1. p 4.1 (a) Find principal stresses and $\tau_{\text{max}}$ analytically and verify w/ Mohr’s circle

Stress Element

![Stress Diagram]

Using principal stress on right:

$$\tau_{13} = \frac{1}{2} \left( \sigma_1 - \sigma_3 \right) = 707.1$$

$$\tau_{12} = \frac{1}{2} \left( \sigma_1 - \sigma_2 \right) = 603.55$$

$$\tau_{23} = \frac{1}{2} \left( \sigma_2 - \sigma_3 \right) = 103.55$$

Solving eq. 4.4 (c) analytically:

$$\sigma^3 - C_2 \sigma^2 - C_1 \sigma - C_0 = 0$$

$$C_1 = \sigma_x + \sigma_y + \sigma_z = 1000$$

$$C_2 = \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 - \sigma_x \sigma_y$$

$$- \sigma_y \sigma_z - \sigma_z \sigma_x = (500)^2$$

$$C_0 = \sigma_x \sigma_y \sigma_z + 2 \tau_{xy} \tau_{yz} \tau_{zx}$$

$$- \sigma_x \tau_{yz}^3 - \sigma_y \tau_{zx}^3 - \sigma_z \tau_{xy}^3 = 0$$

$$\sigma^3 - 1000 \sigma^2 - 250,000 \sigma = 0$$

Factoring:

$$\sigma \left( \sigma^2 - 1000 \sigma - 250,000 \right) = 0$$

$$\sigma = 0$$ is one solution (principal stress)

$$\sigma = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{500 \pm \sqrt{1000^2 + 4(500)^2}}{2}$$

$$\sigma = 1207.1, -207.1$$

So:

$$\sigma_1 = 1207.1, \sigma_2 = 0, \sigma_3 = -207.1$$

Note: For the convention where $\sigma_1 \geq \sigma_2 \geq \sigma_3$
Mohr's Circle

\[ \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} 1000 & 500 & 0 \\ 500 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

Center of Mohr's circle:
\[ C = \frac{\sigma_x + \sigma_y}{2} = 500 \]

Radius of Mohr's Circle:
\[ r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]
\[ r = 707.1 \]

\[ \sigma_1 = C + r = 1207.1 \]
\[ \sigma_3 = C - r = -207.1 \]
\[ \sigma_2 = 0 \]

Answers check.

\[ \tau_{13} = r = 707.1 \]
\[ \tau_{12} = \left| \frac{\sigma_1 + \sigma_3}{2} \right| = 603.55 \]
\[ \tau_{23} = \left| \frac{\sigma_2 - \sigma_3}{2} \right| = 103.55 \]
Find principal stresses and $T_{\text{max}}$ analytically and verify with Mohr's Circle.

Similar to (a):

$$[\sigma] = \begin{bmatrix} -500 & 750 & 0 \\ 750 & 1000 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solving analytically:

$$\sigma^2 - C_2 \sigma - C_1 \sigma - C_0 = 0$$

$$C_2 = -500 + 1000 = 500$$
$$C_1 = 750^2 - (500)(1000) = 10,625,000$$
$$C_0 = 0$$

$$\sigma^2 - 500 \sigma - 10,625,000 = 0$$

$$\sigma = \frac{1310.7}{2}$$
$$\sigma = -810.7$$

So:

$$\sigma_1 = 1310.7$$
$$\sigma_2 = 0$$
$$\sigma_3 = -810.7$$

and

$$T_{12} = 655.35$$
$$T_{13} = 1060.7$$
$$T_{32} = 405.35$$

Mohr's Circle:

$$C = \frac{\sigma_x + \sigma_y}{2} = 250$$

$$\Gamma = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 1060.7$$

$$\sigma_1 = C + \Gamma = 1310.7$$
$$\sigma_2 = 0$$
$$\sigma_3 = -810.7$$

$$T_{12} = 655.35$$
$$T_{13} = 1060.7$$
$$T_{32} = 405.35$$
Find principal stresses and $\tau_{\text{max}}$ analytically and verify with Mohr's circle.

\[
[\sigma] = \begin{bmatrix}
-500 & 100 & 1000 \\
100 & 750 & 250 \\
1000 & 250 & 250 \\
\end{bmatrix}
\]

Solving analytically:

\[
\sigma^3 - C_2 \sigma^2 - C_1 \sigma - C_0 = 0
\]

\[
C_2 = -500 + 750 + 250 = 500
\]

\[
C_1 = 100^2 + 1000^2 + 250^2 - (-500)(750) - (750)(250)
- (250)(-500) = 1,385,000
\]

\[
C_0 = (-500)(750)(250) + 2(100)(1000)(250)
- (-500)(250)^3 - (750)(1000)^2 - 250(100)^3
= -765,000,000
\]

Using MATLAB:

```
>> roots([1 -500 -1385000 -765000000])
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\[
\begin{align*}
\sigma_1 &= 1126.4 \\
\tau_{13} &= 1160.8 \\
\tau_{13} &= 1266.4 \\
\sigma_2 &= 568.3 \\
\tau_{31} &= 279.15 \\
\sigma_3 &= -1194.9 \\
\tau_{32} &= 890.6
\end{align*}
\]

3D Mohr's Circle:

- You have to know at least 1 principal stress to draw Mohr's circle.
- Since we don't know any from inspection, we cannot solve this problem using 3D Mohr's circle.
- The final circle is below.

[Diagram of 3D Mohr's Circle]
4. Find $\tau_{max}$, $\sigma_1$, $\sigma_2$, $\sigma_3$ analytically and verify Mohr's circle.

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_3 & 0 \\ 0 & 0 & \sigma_2 \end{bmatrix}$$

From inspection we see that $\sigma_1$ is a principal stress (no shear).

Mohr's circle:

$$C = \frac{\sigma_1 - \sigma_3}{2} \quad r = \sqrt{(\frac{\sigma_1}{2})^2 + \sigma_2} = \frac{\sigma_2}{2}$$

$\sigma_1 = C + r = \sigma_1 \quad \sigma_2 = 0$

$\sigma_3 = C - r = 0$

5. [\begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix}]

Again, no shear, so from inspection these are principal stresses.

$C_2 = -3p \quad C_1 = -3p^2 \quad C_0 = -p^3$

$\sigma^3 + 3\sigma^2 p + 3\sigma p^2 + p^3 = 0$

Factoring:

$(\sigma_1 + p)(\sigma_2 + p)(\sigma_3 + p) = 0$

$\sigma_1 = \sigma_2 = \sigma_3 = -p$

$\tau_{13} = \tau_{31} = \tau_{32} = 0$ No shears!

$C = \frac{-2p}{2} = -p \quad r = \sqrt{(\frac{-p}{2})^2 + 0} \quad r = 0$

A point