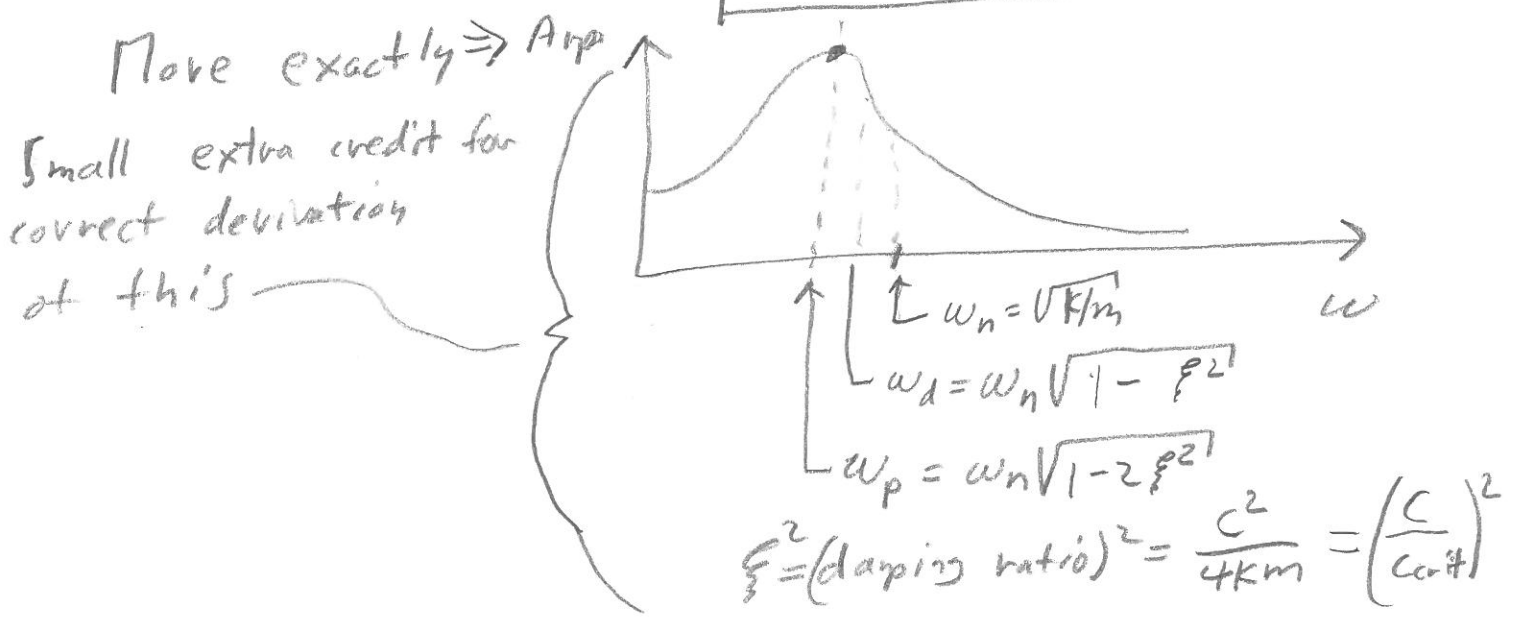
$$m \ddot{x}_{ss} + c \dot{x}_{ss} + k x_{ss} = F_0 \sin(\omega t)$$

No need to find C, D

c) Fastest return to equilib. is for critical damping  $\Rightarrow c^2 = 4km$   
 $c = 2\sqrt{km} = c_{crit}$

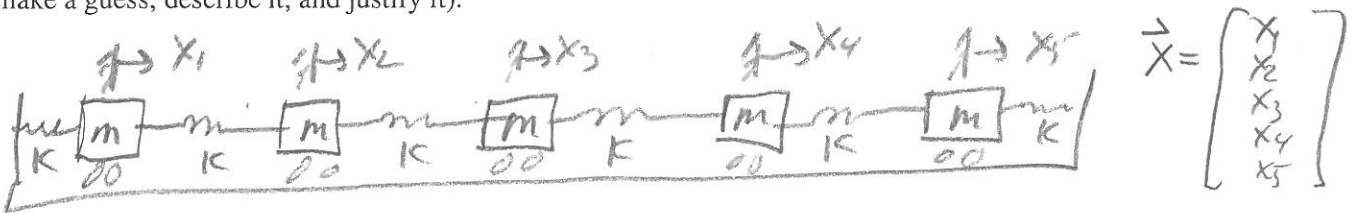


d) Basically at/near resonance = nat. freq. of undamped osc.  $\Rightarrow \omega \approx \sqrt{k/m} = \omega_n$



3) Five equal masses  $m$  are in a line held apart and between two walls with 6 equal springs  $k$ . Assume the springs are relaxed in the equilibrium position. Measure the  $x_i$  relative to the equilibrium position.

- Write out the matrices  $M$  and  $K$  so that  $M\ddot{\vec{x}} + K\vec{x} = \vec{0}$ .
- One normal mode is  $[1 \ -1 \ 0 \ 1 \ -1]^T$ . What is its frequency of oscillation? (use whatever definition of frequency you like and define).
- Find another normal mode and give its frequency.
- Find still another and give its frequency.
- Write a matlab command(s) to find the fifth (as sequenced by Matlab) normal mode and frequency. Just one mode and associated frequency are desired (Why just one? So you can show that you understand the Matlab output).
- Of the five normal modes, what, *approximately*, is the mode shape associated with the lowest frequency (just make a guess, describe it, and justify it).



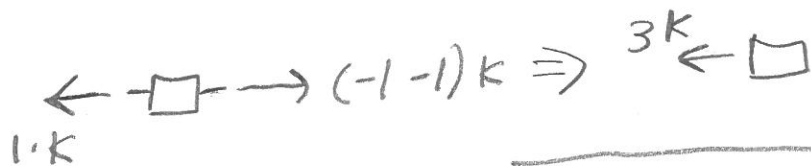
a)

$$m \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \ddot{\vec{x}} + K \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \vec{x} = \vec{0}$$

$M$   $K$



FBD of mass 1 for disp = 1



$$K_{\text{eff}} = \frac{\text{net } F}{\text{disp}} = \frac{3k}{1} = 3k \Rightarrow \omega_1 = \sqrt{\frac{3k}{m}}$$

3)(cont'd)

c)



$$\Rightarrow \vec{v}^2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ -1 \end{bmatrix} \Rightarrow \text{Each mass stretches just one spring} \\ \Rightarrow \omega_2 = \sqrt{k/m}$$

d)



3 masses each effectively sep

no net force



$$\Rightarrow \vec{v}^3 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \quad \omega_3 = \sqrt{2k/m}$$

e)

$$M = m * \text{eye}(5);$$

$$K = k * \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ \text{etc} & & & & \end{bmatrix};$$

$$[V \ 0] = \text{eig}(M^{-1} * K); \quad \text{or } \text{eig}(K, M) \\ \forall s = V(:, s); \\ \text{omega } s = \sqrt{D(s, s)};$$