MAE 325 Fall 1999 1/6

HW 8 Solutions Due 10/27/99

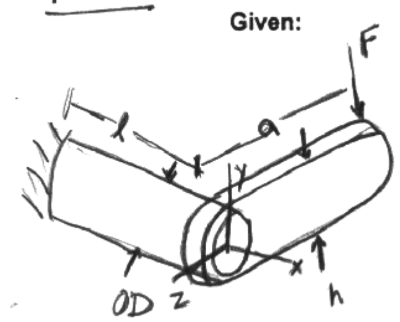
PROBLEM 4-33a

Statement: **For the bracket shown in Figure P4-14 and the data in row *a* of Table P4-3, determine the bending stress at point *A* and the shear stress due to transverse loading at point *B*. Also the torsional shear stress at both points. Then determine the principal stresses at points *A* and *B*.**

FBD

Units: $N := newton$ $\quad MPa := 10^6 \cdot Pa$ $\quad GPa := 10^9 \cdot Pa$

Given:



Tube length	$L := 100 \cdot mm$
Arm length	$a := 400 \cdot mm$
Arm thickness	$t := 10 \cdot mm$
Arm depth	$h := 20 \cdot mm$
Applied force	$F := 50 \cdot N$
Tube OD	$OD := 20 \cdot mm$
Tube ID	$ID := 14 \cdot mm$
Modulus of elasticity	$E := 207 \cdot GPa$

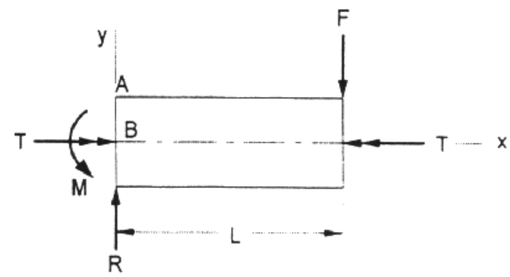


FIGURE 4-33
Free Body Diagram of Tube for Problem 4-33

Solution: See Figure 4-33 and Mathcad file P0433a.

1. Determine the bending stress at point A. From the FBD of the tube in Figure 4-33 we see that

Reaction force	$R := F$	$R = 50.0 \, N$
Reaction moment	$M := F \cdot L$	$M = 5.00 \, N \cdot m$
Distance from NA to outside of tube	$c_t := 0.5 \cdot OD$	$c_t = 10.0 \, mm$
Moment of inertia	$I_t := \frac{\pi}{64} \cdot \left(OD^4 - ID^4\right)$	$I_t = 5968 \, mm^4$
Bending stress at point A	$\sigma_{xA} := \frac{M \cdot c_t}{I_t}$	$\sigma_{xA} = 8.38 \, MPa$

2. Determine the shear stress due to transverse loading at B.

Cross-section area	$A := \frac{\pi}{4} \cdot \left(OD^2 - ID^2\right)$	$A = 160.2 \, mm^2$
Maximum shear	$V := R$	
Maximum shear stress (Equation 4.15d)	$\tau_{Vmax} := 2 \cdot \frac{V}{A}$	$\tau_{Vmax} = 0.624 \, MPa$

3. Determine the torsional shear stress at both points. Using equation 4.23b and the FBD above

Torque on tube	$T := F \cdot a$	$T = 20.0 \, N \cdot m$
Polar moment of inertia	$J := \frac{\pi}{32} \cdot \left(OD^4 - ID^4\right)$	$J = 11936 \, mm^4$
Maximum torsional stress at surface	$\tau_{Tmax} := \frac{T \cdot c_t}{J}$	$\tau_{Tmax} = 16.76 \, MPa$

4. Determine the principal stress at point A.

Stress components	$\sigma_{xA} = 8.378 \, MPa$	$\sigma_{zA} := 0 \cdot MPa$

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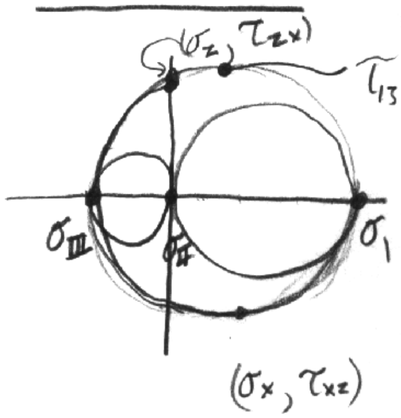
$$\tau_{xz} := \tau_{Tmax} \qquad \tau_{xz} = 16.76 \cdot MPa$$

Mohr's Circle



Principal stresses

$$\sigma_1 := \frac{\sigma_{xA} + \sigma_{zA}}{2} + \sqrt{\left(\frac{\sigma_{xA} - \sigma_{zA}}{2}\right)^2 + \tau_{xz}^2} \qquad \sigma_1 = 21.46 \cdot MPa$$

$$\sigma_2 := 0 \cdot MPa$$

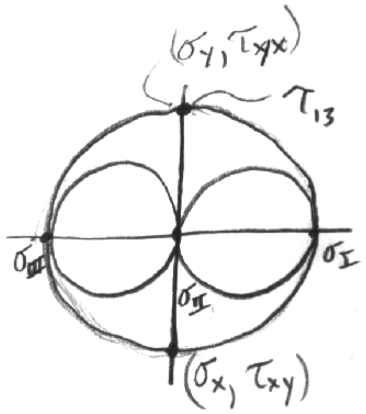
$$\sigma_3 := \frac{\sigma_{xA} + \sigma_{zA}}{2} - \sqrt{\left(\frac{\sigma_{xA} - \sigma_{zA}}{2}\right)^2 + \tau_{xz}^2} \qquad \sigma_3 = -13.08 \cdot MPa$$

$$\tau_{13} := \frac{\sigma_1 - \sigma_3}{2} \qquad \tau_{13} = 17.27 \cdot MPa$$

4. Determine the principal stress at point *B*.

Stress components $\quad \sigma_{xB} := 0 \cdot MPa \qquad \sigma_{yB} := 0 \cdot MPa$

$$\tau_{xy} := \tau_{Tmax} - \tau_{Vmax} \qquad \tau_{xy} = 16.13 \cdot MPa$$



Principal stresses

$$\sigma_1 := \frac{\sigma_{xB} + \sigma_{yB}}{2} + \sqrt{\left(\frac{\sigma_{xB} - \sigma_{yB}}{2}\right)^2 + \tau_{xy}^2} \qquad \sigma_1 = 16.13 \cdot MPa$$

$$\sigma_2 := 0 \cdot MPa$$

$$\sigma_3 := \frac{\sigma_{xB} + \sigma_{yB}}{2} - \sqrt{\left(\frac{\sigma_{xB} - \sigma_{yB}}{2}\right)^2 + \tau_{xy}^2} \qquad \sigma_3 = -16.13 \cdot MPa$$

$$\tau_{13} := \frac{\sigma_1 - \sigma_3}{2} \qquad \tau_{13} = 16.13 \cdot MPa$$

PROBLEM 4-34a

Statement: **For the bracket shown in Figure P4-14 and the data in row *a* of Table P4-3, determine the deflection at load *F*.**

Units: $N := newton$ $MPa := 10^6 \cdot Pa$ $GPa := 10^9 \cdot Pa$

Given:

Tube length	$L := 100 \cdot mm$	Applied force	$F := 50 \cdot N$
Arm length	$a := 400 \cdot mm$	Tube OD	$OD := 20 \cdot mm$
Arm thickness	$t := 10 \cdot mm$	Tube ID	$ID := 14 \cdot mm$
Arm depth	$h := 20 \cdot mm$	Modulus of elasticity	$E := 207 \cdot GPa$
		Modulus of rigidity	$G := 80.8 \cdot GPa$

Solution: See Figure 4-34 and Mathcad file P0434a.

1. The deflection at load *F* can be determined by superimposing the rigid-body deflection of the arm due to the twisting of the tube with the beam deflection of the tube and the arm alone.

2. Determine the rigid-body deflection due to twisting of the tube. Refering to Figure 4-34, the torque in the tube is

Note: parts 2, 3, and 4 can also be done using singularity functions

Torque on tube	$T := F \cdot a$	$T = 20.0 \, N \cdot m$
Polar moment of inertia	$J_t := \frac{\pi}{32} \cdot \left(OD^4 - ID^4\right)$	$J_t = 11936 \, mm^4$
Tube angle of twist	$\theta := \frac{T \cdot L}{J_t \cdot G}$	$\theta = 2.07368 \cdot 10^{-3} \, rad$
		$\theta = 0.119 \, deg$
Deflection at F due to θ	$\delta_\theta := a \cdot \theta$	$\delta_\theta = 0.829 \, mm$

3. Determine the rigid-body deflection due to bending of the tube.

Moment of inertia	$I_t := \frac{J_t}{2}$	$I_t = 5968 \, mm^4$
Deflection of tube end and arm end (see Appendix D)	$\delta_{tb} := \frac{F \cdot L^3}{3 \cdot E \cdot I_t}$	$\delta_{tb} = 0.013 \, mm$

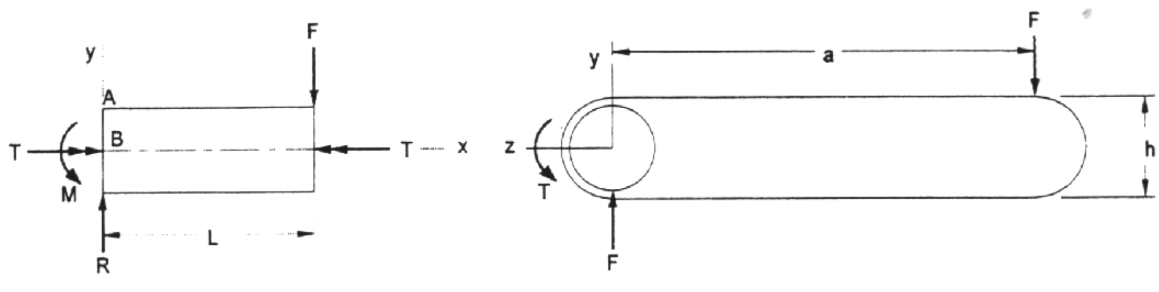


FIGURE 4-34
Free Body Diagrams of Tube and Arm for Problem 4-34

4. Determine the beam bending of arm alone.

Moment of inertia $I_a := \frac{t \cdot h^3}{12}$ $I_a = 6667 \cdot mm^4$

Deflection at F $\delta_a := \frac{F \cdot a^3}{3 \cdot E \cdot I_a}$ $\delta_a = 0.773 \cdot mm$

5. Determine the total deflection by superposition.

$\delta_{tot} := \delta_\theta + \delta_{tb} + \delta_a$ $\boxed{\delta_{tot} = 1.616 \cdot mm}$ downward

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PROBLEM 4-35a

Statement: **For the bracket shown in Figure P4-14 and the data in row *a* of Table P4-3, determine the spring rate of the tube in bending, the spring rate of the arm in bending, and the spring rate of the tube in torsion. Combine these into an overall spring rate in terms of the force *F* and the linear deflection at *F*.**

Units: $N := newton$ $\quad MPa := 10^6 \cdot Pa$ $\quad GPa := 10^9 \cdot Pa$

Given:

Tube length	$L := 100 \cdot mm$	Applied force	$F := 50 \cdot N$
Arm length	$a := 400 \cdot mm$	Tube OD	$OD := 20 \cdot mm$
Arm thickness	$t := 10 \cdot mm$	Tube ID	$ID := 14 \cdot mm$
Arm depth	$h := 20 \cdot mm$	Modulus of elasticity	$E := 207 \cdot GPa$
		Modulus of rigidity	$G := 80.8 \cdot GPa$

Solution: See Figure 4-35 and Mathcad file P0435a.

1. Determine the spring rate due to bending of the tube.

Spring Rate = K

$K = \dfrac{\text{Force}}{\text{Deflection}}$

Moment of inertia $\quad I_t := \dfrac{\pi}{64} \cdot \left(OD^4 - ID^4\right) \quad I_t = 5968 \, mm^4$

Deflection of tube end and arm end (see Appendix D) $\quad \delta_{tb} := \dfrac{F \cdot L^3}{3 \cdot E \cdot I_t} \quad \delta_{tb} = 0.013 \, mm$

Spring rate due to bending in tube $\quad k_{tb} := \dfrac{F}{\delta_{tb}} \quad k_{tb} = 3706 \, \dfrac{N}{mm}$

2. Determine the spring rate due to beam bending of arm alone.

Moment of inertia $\quad I_a := \dfrac{t \cdot h^3}{12} \quad I_a = 6667 \, mm^4$

Deflection at F $\quad \delta_a := \dfrac{F \cdot a^3}{3 \cdot E \cdot I_a} \quad \delta_a = 0.773 \, mm$

Spring rate due to bending in arm $\quad k_a := \dfrac{F}{\delta_a} \quad k_a = 64.7 \, \dfrac{N}{mm}$

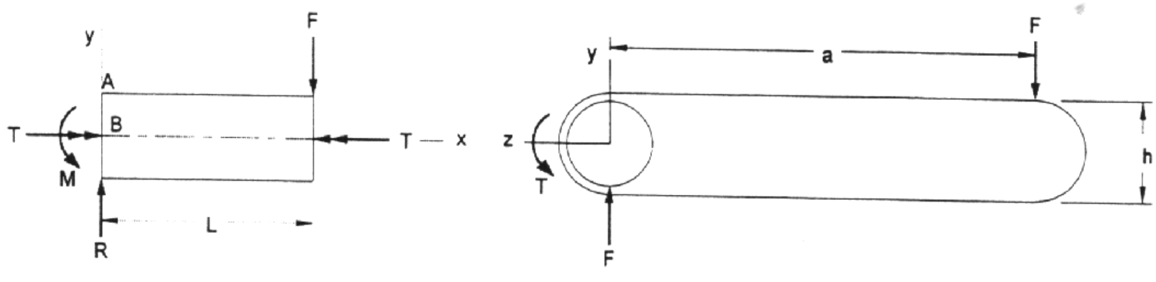

FIGURE 4-35
Free Body Diagrams of Tube and Arm for Problem 4-35

3. Determine the spring rate of the tube in torsion. Refering to Figure 4-35, the torque in the tube is

Torque on tube	$T := F \cdot a$	$T = 20.0 \cdot N \cdot m$
Polar moment of inertia	$J_t := \frac{\pi}{32} \cdot \left(OD^4 - ID^4\right)$	$J_t = 11936 \cdot mm^4$
Tube angle of twist	$\theta := \frac{T \cdot L}{J_t \cdot G}$	$\theta = 2.07368 \cdot 10^{-3} \cdot rad$ $\theta = 0.119 \cdot deg$
Deflection at F due to q	$\delta_\theta := a \cdot \theta$	$\delta_\theta = 0.829 \cdot mm$
Spring rate due to torsion in tube	$k_\theta := \frac{F}{\delta_\theta}$	$k_\theta = 60.28 \cdot \frac{N}{mm}$

4. Determine the overall spring rate. The springs are in series, thus

$$\boxed{\frac{1}{k_{oa}} = \frac{1}{k_\theta} + \frac{1}{k_{tb}} + \frac{1}{k_a}}$$

important

$$k_{oa} := \frac{k_\theta \cdot k_{tb} \cdot k_a}{k_{tb} \cdot k_a + k_\theta \cdot k_a + k_\theta \cdot k_{tb}} \qquad k_{oa} = 30.9 \cdot \frac{N}{mm}$$

Checking, $\delta_{tot} := \frac{F}{k_{oa}}$ $\qquad \delta_{tot} = 1.616 \cdot mm$

which is the same total deflection gotten in Problem 4-34.