1.5.1 Consider the vector equation

$$a\vec{\boldsymbol{A}} + b\vec{\boldsymbol{B}} = \vec{\boldsymbol{C}}$$

with $\vec{\boldsymbol{A}}$, $\vec{\boldsymbol{B}}$, and $\vec{\boldsymbol{C}}$ given. For the cases below find a and b if possible. If there are multiple solutions give at least 2. If there are no solutions explain why.

a) $\vec{\boldsymbol{A}} = \hat{\boldsymbol{\imath}}$, $\vec{\boldsymbol{B}} = \hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{C}} = 3\hat{\boldsymbol{\imath}} + 4\hat{\boldsymbol{\jmath}}$

b) $\vec{\boldsymbol{A}} = \hat{\boldsymbol{\imath}}$, $\vec{\boldsymbol{B}} = 2\hat{\boldsymbol{\imath}}$, $\vec{\boldsymbol{C}} = 3\hat{\boldsymbol{\imath}}$

c) $\vec{\boldsymbol{A}} = \hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{B}} = 2\hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{C}} = 3\hat{\boldsymbol{\imath}}$

d) $\vec{\boldsymbol{A}} = \hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{B}} = -\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{C}} = 2\hat{\boldsymbol{\jmath}}$

e) $\vec{\boldsymbol{A}} = \hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{B}} = 2\hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{C}} = 3\hat{\boldsymbol{\imath}} + 4\hat{\boldsymbol{\jmath}}$

f) $\vec{\boldsymbol{A}} = \pi\hat{\boldsymbol{\imath}} + e\hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{B}} = \sqrt{2}\hat{\boldsymbol{\imath}} + \sqrt{3}\hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{C}} = \hat{\boldsymbol{\imath}}$

$$a\vec{A} + b\vec{B} = \vec{C}$$

a) $a\begin{bmatrix}1\\0\end{bmatrix} + b\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$

$$\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$$

$$a = 3,\ b = 4$$

b) $a\begin{bmatrix}1\\0\end{bmatrix} + b\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}3\\0\end{bmatrix}$

$$\begin{bmatrix}a+2b\\0\end{bmatrix} = \begin{bmatrix}3\\0\end{bmatrix}$$

$$a = 3 - 2b \ \forall b$$

Example : $a = 1,\ b = 1$
$a = -1,\ b = 2$

c) $a\begin{bmatrix}0\\1\end{bmatrix} + b\begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}3\\0\end{bmatrix}$

A solution for a and b are not possible as $\vec{C}$ cannot be expressed as a linear combination of $\vec{A}$ and $\vec{B}$.

d) $a\begin{bmatrix}1\\1\end{bmatrix} + b\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}0\\2\end{bmatrix}$

$$\begin{bmatrix}a-b\\a+b\end{bmatrix} = \begin{bmatrix}0\\2\end{bmatrix}$$

$a - b = 0$ — (i)
$a + b = 2$ — (ii)
Add (i) and (ii)
$2a = 2$
$a = 1$
$\Rightarrow b = 1$

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e) $$a\begin{bmatrix}1\\2\end{bmatrix} + b\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$$

$$\begin{bmatrix}a+2b\\2a+3b\end{bmatrix} = \begin{bmatrix}3\\4\end{bmatrix}$$

$$a+2b=3 \quad - \text{(iii)}$$
$$2a+3b=4 \quad - \text{(iv)}$$

$$(2\times\text{(iii)}) - \text{(iv)}$$

$$b=2$$
$$\Rightarrow a=-1$$

f) $$a\begin{bmatrix}\pi\\e\end{bmatrix} + b\begin{bmatrix}\sqrt{2}\\\sqrt{3}\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$\begin{bmatrix}a\pi+b\sqrt{2}\\ae+b\sqrt{3}\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$a\pi+b\sqrt{2} = 1 \quad - \text{(v)}$$
$$ae+b\sqrt{3} = 0 \quad - \text{(vi)}$$

$$(\text{(v)}\times e) - (\text{(vi)}\times\pi)$$

$$b(e\sqrt{2}-\pi\sqrt{3}) = e$$

$$b = \frac{e}{(e\sqrt{2}-\pi\sqrt{3})}$$

$$a = -\frac{b}{e}\sqrt{3}$$

$$a = \frac{-\sqrt{3}}{(e\sqrt{2}-\pi\sqrt{3})}$$

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1.5.2 Consider the vector equation

$$a\vec{A} + b\vec{B} + c\vec{C} = \vec{D}$$

with $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ given. For the cases below find a if possible, there is no need to find b and c.

a) $\vec{A} = \hat{\imath}, \vec{C} = \hat{k}$
$\vec{B} = \hat{\jmath}, \vec{D} = 3\hat{\imath} + 4\hat{\jmath} + 19\hat{k}$

b) $\vec{A} = 2\hat{\imath}, \vec{C} = 15\hat{\jmath} + 360\hat{k}$
$\vec{B} = 3\hat{\imath} + 4\hat{\jmath}, \vec{D} = 2\hat{\imath} + 17\hat{\jmath} + 37\hat{k}$

c) $\vec{A} = \hat{k}, \vec{C} = \hat{\imath} + \hat{\jmath} + \hat{k}$
$\vec{B} = \hat{\jmath} + \hat{k}, \vec{D} = \hat{\jmath}$

d) $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{C} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$
$\vec{B} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}, \vec{D} = 4\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$

e) $\vec{A} = \sqrt{1}\hat{\imath} + \sqrt{2}\hat{\jmath} + \sqrt{3}\hat{k}$,
$\vec{C} = \sqrt{7}\hat{\imath} + \sqrt{8}\hat{\jmath} + \sqrt{9}\hat{k}$,
$\vec{B} = \sqrt{4}\hat{\imath} + \sqrt{5}\hat{\jmath} + \sqrt{6}\hat{k}$,
$\vec{D} = -\hat{\imath} + \pi\hat{\jmath} + e\hat{k}$

a)
$$a\begin{bmatrix}1\\0\\0\end{bmatrix} + b\begin{bmatrix}0\\1\\0\end{bmatrix} + c\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}3\\4\\19\end{bmatrix}$$

$$\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}3\\4\\19\end{bmatrix}$$

$$a = 3$$

b)
$$a\begin{bmatrix}2\\0\\0\end{bmatrix} + b\begin{bmatrix}3\\4\\0\end{bmatrix} + c\begin{bmatrix}0\\15\\360\end{bmatrix} = \begin{bmatrix}2\\17\\37\end{bmatrix}$$

$$c = \frac{37}{360}$$

$$4b + \frac{37}{24} = 17$$

$$b = \frac{371}{96}$$

$$2a + \frac{371}{32} = 2$$

$$a = \frac{-307}{64}$$

c)
$$a\begin{bmatrix}0\\0\\1\end{bmatrix} + b\begin{bmatrix}0\\1\\1\end{bmatrix} + c\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\begin{bmatrix}c\\b+c\\a+b+c\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$c = 0$$
$$b = 1$$
$$a = -1$$

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d)
$$a\begin{bmatrix}1\\1\\1\end{bmatrix} + b\begin{bmatrix}2\\3\\4\end{bmatrix} + c\begin{bmatrix}3\\4\\5\end{bmatrix} = \begin{bmatrix}4\\5\\7\end{bmatrix}$$

$$\begin{bmatrix}a+2b+3c\\a+3b+4c\\a+4b+5c\end{bmatrix} = \begin{bmatrix}4\\5\\7\end{bmatrix}$$

$a + 2b + 3c = 4$ - (i)

$a + 3b + 4c = 5$ - (ii)

$a + 4b + 5c = 7$ - (iii)

(ii) - (i)

$b + c = 1$

(iii) - (ii)

$b + c = 2$

Hence, it is not possible to determine a as b+c takes the value of both 1 and 2 as per the equations obtained.

e)
$$a\begin{bmatrix}1\\\sqrt{2}\\\sqrt{3}\end{bmatrix} + b\begin{bmatrix}\sqrt{4}\\\sqrt{5}\\\sqrt{6}\end{bmatrix} + c\begin{bmatrix}\sqrt{7}\\\sqrt{8}\\\sqrt{9}\end{bmatrix} = \begin{bmatrix}-1\\\pi\\e\end{bmatrix}$$

$a + 2b + \sqrt{7} = -1$ - (iv)

$\sqrt{2}a + \sqrt{5}b + 2\sqrt{2}c = \pi$ - (v)

$\sqrt{3}a + \sqrt{6}b + 3c = e$ - (vi)

$\sqrt{2}$ (iv) - (v)

$(2\sqrt{2} - \sqrt{5})b + (\sqrt{14} - 2\sqrt{2})c = -\sqrt{2} - \pi$ - (vii)

$\sqrt{3}$ (iv) - (vi)

$(2\sqrt{3} - \sqrt{6})b + (\sqrt{21} - 3)c = \sqrt{3} - e \quad - (viii)$

$(2\sqrt{3} - \sqrt{6})(vii) - (2\sqrt{2} - \sqrt{6})(viii)$

$[(2\sqrt{3} - \sqrt{6})(\sqrt{14} - 2\sqrt{2}) - (2\sqrt{2} - \sqrt{6})(\sqrt{21} - 3)]c = (2\sqrt{3} - \sqrt{6})(-\sqrt{2} \cdot \pi) - (2\sqrt{2} - \sqrt{6})(-\sqrt{3} - e)$

$c = -8.98$

$b = 3.64$

$a = 15.47$

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1.5.3 In the problems below use matrix algebra on a computer to find a, b and c uniquely if possible. If not possible explain why not. You are given that

$$a\vec{A} + b\vec{B} + c\vec{C} = \vec{D}$$

and that

a) $\vec{A} = \hat{\imath}, \vec{B} = \hat{\jmath}$,
$\vec{C} = \hat{k}, \vec{D} = -2\hat{\imath} + 5\hat{\jmath} + 10\hat{k}$

b) $\vec{A} = \hat{\imath} + \hat{\jmath}, \vec{B} = -\hat{\imath} + \hat{\jmath}$,
$\vec{C} = \hat{k}, \vec{D} = 2\hat{\imath}$

c) $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{B} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$,
$\vec{C} = \hat{\imath} + 2\hat{\jmath} + \hat{k}, \vec{D} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$

d) $\vec{A} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{B} = 4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$,
$\vec{C} = 7\hat{\imath} + 8\hat{\jmath} + 9\hat{k}, \vec{D} = 10\hat{k}$

e) $\vec{A} = \sqrt{1}\hat{\imath} + \sqrt{2}\hat{\jmath} + \sqrt{3}\hat{k}, \vec{B} = \sqrt{4}\hat{\imath} + \sqrt{5}\hat{\jmath} + \sqrt{6}\hat{k}$,
$\vec{C} = \sqrt{7}\hat{\imath} + \sqrt{8}\hat{\jmath} + \sqrt{9}\hat{k}, \vec{D} = \sqrt{10}\hat{k}$

Code:

```
syms a b c
% a) %
disp("a)")
A = a*[1; 0; 0];
B = b*[0; 1; 0];
C = c*[0; 0; 1];
D = [-2; 5; 10];
solve(A+B+C==D)

% b) %
disp("b)")
A = a*[1; 1; 0];
B = b*[-1; 1; 0];
C = c*[0; 0; 1];
D = [2; 0; 0];
solve(A+B+C==D)

% c) %
disp("c)")
A = a*[1; 1; 1];
B = b*[2; 1; 1];
C = c*[1; 2; 1];
D = [2; 1; 1];
solve(A+B+C==D)

% d) %
disp("d)")
A = a*[1; 2; 3];
B = b*[4; 5; 6];
C = c*[7; 8; 9];
D = [0; 0; 10];
solve(A+B+C==D)
% This cannot be solved as explained in Question 1.5.2 d) %

% e) %
disp("e)")
A = a*[sqrt(1); sqrt(2); sqrt(3)];
B = b*[sqrt(4); sqrt(5); sqrt(6)];
```

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```
C = c*[sqrt(7); sqrt(8); sqrt(9)];
D = [0; 0; sqrt(10)];
solve(A+B+C==D)
```

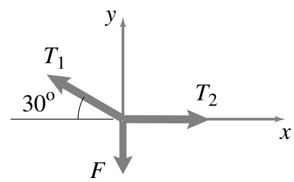
The solution to the question is :

a) $a = -2,\ b = 5,\ c = 10$
b) $a = 1,\ b = -1,\ c = 0$
c) $a = 0,\ b = 1,\ c = 0$
d) This cannot be solved 2 sets of equations are linearly dependant, and we can only obtain 2 sets of linearly independant equations with 3 unkown variables.
e) $$a = \frac{8\sqrt{5} - 5\sqrt{14}}{4\sqrt{6} - 2\sqrt{12} - 6\sqrt{2} + 3\sqrt{5} + \sqrt{84} - \sqrt{105}}$$

$$b = \frac{2\sqrt{35} - 2\sqrt{2}}{4\sqrt{6} - 2\sqrt{12} - 6\sqrt{2} + 3\sqrt{5} + \sqrt{84} - \sqrt{105}}$$

$$c = \frac{-3\sqrt{5}}{4\sqrt{6} - 2\sqrt{12} - 6\sqrt{2} + 3\sqrt{5} + \sqrt{84} - \sqrt{105}}$$

1.5.4 The three forces shown in the figure are in equilibrium, *i.e.*, $\vec{T}_1 + \vec{T}_2 + \vec{F} = \vec{0}$. If $|\vec{F}| = 10\,\text{N}$, find tensions T_1 and T_2 (magnitudes of $\vec{T}_1$ and $\vec{T}_2$).



Problem 1.4

$$T_1 \begin{bmatrix} \cos 150 \\ \sin 150 \end{bmatrix} + T_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -10\text{N} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\sqrt{3}/2\, T_1 + T_2 + 0 \\ T_1/2 \quad -10\text{N} \end{bmatrix} = 0$$

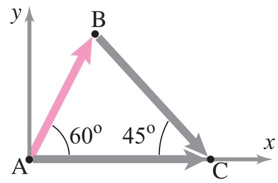
$$T_1 = 20\text{N}$$

Substitute Value of T_1,

$$T_2 = \frac{\sqrt{3}}{2} \times 20\text{N}$$

$$T_2 = 10\sqrt{3}\ \text{N}$$

1.5.5 Points A, B, and C are located in the xy plane as shown in the figure. For position vectors, we can write, $\vec{r}_B + \vec{r}_{C/B} = \vec{r}_C$. Find $|\vec{r}_B|$ and $|\vec{r}_{C/B}|$ if $\vec{r}_C = 10\,\text{m}\,\hat{\imath}$.



Problem 1.5

$\vec{r}_C = 10\text{m}\,\hat{\imath}$

$\vec{r}_B = |\vec{r}_B|\,(\cos 60\,\hat{\imath} + \sin 60\,\hat{\jmath})$

$\vec{r}_{C/B} = |\vec{r}_{B/C}|\,(\cos 315\,\hat{\imath} + \sin 315\,\hat{\jmath})$

$\vec{r}_B + \vec{r}_{C/B} = \vec{r}_C$

$$\begin{bmatrix} |\vec{r}_B|/2 + \frac{\sqrt{2}\,|\vec{r}_{C/B}|}{2} \\ \sqrt{3}\,|\vec{r}_B|/2 + \frac{\sqrt{2}\,|\vec{r}_{C/B}|}{2} \end{bmatrix} = \begin{vmatrix} 10\text{m} \\ 0 \end{vmatrix}$$

$|\vec{r}_B| + \sqrt{2}\,|\vec{r}_{C/B}| = 20\text{m} \quad -\text{(i)}$

$\sqrt{3}\,|\vec{r}_B| + \sqrt{2}\,|\vec{r}_{C/B}| = 0 \quad -\text{(ii)}$

(i) - (ii)

$|\vec{r}_B|\,(1-\sqrt{3}) = 20\text{m}$

$|\vec{r}_B| = \dfrac{20\text{m}}{(1-\sqrt{3})}$

Substitute value of $|\vec{r}_B|$,

$|\vec{r}_{C/B}| = 20\text{m} - |\vec{r}_B|$

$= 20\text{m}\left[\dfrac{1-\sqrt{3}-1}{1-\sqrt{3}}\right]$

$= \dfrac{20}{(1-\sqrt{3})}\ \text{m}$

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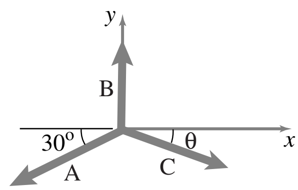
1.5.6 Three vectors, $\vec{\boldsymbol{A}}, \vec{\boldsymbol{B}}$, and $\vec{\boldsymbol{C}}$, (shown in the figure) are such that $\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} + \vec{\boldsymbol{C}} = \vec{\boldsymbol{0}}$. You are given that $A = |\vec{\boldsymbol{A}}| = 5$ and $C = |\vec{\boldsymbol{C}}| = 8$. Find θ.



Problem 1.6

$$\vec{A} = 5\begin{bmatrix} \cos 210 \\ 0 \\ \sin 210 \end{bmatrix}$$

$$\vec{B} = b\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{C} = 8\begin{bmatrix} \cos(360-\theta) \\ 0 \\ \sin(360-\theta) \end{bmatrix}$$

$$\vec{A} + \vec{B} + \vec{C} = \begin{bmatrix} -\frac{5\sqrt{3}}{2} + 8\cos(360-\theta) \\ b \\ -5/2 + 8\sin(360-\theta) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Find value of θ,

$$\cos(360-\theta) = \frac{5\sqrt{3}}{16}$$

$$\cos\theta = \frac{5\sqrt{3}}{16}$$

$$\theta = 57.23^\circ$$

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1.5.7 Let $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$, where $\vec{F}_3 = 10\,\mathrm{N}(\hat{\imath} - \hat{\jmath})$, and $\vec{F}_1/|\vec{F}_1| = 0.250\hat{\imath} + 0.968\hat{\jmath}$ and $\vec{F}_2/|\vec{F}_2| = -0.425\hat{\imath} - 0.905\hat{\jmath}$. Find $|\vec{F}_1|$ from a single scalar equation.

$$\vec{F}_3 = 10\,\mathrm{N}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\hat{F}_1 = \begin{bmatrix} 0.25 \\ 0.968 \end{bmatrix}$$

$$\hat{F}_2 = \begin{bmatrix} -0.425 \\ -0.905 \end{bmatrix}$$

$$|\vec{F}_1|\hat{F}_1 + |\vec{F}_2|\hat{F}_2 + \vec{F}_3 = 0$$

$$\begin{pmatrix} |\vec{F}_1|0.25 - |\vec{F}_2|0.425 + 10\mathrm{N} \\ |\vec{F}_1|0.968 - |\vec{F}_2|0.905 - 10\mathrm{N} \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$|\vec{F}_1| = 4(10\mathrm{N} + |\vec{F}_2|0.425)$$

$$(0.968 \times 4)(10\mathrm{N} + |\vec{F}_2|0.425) - 10\mathrm{N} = 0$$

$$|\vec{F}_2| = \left(\frac{10\,\mathrm{N}}{0.968 \times 4} - 10\mathrm{N}\right) \times \frac{1}{0.425}$$

$$= -11.375\,\mathrm{N}$$

Substitute Value of $|\vec{F}_2|$,

$$|\vec{F}_1| = 20.66\mathrm{N}$$

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1.5.8 To evaluate the equation $\sum \vec{F} = m\vec{a}$ for some problem, a student writes $\sum \vec{F} = F_x\hat{\imath} - (F_y - 30\,\mathrm{N})\hat{\jmath} + 50\,\mathrm{N}\hat{k}$ in the xyz coordinate system, but $\vec{a} = 2.5\,\mathrm{m/s^2}\hat{\imath}' + 1.8\,\mathrm{m/s^2}\hat{\jmath}' - a_z\hat{k}'$ in a rotated $x'y'z'$ coordinate system. If $\hat{\imath}' = \cos 60^\circ\,\hat{\imath} + \sin 60^\circ\,\hat{\jmath}$, $\hat{\jmath}' = -\sin 60^\circ\,\hat{\imath} + \cos 60^\circ\,\hat{\jmath}$ and $\hat{k}' = \hat{k}$, find the scalar equations for the x', y', and z' directions.

$$\hat{\imath}' = \cos 60\,\hat{\imath} + \sin 60\,\hat{\jmath} \qquad -(i)$$

$$\hat{\jmath}' = -\sin 60\,\hat{\imath} + \cos 60\,\hat{\jmath} \qquad -(ii)$$

$\sin 60\,(i) + \cos 60\,(ii)$

$$\sin 60\,\hat{\imath}' + \cos 60\,\hat{\jmath}' = \hat{\jmath}$$

$$\hat{\jmath} = \frac{\sqrt{3}}{2}\hat{\imath}' + \frac{1}{2}\hat{\jmath}'$$

$$\cos 60\,\hat{\imath} = \hat{\imath}' - \sin 60\,\hat{\jmath}$$

$$= \hat{\imath}' - \sin^2 60\,\hat{\imath}' - \sin 60 \cos 60\,\hat{\jmath}'$$

$$\hat{\imath} = \cos 60\,\hat{\imath}' - \sin 60\,\hat{\jmath}'$$

$$= \frac{1}{2}\hat{\imath}' - \frac{\sqrt{3}}{2}\hat{\jmath}'$$

Find Sum of Forces,

$$\sum \vec{F} = F_x\hat{\imath} - (F_y - 30\,\mathrm{N})\hat{\jmath} + 50\,\mathrm{N}\hat{k}$$

$$= F_x\left(\frac{1}{2}\hat{\imath}' - \frac{\sqrt{3}}{2}\hat{\jmath}'\right) - (F_y - 30\,\mathrm{N})\left(\frac{\sqrt{3}}{2}\hat{\imath}' + \frac{1}{2}\hat{\jmath}'\right) + 50\,\mathrm{N}\hat{k}'$$

$$= \left[\frac{F_x}{2} - \frac{F_y\sqrt{3}}{2} + 15\sqrt{3}\,\mathrm{N}\right]\hat{\imath}' - \left[\frac{F_x\sqrt{3}}{2} + \frac{F_y}{2} - 15\,\mathrm{N}\right]\hat{\jmath}' + 50\,\mathrm{N}\hat{k}'$$

$$\vec{F} = m\vec{a}$$

$$\frac{1}{2}\begin{pmatrix} F_x - F_y\sqrt{3} + 30\sqrt{3}\,\mathrm{N} \\ F_x\sqrt{3} + F_y - 30\,\mathrm{N} \\ 100\,\mathrm{N} \end{pmatrix} = m\begin{pmatrix} 2.5\,\mathrm{ms^{-2}} \\ 1.8\,\mathrm{ms^{-2}} \\ -a_z \end{pmatrix}$$

$$x' = \frac{1}{2}(F_x - F_y\sqrt{3} + 30\sqrt{3}\,\mathrm{N})$$

$$y' = \frac{1}{2}(F_x\sqrt{3} + F_y - 30\,\mathrm{N})$$

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$Z' = 50N$

1.5.9 A car travels straight north-east for a while on a dirt road that leads to a north-south highway. The car travels on the highway due north for a while. When the driver stops, the GPS system indicates that the car is 60 miles north and 30 miles east from the starting point. Find the distance travelled on the dirt road.

$$\vec{N}E = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\vec{N} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

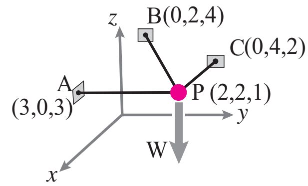
$$a\begin{bmatrix} 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 30\ mi \\ 60\ mi \end{bmatrix}$$

$$\begin{bmatrix} a \\ a+b \end{bmatrix} = \begin{bmatrix} 30mi \\ 60mi \end{bmatrix}$$

$$a = 30mi$$

Hence the car travelled on the dirt road for 30 miles.

1.5.10 A particle is held at point P with the help of three rods PA, PB, and PC. Let the tensions in the three strings be T_A, T_B, and T_C, respectively (so that T_A acts along line PA and so on). The equilibrium of the particle requires that $\vec{T}_A + \vec{T}_B + \vec{T}_C + \vec{W} = \vec{0}$ where $\vec{W} = -10\,\mathrm{N}\,\hat{k}$ is the weight of the particle. Find the magnitudes of tensions in the three rods (note, tension can be negative).



Problem 1.10

$$\vec{T}_A = T_A \begin{bmatrix}-1\\2\\-2\end{bmatrix} \quad \vec{T}_B = T_B \begin{bmatrix}2\\0\\-3\end{bmatrix} \quad \vec{T}_C = T_C \begin{bmatrix}2\\-2\\-1\end{bmatrix} \quad \vec{W} = \begin{bmatrix}0\\0\\-10\,\mathrm{N}\end{bmatrix}$$

$$\vec{T}_A + \vec{T}_B + \vec{T}_C + \vec{W} = 0$$

$$\begin{bmatrix}-T_A + 2T_B + 2T_C\\ 2T_A - 2T_C\\ -2T_A - 3T_B - T_C - 10\mathrm{N}\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$T_A = T_C$$

$$T_B = T_C/2$$

Substitute Value of T_A and T_B,

$$-2T_C - \frac{3T_C}{2} - T_C = 10\mathrm{N}$$

$$-9T_C = 20\mathrm{N}$$

$$T_C = \frac{-20}{9}\mathrm{N} = T_A$$

$$T_B = -\frac{10}{9}\mathrm{N}$$

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1.5.11 You are given that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 5\,\text{kN}\,\hat{\boldsymbol{\jmath}}$ where $\vec{F}_1 = (2\hat{\boldsymbol{\imath}} - 3\hat{\boldsymbol{\jmath}} + 4\hat{\boldsymbol{k}})\,\text{kN}$, $\vec{F}_2 = (\hat{\boldsymbol{\imath}} + 5\hat{\boldsymbol{k}})\,\text{kN}$. Find the direction of $\vec{F}_3$ (An angle measured CCW from the $+x$ axis to the direction of positive $\vec{F}_3$).

$$\vec{F}_1 = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} \text{kN} \qquad \vec{F}_2 = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} \text{kN} \qquad \vec{F}_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 5\,\text{kN}\,\hat{\jmath}$$

$$\begin{bmatrix} 2 + 1 + x \\ -3 + y \\ 4 + 5 + z \end{bmatrix} \text{kN} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} \text{kN}$$

$$x = -3\,\text{kN}$$
$$y = 8\,\text{kN}$$
$$z = -9\,\text{kN}$$

Substitute Value of x, y, z,

$$\vec{F}_3 = \begin{bmatrix} -3 \\ 8 \\ -9 \end{bmatrix} \text{kN}$$

1.5.12 A plane intersects the x, y, and z axis at 3,4, and 5 respectively. What point on the plane is in the direction $\hat{\imath}+2\hat{\jmath}+3\hat{k}$ from the point (10,10,10)? (Find the x, y and z components of the point.).

To find the normal vector to the plane

$$\begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix} \times \begin{bmatrix} 3 \\ 0 \\ -5 \end{bmatrix} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -4 & 0 \\ 3 & 0 & -5 \end{vmatrix}$$

$$= 20\hat{\imath} + 15\hat{\jmath} + 12\hat{k}$$

Point on the Plane $= \begin{bmatrix} 20 \\ 15 \\ 12 \end{bmatrix} \times \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 20 & 15 & 12 \\ 10 & 10 & 10 \end{vmatrix}$$

$$= 30\hat{\imath} - 80\hat{\jmath} + 50\hat{k}$$

$$= 10(3\hat{\imath} - 8\hat{\jmath} + 5\hat{k})$$

Hence the point on the plane is : (30, -80, 50).

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1.5.13 Write the following equations in matrix form to solve for x, y, and z:

$$
\begin{aligned}
2x - 3y + 5 &= 0, \\
y + 2\pi z &= 21, \\
\frac{1}{3}x - 2y + \pi z - 11 &= 0.
\end{aligned}
$$

$$2x - 3y = -5$$

$$y + 2\pi z = 21$$

$$\frac{1}{3}x - 2y + \pi z = 11$$

$$x\begin{bmatrix} 2 \\ 0 \\ 1/3 \end{bmatrix} + y\begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} + z\begin{bmatrix} 0 \\ 2\pi \\ \pi \end{bmatrix} = \begin{bmatrix} -5 \\ 21 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 1/3 \\ -3 & 1 & -2 \\ 0 & 2\pi & \pi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 21 \\ 11 \end{bmatrix}$$

Find Inverse of the co-efficients matrix

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5/8 & 1/12 & -113/8520 \\ 3/8 & 1/4 & 339/2840 \\ -3/4 & -1/2 & 113/1420 \end{bmatrix} \begin{bmatrix} -5 \\ 21 \\ 11 \end{bmatrix}$$

$$= \begin{bmatrix} -1.52 \\ 4.68 \\ -5.87 \end{bmatrix}$$

$x = -1.52$, $y = 4.68$, $z = -5.87$

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1.5.14 Are the following equations linearly independent?

a) $x_1 + 2x_2 + x_3 = 30$

b) $3x_1 + 6x_2 + 9x_3 = 4.5$

c) $2x_1 + 4x_2 + 15x_3 = 7.5$.

$x_1 + 2x_2 + x_3 = 30$

$3x_1 + 6x_2 + 9x_3 = 4.5$

$2x_1 + 4x_2 + 15x_3 = 7.5$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 1 & 9 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 4.5 \\ 7.5 \end{bmatrix}$$

Find the rref of the co-efficient matrix

$$\text{rref} = \begin{bmatrix} 1 & 0 & -4.5 \\ 0 & 1 & 2.16 \\ 0 & 0 & 0 \end{bmatrix}$$

The equations are linearly dependant as the geometric multiplicity is 2, not 3.

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1.5.15 Write computer commands (or a program) to solve for x, y and z from the following equations with r as an input variable. Your program should display an error message if, for a particular r, the equations are not linearly independent.

a) $5x + 2r\,y + z = 2$

b) $3x + 6y + (2r - 1)z = 3$

c) $2x + (r - 1)y + 3r\,z = 5.$

Find the solutions for $r = 3$, 4.99, and 5.

Code :

```
syms r x y z

A = [5 3 2; 2*r 6 (r-1); 1 (2*r)-1 3*r];

B = [x; y; z];

C = [2; 3; 5];

% For r = 3 %

disp("r = 3")
solve(subs(A,r,3)*B==C)

% For r = 4.99 %

disp("r = 4.99")
solve(subs(A,r,4.99)*B==C)

% For r = 5 %
disp("r = 5")
solve(subs(A,r,5)*B==C)
disp("Hence this cannot be solved for r = 5")
```

The solution to the question is :

For r $=$ 3 :

$$x = \frac{1}{16},\ y = \frac{5}{16},\ z = \frac{3}{8}$$

For r $=$ 4.99 :

$$x = \frac{67499}{2395},\ y = \frac{-180553}{2395},\ z = \frac{104602}{2395}$$

For r = 5 :

No solution can be found.

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1.5.16 An exam problem in statics has three unknown forces. A student writes the following three equations (he knows that he needs three equations for three unknowns!) — one for the force balance in the x-direction and the other two for the moment balance about two different points.

a) $F_1 - \frac{1}{2}F_2 + \frac{1}{\sqrt{2}}F_3 = 0$

b) $2F_1 + \frac{3}{2}F_2 = 0$

c) $\frac{5}{2}F_2 + \sqrt{2}F_3 = 0.$

Can the student solve for F_1, F_2, and F_3 uniquely from these equations?

Answer:
Yes.

$$\begin{bmatrix} 1 & 2 & 0 \\ -1/2 & 3/2 & 5/2 \\ 1/\sqrt{2} & 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Find and multiply the equation with the inverse of the co-efficients matrix.

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 0.3 & -0.4 & 0.7071 \\ 0.35 & 0.2 & -0.3536 \\ -0.15 & 0.2 & 0.3536 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow F_1 = F_2 = F_3 = 0$$

Hence, the student cannot solve for these forces unless the magnitude of all 3 forces are 0.

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1.5.17 What is the solution to the set of equations:

$$
\begin{aligned}
x + y + z + w &= 0 \\
x - y + z - w &= 0 \\
x + y - z - w &= 0 \\
x + y + z - w &= 2?
\end{aligned}
$$

$$
\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} =
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}
$$

Find and mutiply vector equation with the inverse of the coefficient matrix.

$$
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} =
\begin{bmatrix} 1/2 & 0 & 0 & 1/2 \\ 1/2 & -1/2 & 0 & 0 \\ 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1/2 & -1/2 \end{bmatrix}
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}
$$

$$
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} =
\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}
$$

Hence, $x = 1$, $y = z = 0$, $w = -1$

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