1.2 The dot product of two vectors

1.2.1 Find the dot product of $\vec{\boldsymbol{a}} = 2\hat{\imath} + 3\hat{\jmath} - \hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{b}} = 2\hat{\imath} + \hat{\jmath} + 2\hat{\boldsymbol{k}}$.

1.2.2 Find the dot product of $\vec{\boldsymbol{F}} = 0.5\,\text{N}\hat{\imath} + 1.2\,\text{N}\hat{\jmath} + 1.5\,\text{N}\hat{\boldsymbol{k}}$ and $\hat{\boldsymbol{\lambda}} = -0.8\hat{\imath} + 0.6\hat{\jmath}$.

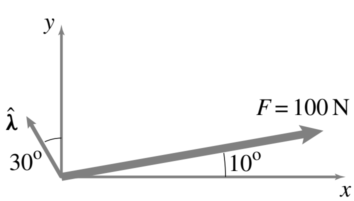
1.2.3 Find the dot product $\vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{r}}$ where $\vec{\boldsymbol{F}} = (5\hat{\imath} + 4\hat{\jmath})\,\text{N}$ and $\vec{\boldsymbol{r}} = (-0.8\hat{\imath} + \hat{\jmath})\,\text{m}$. Draw the two vectors and justify your answer for the dot product.

1.2.4 Two vectors, $\vec{\boldsymbol{a}} = -4\sqrt{3}\hat{\imath} + 12\hat{\jmath}$ and $\vec{\boldsymbol{b}} = \hat{\imath} - \sqrt{3}\hat{\jmath}$ are given. Find the dot product of the two vectors. How is $\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{b}}$ related to $|\vec{\boldsymbol{a}}||\vec{\boldsymbol{b}}|$ in this case?

1.2.5 Find the dot product of two vectors $\vec{\boldsymbol{F}} = 10\,\text{lbf}\hat{\imath} - 20\,\text{lbf}\hat{\jmath}$ and $\hat{\boldsymbol{\lambda}} = 0.8\hat{\imath} + 0.6\hat{\jmath}$. Sketch $\vec{\boldsymbol{F}}$ and $\hat{\boldsymbol{\lambda}}$ and show what their dot product represents .

1.2.6 The position vector of a point A is $\vec{\boldsymbol{r}}_A = 30\,\text{cm}\hat{\imath}$. Find the dot product of $\vec{\boldsymbol{r}}_A$ with $\hat{\boldsymbol{\lambda}} = \frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$.

1.2.7 From the figure below, find the component of force $\vec{\boldsymbol{F}}$ in the direction of $\hat{\boldsymbol{\lambda}}$.



Problem 1.2.7

1.2.8 Find the angle between $\vec{\boldsymbol{F}}_1 = 2\,\text{N}\hat{\imath} + 5\,\text{N}\hat{\jmath}$ and $\vec{\boldsymbol{F}}_2 = -2\,\text{N}\hat{\imath} + 6\,\text{N}\hat{\jmath}$.

1.2.9 Given $\vec{\boldsymbol{\omega}} = 2\,\text{rad/s}\hat{\imath} + 3\,\text{rad/s}\hat{\jmath}$, $\vec{\boldsymbol{H}}_1 = (20\hat{\imath} + 30\hat{\jmath})\,\text{kg}\,\text{m}^2/\,\text{s}$ and $\vec{\boldsymbol{H}}_2 = (10\hat{\imath} + 15\hat{\jmath} + 6\hat{\boldsymbol{k}})\,\text{kg}\,\text{m}^2/\,\text{s}$, find (a) the angle between $\vec{\boldsymbol{\omega}}$ and $\vec{\boldsymbol{H}}_1$ and (b) the angle between $\vec{\boldsymbol{\omega}}$ and $\vec{\boldsymbol{H}}_2$.

1.2.10 The unit normal to a surface is given as $\hat{\boldsymbol{n}} = 0.74\hat{\imath} + 0.67\hat{\jmath}$. If the weight of a block on this surface acts in the $-\hat{\jmath}$ direction, find the angle that a 1000 N normal force makes with the direction of weight of the block.

1.2.11 Vector algebra. For each equation below state whether:

1. The equation is nonsense. If so, why?
2. Is always true. Why? Give an example.
3. Is never true. Why? Give an example.
4. Is sometimes true. Give examples both ways.

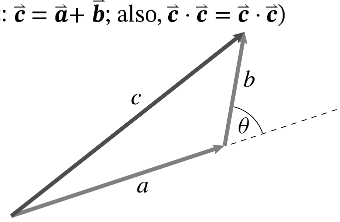
You may use trivial examples.

a) $\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}}$
b) $\vec{\boldsymbol{A}} + b = b + \vec{\boldsymbol{A}}$
c) $\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{B}} = \vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{A}}$
d) $\vec{\boldsymbol{B}}/\vec{\boldsymbol{C}} = B/C$
e) $b/\vec{\boldsymbol{A}} = b/A$
f) $\vec{\boldsymbol{A}} = (\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{B}})\vec{\boldsymbol{B}} + (\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{C}})\vec{\boldsymbol{C}} + (\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{D}})\vec{\boldsymbol{D}}$

1.2.12 Use the dot product to show 'the law of cosines'; i. e.,

$$c^2 = a^2 + b^2 + 2ab\cos\theta.$$

(Hint: $\vec{\boldsymbol{c}} = \vec{\boldsymbol{a}} + \vec{\boldsymbol{b}}$; also, $\vec{\boldsymbol{c}}\cdot\vec{\boldsymbol{c}} = \vec{\boldsymbol{c}}\cdot\vec{\boldsymbol{c}}$)



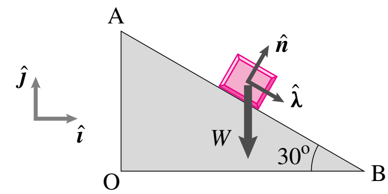
Problem 1.2.12

1.2.13 Find the direction cosines of $\vec{\boldsymbol{F}} = 3\,\text{N}\hat{\imath} - 4\,\text{N}\hat{\jmath} + 5\,\text{N}\hat{\boldsymbol{k}}$.

1.2.14 A force acting on a bead of mass m is given as $\vec{\boldsymbol{F}} = -20\,\text{lbf}\hat{\imath} + 22\,\text{lbf}\hat{\jmath} + 12\,\text{lbf}\hat{\boldsymbol{k}}$. What is the angle between the force and the z-axis?

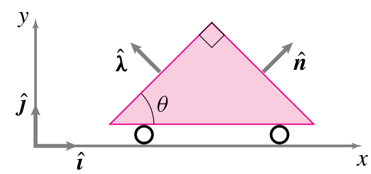
1.2.15 (a) Draw the vector $\vec{\boldsymbol{r}} = 3.5 \in \hat{\imath} + 3.5 \in \hat{\jmath} - 4.95 \in \hat{\boldsymbol{k}}$. (b) Find the angle this vector makes with the z-axis. (c) Find the angle this vector makes with the x-y plane.

1.2.16 In the figure shown, $\hat{\boldsymbol{\lambda}}$ and $\hat{\boldsymbol{n}}$ are unit vectors parallel and perpendicular to the surface AB, respectively. A force $\vec{\boldsymbol{W}} = -50\,\text{N}\hat{\jmath}$ acts on the block. Find the components of $\vec{\boldsymbol{W}}$ along $\hat{\boldsymbol{\lambda}}$ and $\hat{\boldsymbol{n}}$.



Problem 1.2.16

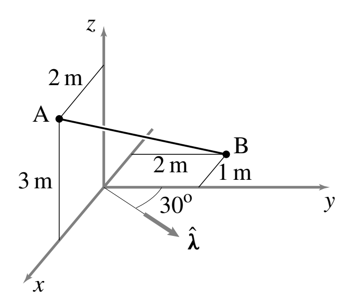
1.2.17 Express the unit vectors $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{\lambda}}$ in terms of $\hat{\imath}$ and $\hat{\jmath}$ shown in the figure. What are the x and y components of $\vec{\boldsymbol{r}} = 3.0\,\text{ft}\hat{\boldsymbol{n}} - 1.5\,\text{ft}\hat{\boldsymbol{\lambda}}$? *



Problem 1.2.17

1.2.18 Find the projection of vector $\vec{\boldsymbol{r}}_{AB}$ (you have to first find this position vector)

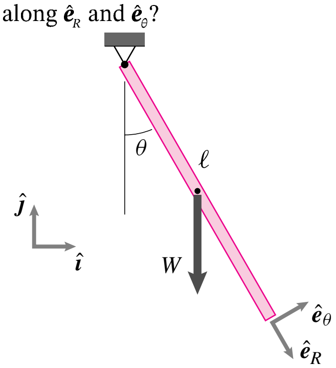
1. on the y-axis, and
2. in the $\hat{\boldsymbol{\lambda}}$ direction.



Problem 1.2.18

1.2.19 The net force acting on a particle is $\vec{\boldsymbol{F}} = 2\,\text{N}\hat{\imath} + 10\,\text{N}\hat{\jmath}$. Find the components of this force in another coordinate system with basis vectors $\hat{\imath}' = -\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}$ and $\hat{\jmath}' = -\sin\theta\hat{\imath} - \cos\theta\hat{\jmath}$. For $\theta = 30^\circ$, sketch the vector $\vec{F}$ and show its components in the two coordinate systems.

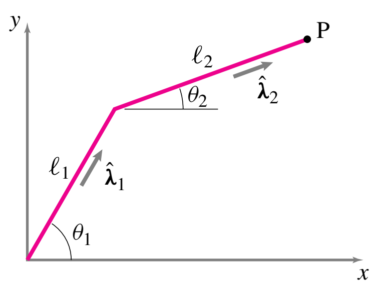
1.2.20 Find the unit vectors $\hat{e}_R$ and $\hat{e}_\theta$ in terms of $\hat{\imath}$ and $\hat{\jmath}$ with the geometry shown in the figure. What are the components of $\vec{W}$ along $\hat{e}_R$ and $\hat{e}_\theta$?



Problem 1.2.20

1.2.21 Write the position vector of point P in terms of $\hat{\lambda}_1$ and $\hat{\lambda}_2$ and

1. find the y-component of $\vec{r}_P$,
2. find the component of $\vec{r}_P$ along $\hat{\lambda}_1$.



Problem 1.2.21

1.2.22
Let $\vec{F}_1 = 30\,\mathrm{N}\hat{\imath} + 40\,\mathrm{N}\hat{\jmath} - 10\,\mathrm{N}\hat{k}$, $\vec{F}_2 = -20\,\mathrm{N}\hat{\jmath} + 2\,\mathrm{N}\hat{k}$, and $\vec{F}_3 = F_{3_x}\hat{\imath} + F_{3_y}\hat{\jmath} - F_{3_z}\hat{k}$. If the sum of all these forces must equal zero, find the required scalar equations to solve for the components of $\vec{F}_3$.

1.2.23 A force $\vec{F}$ is directed from point A(3,2,0) to point B(0,2,4). If the x-component of the force is 120 N, find the y- and z-components of $\vec{F}$.

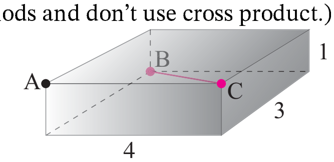
1.2.24 A vector equation for the sum of forces results in the following equation:

$$\frac{F}{2}(\hat{\imath} - \sqrt{3}\hat{\jmath}) + \frac{R}{5}(3\hat{\imath} + 6\hat{\jmath}) = 25\,\mathrm{N}\hat{\lambda}$$

where $\hat{\lambda} = 0.30\hat{\imath} - 0.954\hat{\jmath}$. Find two scalar equations by dotting both sides of the equation first with $\hat{\lambda}$ and then separately with a vector orthogonal to $\hat{\lambda}$.

1.2.25 Write a computer program (or use a canned program) to find the dot product of two 3-D vectors. Test the program by computing the dot products $\hat{\imath}\cdot\hat{\imath}, \hat{\imath}\cdot\hat{\jmath}$, and $\hat{\jmath}\cdot\hat{k}$. Now use the program to find the projections of $\vec{F} = (2\hat{\imath} + 2\hat{\jmath} - 3\hat{k})\,\mathrm{N}$ along the line $\vec{r}_{AB} = (0.5\hat{\imath} - 0.2\hat{\jmath} + 0.1\hat{k})\,\mathrm{m}$.

1.2.26 What is the shortest distance between the point A and the diagonal BC of the parallelepiped shown? (Use vector methods and don't use cross product.)



Problem 1.2.26