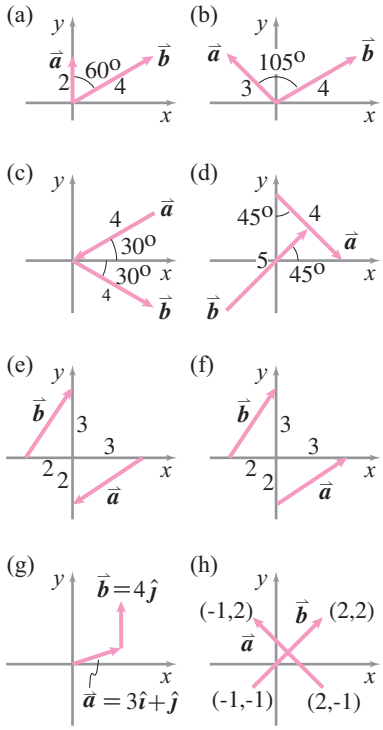
1.3 Vector Cross Product

1.3.1 Find the cross product of the two vectors shown in the figures below from the information given in the figures.



Problem 1.3.1

1.3.2 Vector algebra. For each equation below state whether:

1. The equation is nonsense. If so, why?
2. Is always true. Why? Give an example.
3. Is never true. Why? Give an example.
4. Is sometimes true. Give examples both ways.

You may use trivial examples.

a) $\vec{B}\times\vec{C}=\vec{C}\times\vec{B}$

b) $\vec{B}\times\vec{C}=\vec{C}\cdot\vec{B}$

c) $\vec{C}\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\vec{C}\times\vec{A})$

d) $\vec{A}\times(\vec{B}\times\vec{C})=(\vec{A}\cdot\vec{C})\vec{B}-(\vec{A}\cdot\vec{B})\vec{C}$

1.3.3 What do you get when you cross a vector and a scalar? *

1.3.4 Carry out the following cross products in different ways and determine which method takes the least amount of time for you.

a) $\vec{r}=2.0\,\text{ft}\,\hat{\imath}+3.0\,\text{ft}\,\hat{\jmath}-1.5\,\text{ft}\,\hat{k}$; $\vec{F}=-0.3\,\text{lbf}\,\hat{\imath}-1.0\,\text{lbf}\,\hat{k}$; $\vec{r}\times\vec{F}=$?

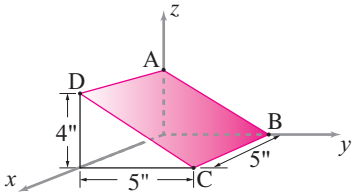
b) $\vec{r}=(-\hat{\imath}+2.0\hat{\jmath}+0.4\hat{k})\,\text{m}$; $\vec{L}=(3.5\hat{\jmath}-2.0\hat{k})\,\text{kg m/s}$; $\vec{r}\times\vec{L}=$?

c) $\vec{\omega}=(\hat{\imath}-1.5\hat{\jmath})\,\text{rad/s}$; $\vec{r}=(10\hat{\imath}-2\hat{\jmath}+3\hat{k})\in$; $\vec{\omega}\times\vec{r}=$?

1.3.5 Cross Product program Write a program that will calculate cross products. The input to the function should be the components of the two vectors and the output should be the components of the cross product. As a model, here is a function file that calculates dot products in pseudo code.

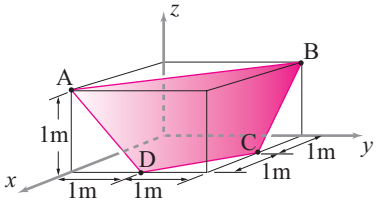
```
%program definition
z(1)=a(1)*b(1);
z(2)=a(2)*b(2);
z(3)=a(3)*b(3);
w=z(1)+z(2)+z(3);
```

1.3.6 Find a unit vector normal to the surface ABCD shown in the figure.



Problem 1.3.6

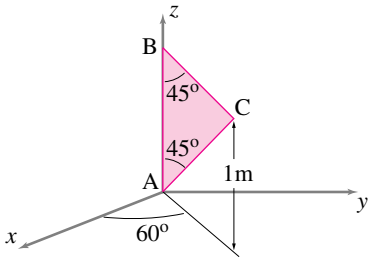
1.3.7 If the magnitude of a force $\vec{N}$ normal to the surface ABCD in the figure is 1000 N, write $\vec{N}$ as a vector. *



Problem 1.3.7

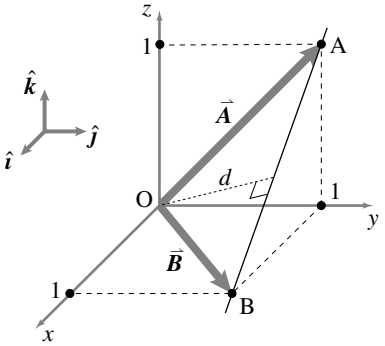
1.3.8 The equation of a surface is given as $z=2x-y$. Find a unit vector $\hat{n}$ normal to the surface.

1.3.9 In the figure, a triangular plate ACB, attached to rod AB, rotates about the z-axis. At the instant shown, the plate makes an angle of 60° with the x-axis. Find and draw a vector normal to the surface ACB.



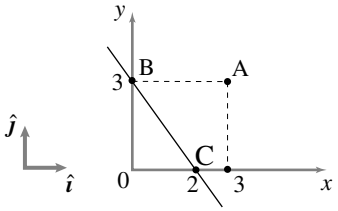
Problem 1.3.9

1.3.10 What is the distance d between the origin and the line AB shown? (You may write your solution in terms of $\vec{A}$ and $\vec{B}$ before doing any arithmetic). *



Problem 1.3.10

1.3.11 What is the perpendicular distance between point A and line BC shown? (There are at least 3 ways to do this using various vector products, how many ways can you find?)



Problem 1.3.11

1.3.12 Given a force, $\vec{F}_1=(-3\hat{\imath}+2\hat{\jmath}+5\hat{k})$ N acting at a point P whose position is given by $\vec{r}_{P/O}=(4\hat{\imath}-2\hat{\jmath}+7\hat{k})$ m, what is the moment about an axis through the origin O with direction $\hat{\lambda}=\frac{2}{\sqrt{5}}\hat{\imath}+\frac{1}{\sqrt{5}}\hat{\jmath}$?

1.3.13 A, B, and C are located by position vectors $\vec{r}_A = (1, 2, 3)$, $\vec{r}_B = (4, 5, 6)$, and $\vec{r}_C = (7, 8, 10)$.

a) Use the vector dot product to find the angle BAC (A is at the vertex of this angle).

b) Use the vector cross product to find the angle BCA (C is at the vertex of this angle).

c) Find a unit vector perpendicular to the plane ABC.

d) How far is the infinite line defined by AB from the origin? (That is, how close is the closest point on this line to the origin?)

e) Is the origin co-planar with the points A, B, and C?

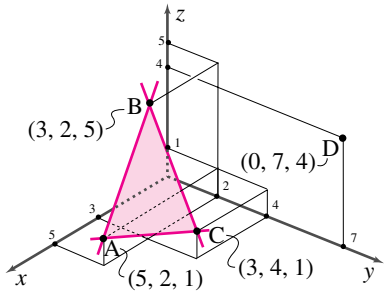
1.3.14 Points A, B, and C in the figure define an infinite plane.

a) Find a unit normal vector to the plane. *

b) Find the perpendicular distance from point D to this infinite plane (not necessarily inside the triangle ABC). *

c) What are the coordinates of the point on the plane closest to point D? *

d) Is this point on or off the triangle used to define the plane?



Problem 1.3.14

1.3.15 What point on the line that goes through the points (1,2,3) and (7,12,15) is closest to the origin?

1.3.16 A regular tetrahedron is a triangular-based pyramid where all 6 edges have the same length ℓ. What is the perpendicular distance between a pair of non-touching edges? (There are many ways to solve this problem). *

1.3.17 Why did the chicken cross the road? *