

Your TA, Section # and Section time:

"SOLUTIONS"

Your name:

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TAM/ENGRD 2030

Prelim 1

February 27, 2014

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

## How to get the highest score?

Please do these things:

↖ ↗ : Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.

↖ : Use correct **vector notation**.

A+ : Be (I) neat, (II) clear and (III) well organized.

□ : TIDILY REDUCE and **box in** your answers (Don't leave simplifyable algebraic expressions).

>> : Make appropriate Matlab code clear and correct.

You can use shortcut notation like " $T_7 = 18$ " instead of, say, " $T(7) = 18$ ".

Small syntax errors will have small penalties.

↖ ↗ : Clearly **define** any needed dimensions ( $\ell, h, d, \dots$ ), coordinates ( $x, y, r, \theta \dots$ ), variables ( $v, m, t, \dots$ ), base vectors ( $\hat{i}, \hat{j}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \dots$ ) and signs ( $\pm$ ) with sketches, equations or words.

→ : **Justify** your results so a grader can distinguish an informed answer from a guess.

☛ : If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).

≈ : Work for **partial credit** (from 60–100%, depending on the problem)

– Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.

– Reduce the problem to a clearly defined set of equations to solve.

– Provide Matlab code which would generate the desired answer (and explain the nature of the output).

□ □ **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Problem 1:          /25

Problem 2:          /25

Problem 3:          /25

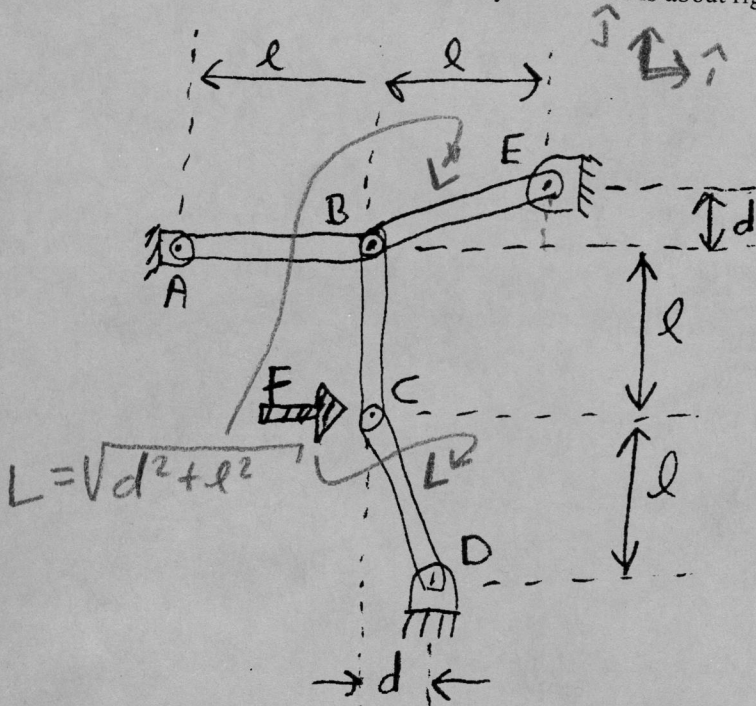
**Hint:** No part of this test depends on, or will even probably be helped by, extensive calculations.

1) Statics. Consider the small structure below. Assume rigid bars and neglect gravity. [Hint: this problem is prone to sign confusion, so take care.]

- a) Read the cover page. Write "I have read the cover page and I understand it."  
 b) Find  $T_{AB}$ , the 'bar force' in bar AB, in terms of  $F$ ,  $d$  and  $\ell$ .

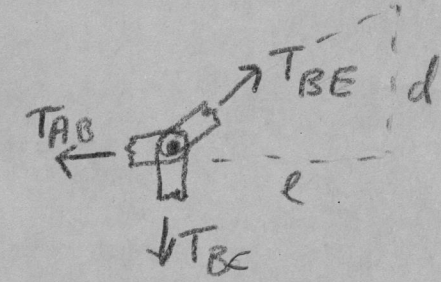
I wrote it!

- c) If  $F = 1 \text{ N}$ ,  $d = 1 \text{ cm}$  and  $\ell = 10 \text{ cm}$  what is  $T_{AB}$ ? If you don't trust your formula for  $T_{AB}$  from part (b), you can get full credit here if your answer is about right and justified with some reasonable words.

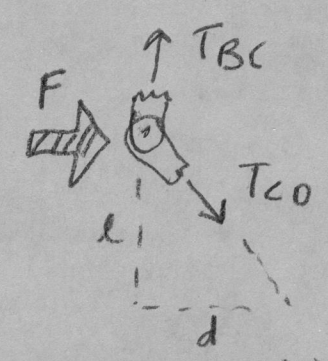


FBDs

joint B:



joint C:



joint C:  $\sum \vec{F}_i = \vec{0} \Rightarrow \{F\hat{i} + T_{BC}\hat{j} + T_{CD}\left(\frac{d}{L}\hat{i} - \frac{\ell}{L}\hat{j}\right) = \vec{0}\}$

$\{ \} \cdot \hat{i} \Rightarrow T_{CD} = \frac{-FL}{d}$

$\{ \} \cdot \hat{j} \Rightarrow T_{BC} = \frac{\ell}{L} T_{CD} = \frac{\ell}{L} \frac{-L}{d} F \Rightarrow \boxed{T_{BC} = \frac{-\ell}{d} F}$

joint B: Exactly the same geometry as joint C, rotated  $90^\circ$  & with  $-T_{BC}$  playing the role of  $F$ .

$\Rightarrow T_{AB} = \frac{-\ell}{d} (-T_{BC}) = \frac{\ell}{d} \cdot \frac{-\ell}{d} F$

$\boxed{T_{AB} = \frac{-\ell^2}{d^2} F}$  (b)

Why?

Each toggle mechanism multiplies force by about 10.

$\ell = 10 \text{ cm}$   
 $d = 1 \text{ cm} \Rightarrow \boxed{T_{AB} = -100 \text{ N}}$  (c)  
 $F = 1 \text{ N}$   
 bar force = 100N compression

2) What's the problem? That is the problem. The computer code on the next page is poorly documented. But once the missing line of code is filled in (at the "% !\*") it runs without error.

a) Describe below *as precisely as you can*, with pictures, words and equations, a mechanics question for which this code gives the answer. Then set up the governing equations.

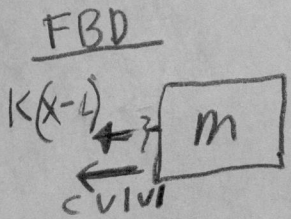
b) Fill in the missing line of code.

c) As precisely as you can, draw the plot that the code at the right makes (assuming you have correctly filled in the missing line).

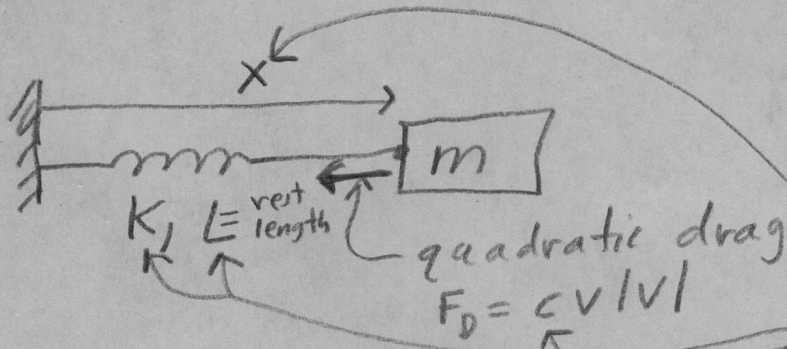
From code: (1)  $\Rightarrow \dot{x} = v$

(2)  $\Rightarrow \dot{v} = (-k(x-L) - c|v|v)/m$

$\Rightarrow ma = -k(x-L) - \underbrace{c|v|v}_{\text{quadratic drag}}$   
 (rest length)



1D



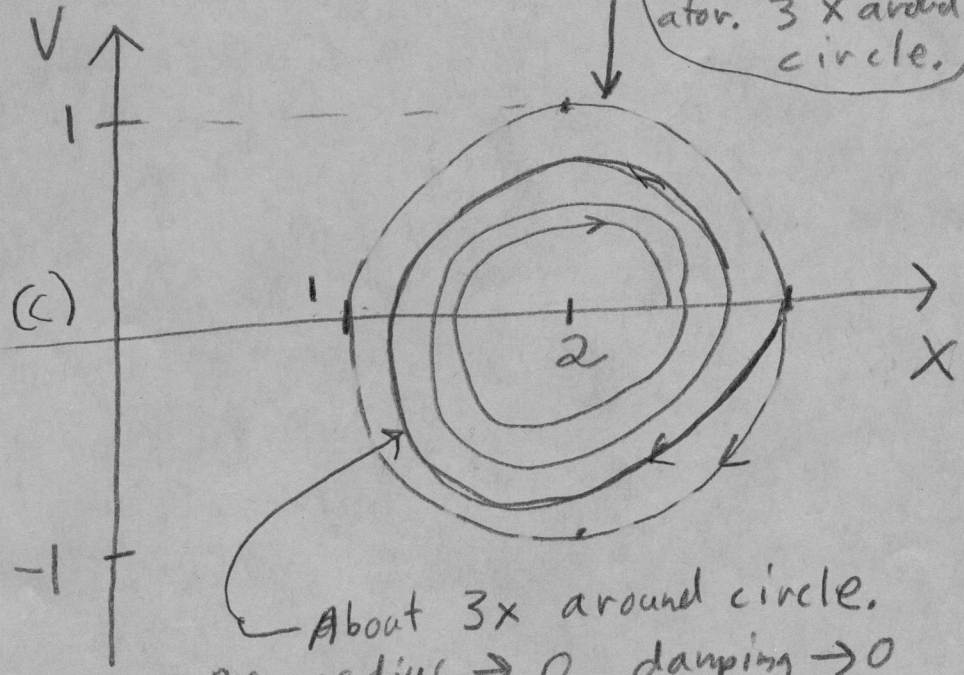
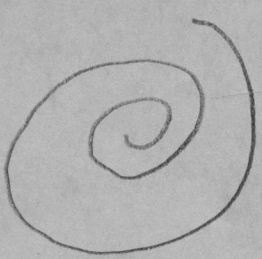
$m=1$   
 $k=1$   
 $L=2$   
 $c=0.1$

assume consistent units

ICs:  $x_0 = 3, v_0 = 0$   
 Plot  $v$  vs  $x$  for  $0 \leq t \leq 6\pi$

If no damping this would be harmonic oscillator. 3 x around circle.

If linear damping  
 $\Rightarrow$  exponential spiral



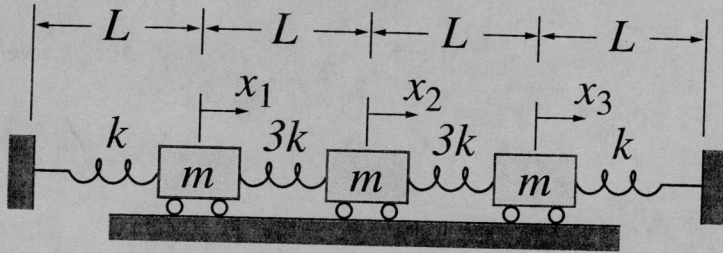
About 3x around circle.  
 AS radius  $\rightarrow 0$  damping  $\rightarrow 0$   
 (because of " $v^2$ ")  $\Rightarrow$  more like circles than exponential spiral



3) For the three-mass system shown assume that at the equilibrium position all of the springs are relaxed and that  $x_1 = x_2 = x_3 = 0$ .

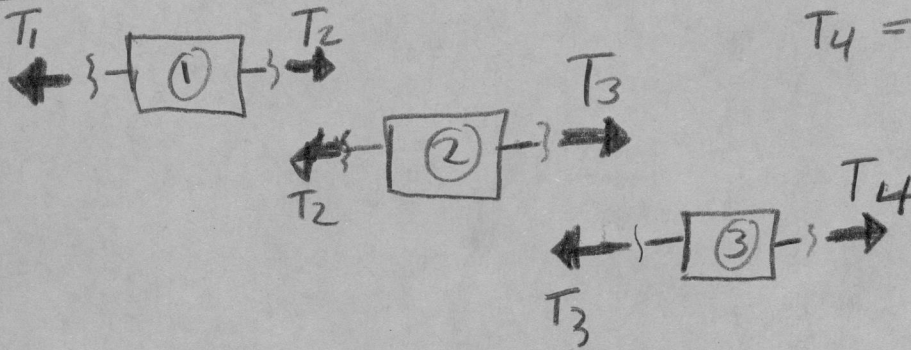
a) Write the governing equations in matrix form (Use the laws of mechanics to find these).

b) As precisely as you can describe one normal mode of this system (for example, find the frequency of vibration for this mode). [Please do *not* use eigen-values and eigen-vectors to find the mode and its frequency unless you know no other way.]



$$\begin{aligned} T_1 &= kx_1 \\ T_2 &= 3k(x_2 - x_1) \\ T_3 &= 3k(x_3 - x_2) \\ T_4 &= -kx_3 \end{aligned}$$

FBD,



LMB:

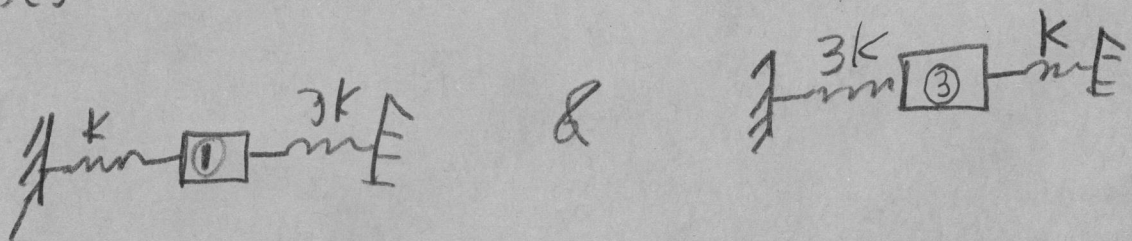
$$\begin{aligned} 1: \quad m \ddot{x}_1 &= T_2 - T_1 = 3k(x_2 - x_1) - kx_1 \\ 2: \quad m \ddot{x}_2 &= T_3 - T_2 = 3k(x_3 - x_2) - 3k(x_2 - x_1) \\ 3: \quad m \ddot{x}_3 &= T_4 - T_3 = -kx_3 - 3k(x_3 - x_2) \end{aligned}$$

$$\Rightarrow \underbrace{m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_M \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \underbrace{k \begin{bmatrix} 4 & -3 & 0 \\ -3 & 6 & -3 \\ 0 & -3 & 4 \end{bmatrix}}_K \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By symmetry, one normal mode is!

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_0 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \sin(\omega t)$$

Middle mass is still, so the side masses see this



$$\Rightarrow \text{---} \overset{4k}{\text{---}} \text{---} \boxed{m} \Rightarrow m \ddot{x} + 4kx = 0$$

$$\Rightarrow x = x_0 \sin \omega t$$

$\omega = \sqrt{4k/m} = 2\sqrt{k/m}$

(b) Mode shape =  $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ , frequency  $\omega = 2\sqrt{k/m}$