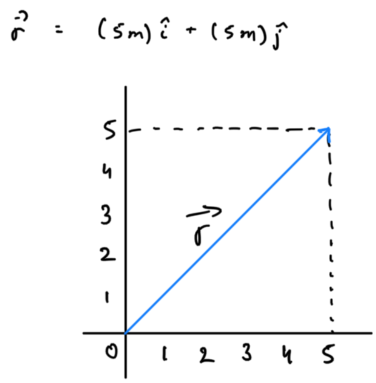
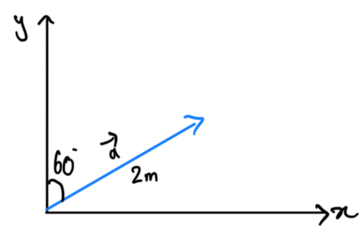
1.1.1 Draw the vector $\vec{\boldsymbol{r}} = (5\,\mathrm{m})\hat{\boldsymbol{\imath}}+(5\,\mathrm{m})\,\hat{\boldsymbol{\jmath}}$.



1.1.2 A vector $\vec{a}$ is 2 m long and points northwest at an angle 60° from the north. Draw the vector.

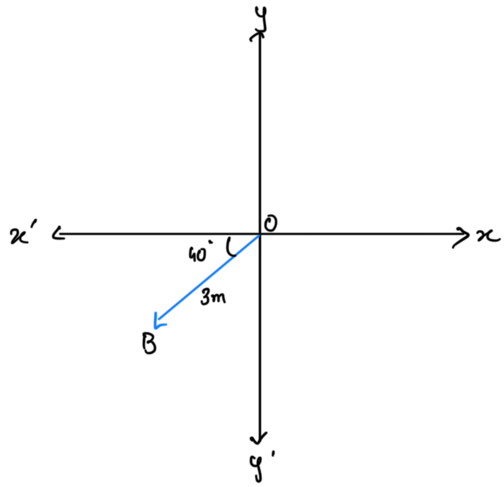
$|\vec{a}| = 2m$

Angle from North : 60°



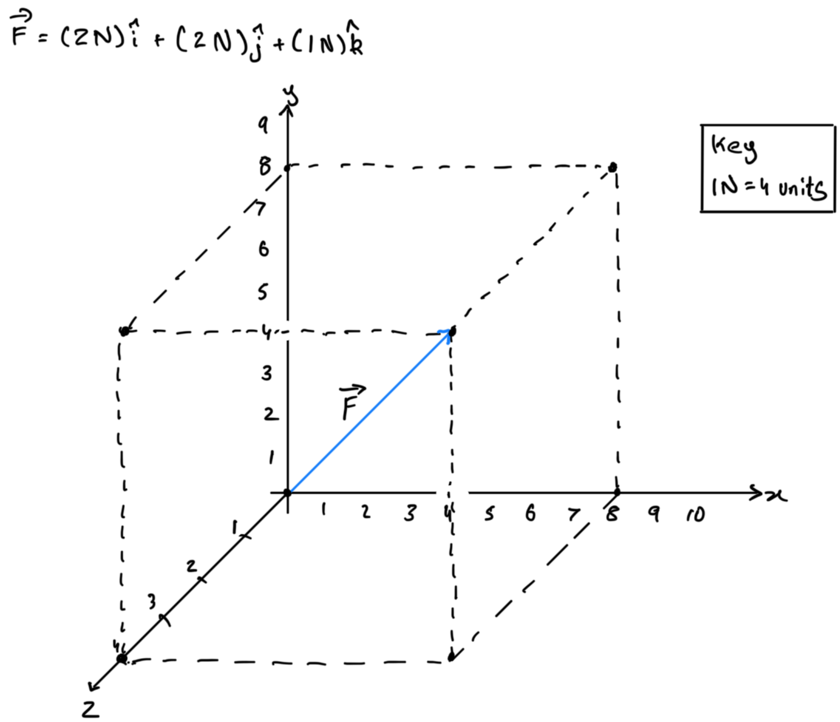
1.1.3 The position vector of a point B measures 3 m and is directed at 40° CCW from the negative x-axis. Show the position vector.

$|\vec{OB}| = 3m$



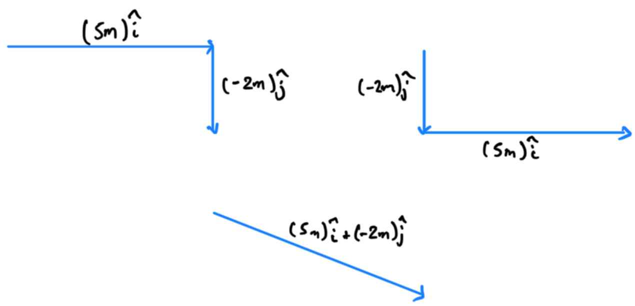
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.4 Draw a force vector that is given as
$\vec{F} = 2\,\text{N}\hat{\imath} + 2\,\text{N}\hat{\jmath} + 1\,\text{N}\hat{k}$.



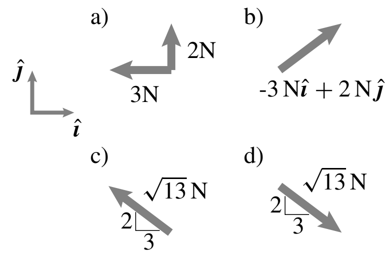
1.1.5 Represent the vector $\vec{r} = 5\,\mathrm{m}\hat{\imath} - 2\,\mathrm{m}\hat{\jmath}$ in three different ways.

$\vec{r} = (5m)\hat{\imath} - (2m)\hat{\jmath}$



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.6 Which one of the following representations of the same vector $\vec{F}$ is wrong and why?



Problem 1.6

In the given question (d) represents a different vector.

In the case of (a) the vector can be written as,

$$\vec{F} = (-3N)\hat{\imath} + (2N)\hat{\jmath}$$
$$|\vec{F}| = \sqrt{9N^2 + 4N^2}$$
$$= \sqrt{13}\ N$$

In the case of (c) the vector can be written as,

$$\vec{F} = (-3N)\hat{\imath} + (2N)\hat{\jmath}$$
$$|\vec{F}| = \sqrt{13}\ N$$

Hence the two represent the same vector.

In the case of (b) the vector can be written as,

$$\vec{F} = (-3N)\hat{\imath} + (2N)\hat{\jmath}$$
$$|\vec{F}| = \sqrt{13}\ N$$

Since, the vector equation is equal to (a) and (c), (b) represents the same vector.

In the case of (d) the vector can be written as,

$$\vec{F} = (3N)\hat{\imath} + (-2N)\hat{\jmath}$$
$$|\vec{F}| = \sqrt{13}\ N$$

Here, the vector equation is not the same as in case (a), (b) and (c), and represents a different vector.

Hence (a), (b) and (c) represent the same vector.

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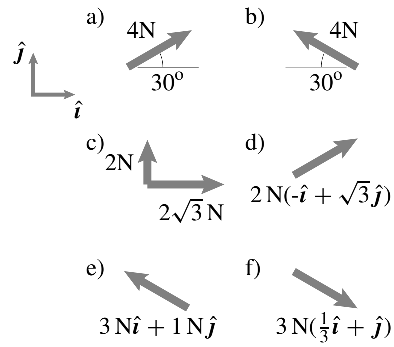
1.1.7 There are exactly two pairs that describe the same vector in the following pictures. The other two don't match. Match the correct pictures into pairs.



Problem 1.7

(a) and (c) correspond to the same vector,

Vector Equation for (a) : $((4N)\cos 30)\hat{\imath} + ((4N)\sin 30)\hat{\jmath}$
$= (2\sqrt{3}N)\hat{\imath} + (2N)\hat{\jmath}$

Vector Equation for (c) : $(2\sqrt{3}N)\hat{\imath} + (2N)\hat{\jmath}$

Since the vector equations are equal, the two represent the same vector.

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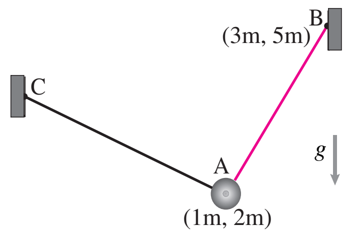
1.1.8 A string connects a particle A at (1m, 2m) to a support B at (3m, 5m). The tension in the string is 10N. There are other strings also holding the particle in place. What is the force of string AB on the particle?



Problem 1.8

For a perfect string, the tension is equal at all points on the string.

Hence, the force applied by the string on the particle is 10N.

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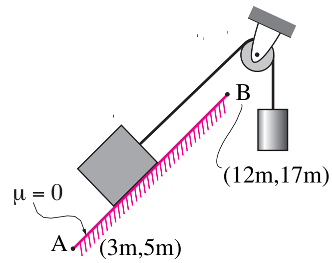
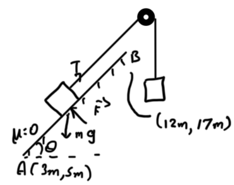
1.1.9 A frictionless ramp connects A at (3m, 5m) to B at (12m, 17m). The ramp pushes a block with a force of 50N normal to the ramp surface. Express the force from the ramp as a vector $\vec{F}$ (ignore the other forces that also act on the block holding it in place).



Problem 1.9



$|\vec{F}| = 50N$

$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

$\vec{r}_a = 3m\hat{i} + 5m\hat{j}$

$\vec{r}_B = 12m\hat{i} + 17m\hat{j}$

$\vec{r}_{AB} = 9m\hat{i} + 12m\hat{j}$

$|\vec{r}_{AB}| = \sqrt{(9m)^2 + (12m)^2}$

$= 15m$

$\hat{\lambda}_{AB} = 3/5\,\hat{i} + 4/5\,\hat{j}$

Find unit vector perpendicular to $\overrightarrow{AB}$,

$\hat{\lambda}^{\perp}_{AB} = -3/5\,\hat{i} + 4/5\,\hat{j}$

$\vec{F} = 50N \times \vec{\lambda}^{\perp}_{AB}$

$= -30N\hat{i} + 40N\hat{j}$

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1.1.10 Find the sum of forces $\vec{F}_1 = 20\,\text{N}\hat{\imath} - 2\,\text{N}\hat{\jmath}$, $\vec{F}_2 = 30\,\text{N}(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath})$, and $\vec{F}_3 = -20\,\text{N}(-\hat{\imath} + \sqrt{3}\hat{\jmath})$.

$$\vec{F}_1 = (20\text{N})\hat{\imath} + (2\text{N})\hat{\jmath}$$

$$\vec{F}_2 = (30\text{N})\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right)$$

$$= (15\sqrt{2}\,\text{N})\hat{\imath} + (15\sqrt{2}\,\text{N})\hat{\jmath}$$

$$\vec{F}_3 = (-20\text{N})(-\hat{\imath} + \sqrt{3}\hat{\jmath})$$

$$= (20\text{N})\hat{\imath} - (20\sqrt{3}\,\text{N})\hat{\jmath}$$

Add $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$,

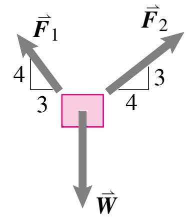
$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= (20\text{N})\hat{\imath} + (2\text{N})\hat{\jmath} + (15\sqrt{2}\,\text{N})\hat{\imath} + (15\sqrt{2}\,\text{N})\hat{\jmath} + (20\text{N})\hat{\imath} - (20\sqrt{3}\,\text{N})\hat{\jmath}$$

$$= (20\text{N} + 15\sqrt{2}\,\text{N} + 20\text{N})\hat{\imath} + (2\text{N} + 15\sqrt{2}\,\text{N} - 20\sqrt{3}\,\text{N})\hat{\jmath}$$

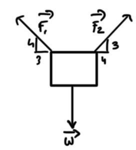
$$= 5\text{N}(8 + 3\sqrt{2})\hat{\imath} + (2 + 15\sqrt{2} - 20\sqrt{3})\text{N}\hat{\jmath}$$

1.1.11 The forces acting on a block of mass $m = 5\,\text{kg}$ are shown in the figure, where $F_1 = 20\,\text{N}$, $F_2 = 50\,\text{N}$, and $W = mg$. Find the sum $\vec{F}$ $(= \vec{F}_1 + \vec{F}_2 + \vec{W})$.



Problem 1.11

Find vectors $\vec{F_1}$ and $\vec{F_2}$



$$\hat{F_1} = \frac{1}{\sqrt{3^2+4^2}}(-3\hat{i} + 4\hat{j})$$

$$= \frac{-3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

$$\vec{F_1} = 20\,\text{N} \times \hat{F_1}$$

$$= -12\,\text{N}\,\hat{i} + 16\,\text{N}\,\hat{i}$$

$$\hat{F_2} = \frac{1}{\sqrt{3^2+4^2}}(4\hat{i} + 3\hat{j})$$

$$= \frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$$

$$\vec{F_2} = 50\,\text{N} \times \hat{F_2}$$

$$= 40\,\text{N}\,\hat{i} + 30\,\text{N}\,\hat{j}$$

$$\vec{w} = -mg\,\hat{j}$$

$$= -5\,\text{kg} \times 10\,\text{m}\,\text{s}^{-2} \times \hat{j}$$

$$= -50\,\text{N}\,\hat{j}$$

Sum Forces,

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{w}$$

$$= -12\,\text{N}\,\hat{i} + 16\,\text{N}\,\hat{j} + 40\,\text{N}\,\hat{i} + 30\,\text{N}\,\hat{j} - 50\,\text{N}\,\hat{j}$$

$$= 28\,\text{N}\,\hat{i} - 4\,\text{N}\,\hat{j}$$

$$= 4\,\text{N}(7\hat{i} - \hat{j})$$

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1.1.12 Given that the sum of four vectors $\vec{F}_i, i = 1$ to 4, is zero, where $\vec{F}_1 = 20\,\mathrm{N}\hat{\imath}, \vec{F}_2 = -50\,\mathrm{N}\hat{\jmath}, \vec{F}_3 = 10\,\mathrm{N}(-\hat{\imath} + \hat{\jmath})$, find $\vec{F}_4$.

$$\vec{F}_1 = 20\mathrm{N}\hat{\imath}$$

$$\vec{F}_2 = -50\mathrm{N}\hat{\jmath}$$

$$\vec{F}_3 = 10\mathrm{N}(-\hat{\imath} + \hat{\jmath})$$

$$= -10\mathrm{N}\hat{\imath} + 10\mathrm{N}\hat{\jmath}$$

Sum Forces and Equate to 0.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

$$\Rightarrow 20\mathrm{N}\hat{\imath} - 50\mathrm{N}\hat{\jmath} - 10\mathrm{N}\hat{\imath} + 10\mathrm{N}\hat{\jmath} + \vec{F}_4 = 0$$

$$\Rightarrow 10\mathrm{N}\hat{\imath} - 40\mathrm{N}\hat{\jmath} + \vec{F}_4 = 0$$

$$\Rightarrow \vec{F}_4 = -10\mathrm{N}(\hat{\imath} - 4\hat{\jmath})$$

1.1.13 Three forces $\vec{F} = 2\,\mathrm{N}\hat{\imath} - 5\,\mathrm{N}\hat{\jmath}$, $\vec{R} = 10\,\mathrm{N}(\cos\theta\hat{\imath} + \sin\theta\hat{\jmath})$ and $\vec{W} = W\,\mathrm{N}\hat{\jmath}$ with $W > 0$, sum up to zero. Determine θ and W and draw the force vector $\vec{R}$ clearly showing its direction.

$$\vec{F} = 2N\hat{i} - 5N\hat{j}$$

$$\vec{R} = 10N(\cos\theta\hat{i} + \sin\theta\hat{j})$$

$$\vec{W} = WN\hat{j}$$

$$\vec{F} + \vec{R} + \vec{W} = 0$$

$$2N\hat{i} - 5N\hat{j} + 10N(\cos\theta\,\hat{i} + \sin\theta\hat{j}) + WN\hat{j}$$

Sum $\hat{i}$ terms and equate to 0

$$2N\hat{i} + 10N\cos\theta\,\hat{i} = 0$$

$$2N(1 + 5\cos\theta)\hat{i} = 0$$

$$1 + 5\cos\theta = 0$$

$$\cos\theta = -1/5$$

$$\theta = \cos^{-1}(1/5)$$

$$= 1.37$$

Sum $\hat{j}$ terms and equate to 0

$$-5N\hat{j} + 10N\sin\theta\hat{j} + WN\hat{j} = \vec{0}$$

$$\left[-5N + 10N\ \sin\left(\cos^{-1}\left(\frac{1}{5}\right)\right) + WN\right]\hat{j} = \vec{0}$$

$$\left(-5N + 10 \times \frac{2\sqrt{6}}{5} + WN\right)\hat{j} = \vec{0}$$

$$W = (5 - 4\sqrt{6})N$$

$$= -4.79N$$

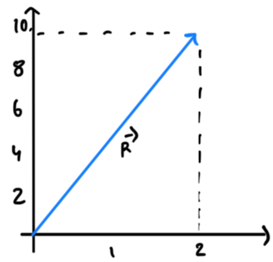
$$\boxed{\begin{aligned} &\cos^2\theta + \sin^2\theta = 1 \\ &\frac{1}{25} + \sin^2\theta = 1 \\ &\sin^2\theta = \frac{24}{25} \\ &\sin\theta = \frac{2\sqrt{6}}{5} \end{aligned}}$$

$$\vec{R} = 10N(\cos\theta\hat{i} + \sin\theta\hat{j})$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$= 10N\left(\frac{1}{5}\hat{i} + \frac{2\sqrt{6}}{5}\hat{j}\right)$$

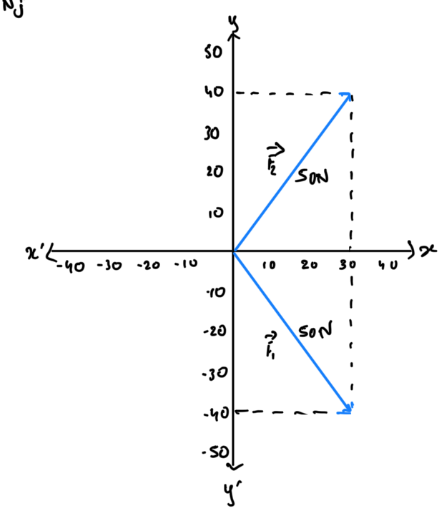
$$= 2N\hat{i} + 4\sqrt{6}\,N\hat{j}$$



1.1.15 Find the magnitudes of the forces $\vec{F}_1 = 30\,\mathrm{N}\hat{\imath} - 40\,\mathrm{N}\hat{\jmath}$ and $\vec{F}_2 = 30\,\mathrm{N}\hat{\imath} + 40\,\mathrm{N}\hat{\jmath}$. Draw the two forces, representing them with their magnitudes.

$\vec{F_1} = 30\,N\hat{i} - 40\,N\hat{j}$

$\vec{F_2} = 30\,N\hat{i} + 40\,N\hat{j}$



Find Magnitude of Forces.

$|\vec{F_1}| = \sqrt{(30N)^2+(-40N)^2}$

$= 50\,N$

$|\vec{F_2}| = \sqrt{(30\,N)^2+(40\,N)^2}$

$= 50N$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.16 Two forces $\vec{\boldsymbol{R}} = 2\,\mathrm{N}(0.16\hat{\boldsymbol{\imath}} + 0.80\hat{\boldsymbol{\jmath}})$ and $\vec{\boldsymbol{W}} = -36\,\mathrm{N}\hat{\boldsymbol{\jmath}}$ act on a particle. Find the magnitude of the net force. What is the direction of this force?

$\vec{R} = 2N\,(0.16N\hat{i} + 0.8N\hat{j})$

$\vec{w} = -36\,N\hat{j}$

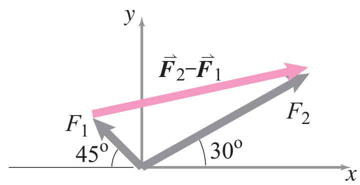
Add Vectors to find net force,

$$\begin{aligned}
\text{Net Force} &= \vec{R} + \vec{w} \\
&= 2N(0.16N\hat{i} + 0.8\hat{j}) - 36\,N\hat{j} \\
&= 0.32N\hat{i} + 1.6N\hat{j} - 36N\hat{j} \\
&= 0.32N\hat{i} - 34.4\,N\hat{j}
\end{aligned}$$

$$\begin{aligned}
\text{Magnitude of Force} &= \sqrt{(0.32\,N)^2 + (-34.4\,N)^2} \\
&= 34.4\,N
\end{aligned}$$

$$\begin{aligned}
\text{Unit Vector of Force} &= \frac{\vec{R} + \vec{w}}{|\vec{R} + \vec{w}|} \\
&= (9.3 \times 10^{-3})\hat{i} + \hat{j}
\end{aligned}$$

1.1.17 In the figure shown, $F_1 = 100\,\mathrm{N}$ and $F_2 = 300\,\mathrm{N}$. Find the magnitude and direction of $\vec{F}_2 - \vec{F}_1$.



Problem 1.17

$|\vec{F}_1| = 100\,\mathrm{N}$

$\theta_{F_1} = 135^\circ$

$|\vec{F}_2| = 300\,\mathrm{N}$

$\theta_{F_2} = 30^\circ$

$$\vec{F}_1 = (100\,\mathrm{N} \cos 135)\hat{\imath} + (100\,\mathrm{N} \sin 135)\hat{\jmath}$$
$$= (-50\sqrt{2}\,\mathrm{N})\hat{\imath} + (50\sqrt{2}\,\mathrm{N})\hat{\jmath}$$

$$\vec{F}_2 = (300\,\mathrm{N} \cos 30)\hat{\imath} + (300\,\mathrm{N} \sin 30)\hat{\jmath}$$
$$= (150\sqrt{3}\,\mathrm{N})\hat{\imath} + (150\,\mathrm{N})\hat{\jmath}$$

Subtract $\vec{F}_1$ from $\vec{F}_2$,

$$\vec{F}_2 - \vec{F}_1 = (150\sqrt{3}\,\mathrm{N})\hat{\imath} + (150\,\mathrm{N})\hat{\jmath} - (-50\sqrt{2}\,\mathrm{N})\hat{\imath} - (50\sqrt{2}\,\mathrm{N})\hat{\jmath}$$
$$= 50\,\mathrm{N}(3\sqrt{3} + \sqrt{2})\hat{\imath} + 50\,\mathrm{N}(3 - \sqrt{2})\hat{\jmath}$$

$$|\vec{F}_2 - \vec{F}_1| = \sqrt{(50\,\mathrm{N}(3\sqrt{3} + \sqrt{2}))^2 + (50(3 - \sqrt{2})\,\mathrm{N})^2}$$
$$= 339.89\,\mathrm{N}$$

Direction: $\dfrac{\vec{F}_2 - \vec{F}_1}{|\vec{F}_2 - \vec{F}_1|} = 0.972\,\hat{\imath} + 0.233\,\hat{\jmath}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

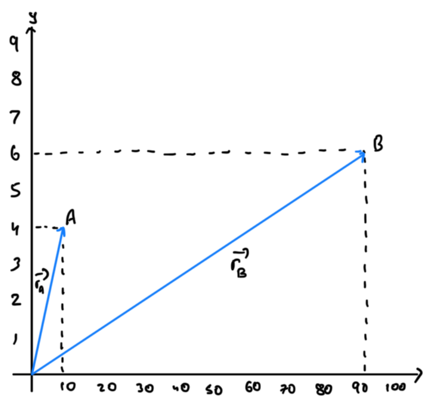
1.1.18 Two points A and B are located in the xy plane. The coordinates of A and B are (4 mm, 8 mm) and (90 mm, 6 mm), respectively.

1. Draw position vectors $\vec{r}_A$ and $\vec{r}_B$.
2. Find the magnitude of $\vec{r}_A$ and $\vec{r}_B$.
3. How far is A from B?

$A \equiv (4mm, 8mm)$

$B \equiv (90mm, 6mm)$

1.



2. Find magnitude of $\vec{r}_A$ and $\vec{r}_B$,

$$|\vec{r}_A| = \sqrt{(4mm)^2 + (8mm)^2}$$

$$= 4\sqrt{5}\ mm$$

$$|\vec{r}_B| = \sqrt{(6mm)^2 + (90mm)^2}$$

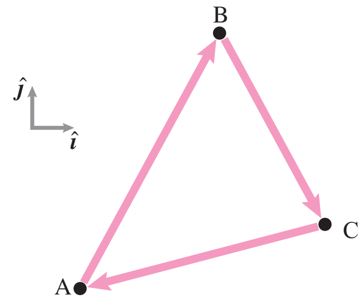
$$= 6\sqrt{226}\ mm$$

3. $|\vec{AB}| = \sqrt{(4mm - 6mm)^2 + (90mm - 8mm)^2}$

$$= 58\sqrt{2}\ mm$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.19 Three position vectors are shown in the figure below. Given that $\vec{r}_{B/A} = 3\,\text{m}(\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath})$ and $\vec{r}_{C/B} = 1\,\text{m}\hat{\imath} - 2\,\text{m}\,\hat{\jmath}$, find $\vec{r}_{A/C}$.



Problem 1.19

$\vec{r}_{B/A} = 3m\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$

$\vec{r}_{C/B} = 1m\hat{i} - 2m\hat{j}$

Find vector C from A,

$$\vec{r}_{C/A} = \vec{r}_{C/B} + \vec{r}_{B/A}$$

$$= 3m\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) + 1m\hat{i} - 2m\hat{j}$$

$$= \frac{3m}{2}\hat{i} + \frac{3\sqrt{3}}{2}m\hat{j} + 1m\hat{i} - 2m\hat{j}$$

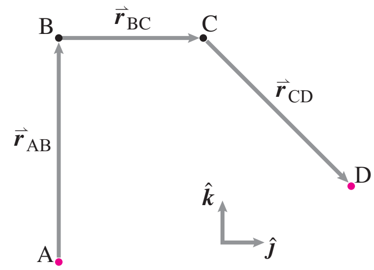
$$= \frac{5m}{2}\hat{i} + \frac{(3\sqrt{3}+2)m}{2}\hat{j}$$

$$\vec{r}_{A/C} = -\vec{r}_{C/A}$$

$$= -\frac{5m}{2}\hat{i} - \frac{(3\sqrt{3}+2)m}{2}\hat{j}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.20 In the figure shown below, the position vectors are $\vec{r}_{AB} = 3\,\text{ft}\,\hat{k}$, $\vec{r}_{BC} = 2\,\text{ft}\,\hat{j}$, and $\vec{r}_{CD} = 2(\hat{j} - \hat{k})\,\text{ft}$. Find the position vector $\vec{r}_{AD}$.



Problem 1.20

$\vec{r}_{AB} = 3\text{ft}\,\hat{k}$

$\vec{r}_{BC} = 2\text{ft}\,\hat{j}$

$\vec{r}_{CD} = 2(\hat{j}-\hat{k})\text{ft}$

Find vector from A to D,

$$\vec{r}_{AD} = \vec{r}_{AB} + \vec{r}_{BC} + \vec{r}_{CD}$$

$$= 3\text{ft}\,\hat{k} + 2\text{ft}\,\hat{j} + 2(\hat{j}-\hat{k})\text{ft}$$

$$= (2+2)\text{ft}\,\hat{j} + (3-2)\text{ft}\,\hat{k}$$

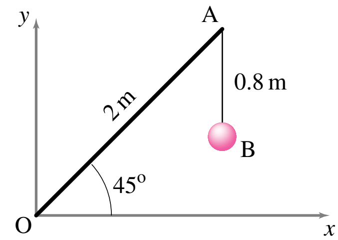
$$= 4\text{ft}\,\hat{j} + 1\text{ft}\,\hat{k}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.21 In the figure shown, a ball is suspended with a 0.8 m long cord from a 2 m long hoist OA.

1. Find the position vector $\vec{r}_B$ of the ball.

2. Find the distance of the ball from the origin.



Problem 1.21

1. $\vec{r}_A = (2m\cos 45)\hat{\imath} + (2m\sin 45)\hat{\jmath}$

$= (\sqrt{2}m)\hat{\imath} + (\sqrt{2}m)\hat{\jmath}$

Add $\vec{r}_{B/A}$ to $\vec{r}_A$,

$\vec{r}_B = \vec{r}_A + (-0.8m)\hat{\jmath}$

$= (\sqrt{2}\,m)\hat{\imath} + (\sqrt{2}m)\hat{\jmath} - (0.8m)\hat{\jmath}$

$= (\sqrt{2}\,m)\hat{\imath} + (\sqrt{2} - 0.8)\,m\hat{\jmath}$

2. Distance of ball from origin $= |\vec{r}_B|$

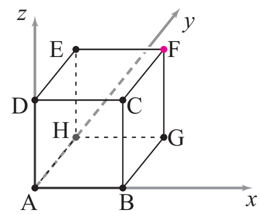
$= \sqrt{(\sqrt{2}m)^2 + (\sqrt{2} - 0.8)^2 m^2}$

$= 1.54\ m$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.22 A cube of side 6 ∈ is shown in the figure.

1. Find the position vector of point F, $\vec{r}_F$, from the vector sum $\vec{r}_F = \vec{r}_D + \vec{r}_{C/D} + \vec{r}_{F/c}$.
2. Calculate $|\vec{r}_F|$.
3. Find $\vec{r}_G$ using $\vec{r}_F$.



Problem 1.22

1. $\vec{r}_F = \vec{r}_D + \vec{r}_{C/D} + \vec{r}_{F/C}$

$\vec{r}_D = 6 \text{ in } \hat{k}$

$\vec{r}_{C/D} = 6 \text{ in } \hat{i}$

$\vec{r}_{F/C} = 6 \text{ in } \hat{j}$

$\vec{r}_F = (6\hat{k} + 6\hat{i} + 6\hat{j}) \text{ in}$

$= 6(\hat{i} + \hat{j} + \hat{k}) \text{ in}$

2. $|\vec{r}_F| = \sqrt{(6 \text{ in})^2 + (6 \text{ in})^2 + (6 \text{ in})^2}$

$= \sqrt{108} \text{ in}$

$= 6\sqrt{3} \text{ in}$

3. Subtract $\vec{r}_{G/F}$ from $\vec{r}_F$

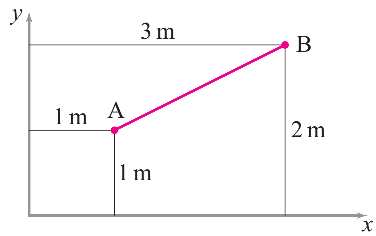
$\vec{r}_G = \vec{r}_F - 6 \text{ in } \hat{k}$

$= (6\hat{i} + 6\hat{j} + \cancel{6\hat{k}} - \cancel{6\hat{k}}) \text{ in}$

$= (6\hat{i} + 6\hat{j}) \text{ in}$

$= 6(\hat{i} + \hat{j}) \text{ in}$

1.1.23 Find the unit vector $\hat{\lambda}_{AB}$, directed from point A to point B shown in the figure.



Problem 1.23

$$\vec{r}_A = (1m)\hat{\imath} + (1m)\hat{\jmath}$$

$$\vec{r}_B = (3m)\hat{\imath} + (2m)\hat{\jmath}$$

Subtract position vector A from B,

$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$$

$$= (3m)\hat{\imath} + (2m)\hat{\jmath} - (1m)\hat{\imath} - (1m)\hat{\jmath}$$

$$= (2m)\hat{\imath} + (1m)\hat{\jmath}$$

$$|\vec{r}_{B/A}| = \sqrt{(2m)^2 + (1m)^2}$$

$$= \sqrt{5}\,m$$

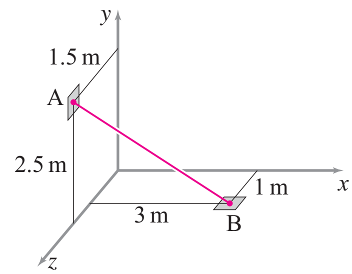
$$\hat{\lambda}_{B/A} = \frac{\vec{r}_{B/A}}{|\vec{r}_{B/A}|}$$

$$= \frac{1}{\sqrt{5}\,m}\left[(2m)\hat{\imath} + (1m)\hat{\jmath}\right]$$

$$= \left(\frac{2}{\sqrt{5}}\right)\hat{\imath} + \left(\frac{1}{\sqrt{5}}\right)\hat{\jmath}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.24 Find a unit vector along string BA and express the position vector of A with respect to B, $\vec{r}_{A/B}$, in terms of the unit vector.



Problem 1.24

$\vec{r}_A = (2.5m)\hat{j} + (1.5m)\hat{k}$

$\vec{r}_B = (3m)\hat{i} + (1m)\hat{k}$

Subtract position vector B from A,

$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$

$= (2.5m)\hat{j} + (1.5m)\hat{k} - (3m)\hat{i} - (1m)\hat{k}$

$= -(3m)\hat{i} + (2.5m)\hat{j} + (0.5m)\hat{k}$

$|\vec{r}_{A/B}| = \sqrt{(-3m)^2 + (2.5m)^2 + (0.5m)^2}$

$= \frac{\sqrt{62}}{2}$

$\hat{\lambda}_{A/B} = \frac{2}{\sqrt{62}}\left[-(3m)\hat{i} + (2.5m)\hat{j} + (0.5m)\hat{k}\right]$

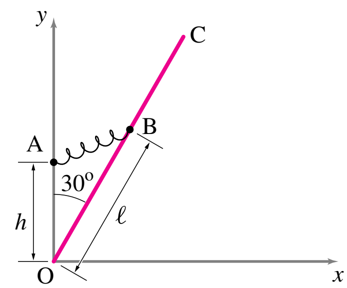
Since $\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$ and $\vec{r}_{A/B} = |\vec{r}_{A/B}|\,\hat{\lambda}_{A/B}$

$|\vec{r}_{A/B}|\,\hat{\lambda}_{A/B} = \vec{r}_A - \vec{r}_B$

$\vec{r}_A = (|\vec{r}_{A/B}|\,\hat{\lambda}_{A/B}) + \vec{r}_B$

$\vec{r}_{A/B} = |\vec{r}_{A/B}|\,\hat{\lambda}_{A/B}$

1.1.25 In the structure shown in the figure, $\ell = 2\,\text{ft}, h = 1.5\,\text{ft}$. The force on B from the spring is $\vec{\boldsymbol{F}} = k\vec{\boldsymbol{r}}_{\text{AB}}$, where $k = 100\,\text{lbf}/\text{ft}$. Find a unit vector $\hat{\boldsymbol{\lambda}}_{\text{AB}}$ along AB and calculate the spring force $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{\lambda}}_{\text{AB}}$.



Problem 1.25

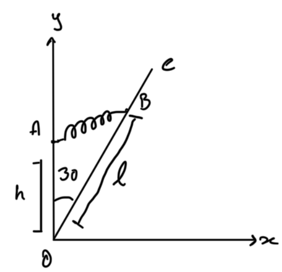
$k = 100\ \text{lbf}/\text{ft}$

$\ell = 2\ \text{ft}$

$h = 1.5\ \text{ft}$

$\vec{F} = k\,\vec{r}_{AB}$

$= k\,|\vec{r}_{AB}|\,\hat{\lambda}_{AB}$



Subtract position vector of A from B,

$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

$\vec{r}_B = (\ell \sin 30)\hat{i} + (\ell \cos 30)\hat{j}$

$= (1\ \text{ft})\hat{i} + (\sqrt{3}\ \text{ft})\hat{j}$

$\vec{r}_A = (1.5\ \text{ft})\hat{j}$

$\vec{r}_{AB} = (1\ \text{ft})\hat{i} + (\sqrt{3}\ \text{ft})\hat{j} - (1.5\ \text{ft})\hat{j}$

$= (1\ \text{ft})\hat{i} + (\sqrt{3} - 1.5)\,\text{ft}\,\hat{j}$

$|\vec{r}_{AB}| = \sqrt{(1\ \text{ft})^2 + (\sqrt{3} - 1.5)^2\,\text{ft}^2}$

$= 1.02\ \text{ft}$

$\hat{\lambda}_{AB} = \dfrac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$

$= (0.97)\hat{i} + (0.227)\hat{j}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$\vec{F} = k|\vec{r}_{AB}|\cancel{\lambda_{AB}} = F\cancel{\lambda_{AB}}$$

$$\Rightarrow k|\vec{r}_{AB}| = F$$

$$\Rightarrow F = 100\ \text{lbf/ft} \times 1.02\ \text{ft}$$
$$= 102\ \text{lbf}$$

$$\vec{F} = 102\ \text{lbf}\,(0.97\hat{\imath} + 0.227\hat{\jmath})$$

1.1.26 Express the vector $\vec{r}_A = 2\,\text{m}\hat{\imath} - 3\,\text{m}\hat{\jmath} + 5\,\text{m}\hat{k}$ in terms of its magnitude and a unit vector indicating its direction.

$$\vec{r}_A = 2m\hat{\imath} - 3m\hat{\jmath} + 5m\hat{k}$$

$$|\vec{r}_A| = \sqrt{(2m)^2 + (-3m)^2 + (5m)^2}$$

$$= \sqrt{38}\ m$$

$$\hat{\lambda}_A = \frac{\vec{r}_A}{|\vec{r}_A|}$$

$$= \left(\frac{2}{\sqrt{38}}\right)\hat{\imath} + \left(\frac{-3}{\sqrt{38}}\right)\hat{\jmath} + \left(\frac{5}{\sqrt{38}}\right)\hat{k}$$

Multiply magnitude with unit vector,

$$\vec{r}_A = |\vec{r}_A|\,\hat{\lambda}_A$$

$$= (\sqrt{38}\ m)\left[\left(\frac{2}{\sqrt{38}}\right)\hat{\imath} + \left(\frac{-3}{\sqrt{38}}\right)\hat{\jmath} + \left(\frac{5}{\sqrt{38}}\right)\hat{k}\right]$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.27 Let $\vec{F} = 10\,\text{lbf}\,\hat{\imath} + 30\,\text{lbf}\,\hat{\jmath}$ and $\vec{W} = -20\,\text{lbf}\,\hat{\jmath}$. Find a unit vector in the direction of the net force $\vec{F} + \vec{W}$, and express the net force in terms of the unit vector.

$\vec{F} = (10\,\text{lbf})\hat{\imath} + (30\,\text{lbf})\hat{\jmath}$

$\vec{W} = -20\,\text{lbf}\,\hat{\jmath}$

Let $\vec{V} = \vec{F} + \vec{W}$

$\vec{V} = (10\,\text{lbf})\hat{\imath} + (30\,\text{lbf})\hat{\jmath} - (20\,\text{lbf})\hat{\jmath}$

$= 10\,\text{lbf}\,(\hat{\imath} + \hat{\jmath})$

$|\vec{V}| = 10\,\text{lbf}\sqrt{1^2 + 1^2}$

$= 10\sqrt{2}\,\text{lbf}$

Find unit vector in the direction of V,

$$\vec{\lambda}_V = \frac{\vec{V}}{|\vec{V}|}$$

$$= \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$$

$$\vec{V} = |\vec{V}|\,\vec{\lambda}_V$$

$$= 10\sqrt{2}\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right)$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.28 Let $\hat{\lambda}_1 = 0.80\hat{\imath} + 0.60\hat{\jmath}$ and $\hat{\lambda}_2 = 0.5\hat{\imath} + 0.866\hat{\jmath}$.

1. Show that $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are unit vectors.
2. Is the sum of these two unit vectors also a unit vector? If not, find a unit vector along the sum of $\hat{\lambda}_1$ and $\hat{\lambda}_2$.

$\hat{\lambda}_1 = 0.8\hat{i} + 0.6\hat{j}$

$\hat{\lambda}_2 = 0.5\hat{i} + 0.866\hat{j}$

1. Find magnitudes of vectors,

$|\hat{\lambda}_1| = \sqrt{0.8^2 + 0.6^2}$

$= 1$

Hence, $\hat{\lambda}_1$ is a unit vector.

$|\hat{\lambda}_2| = \sqrt{0.5^2 + 0.866^2}$

~ 1

Hence, $\hat{\lambda}_2$ is a unit vector.

2. $\hat{\lambda}_1 + \hat{\lambda}_2 = 0.8\hat{i} + 0.6\hat{j} + 0.5\hat{i} + 0.866\hat{j}$

$= 1.3\hat{i} + 1.466\hat{j}$

$|\hat{\lambda}_1 + \hat{\lambda}_2| = \sqrt{1.3^2 + 1.466^2}$

$= 1.959$

Hence $\hat{\lambda}_1 + \hat{\lambda}_2$ is not a unit vector.

Unit vector of $\hat{\lambda}_1 + \hat{\lambda}_2 = \dfrac{\hat{\lambda}_1 + \hat{\lambda}_2}{|\hat{\lambda}_1 + \hat{\lambda}_2|}$

$= \dfrac{1}{1.959}(1.3\hat{i} + 1.466\hat{j})$

$= (0.663)\hat{i} + (0.741)\hat{j}$

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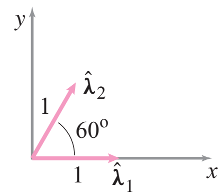
1.1.29 For the unit vectors $\hat{\boldsymbol{\lambda}}_1$ and $\hat{\boldsymbol{\lambda}}_2$ shown below, find the scalars α and β such that $\alpha\hat{\boldsymbol{\lambda}}_1 - 3\hat{\boldsymbol{\lambda}}_2 = \beta\hat{\boldsymbol{\jmath}}$.



Problem 1.29

$$\hat{\lambda}_1 = \hat{\imath}$$

$$\hat{\lambda}_2 = (\cos 60)\hat{\imath} + (\sin 60)\hat{\jmath}$$

$$= \frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}$$

$$\alpha\hat{\lambda}_1 - 3\hat{\lambda}_2 = \beta\hat{\jmath}$$

$$\alpha\hat{\imath} - \frac{3}{2}\hat{\imath} - \frac{3\sqrt{3}}{2}\hat{\jmath} = \beta\hat{\jmath}$$

Separate Equations for $\hat{\imath}$ and $\hat{\jmath}$

$$\alpha\hat{\imath} - \frac{3}{2}\hat{\imath} = 0$$

$$\Rightarrow \alpha = \frac{3}{2}$$

$$-\frac{3\sqrt{3}}{2}\hat{\jmath} = \beta\hat{\jmath}$$

$$\Rightarrow \beta = -\frac{3\sqrt{3}}{2}$$

1.1.30 If a mass slides from point A towards point B along a straight path and the coordinates of points A and B are (0 ∈ , 5 ∈, 0 ∈) and (10 ∈, 0 ∈, 10 ∈), respectively, find the unit vector $\hat{\lambda}_{AB}$ directed from A to B along the path.

$\vec{r}_A = (5\,in)\,\hat{j}$

$\vec{r}_B = (10\,in)\,\hat{i} + (10\,in)\,\hat{k}$

Subtract position vector of A from B

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$

$$= (10\,in)\,\hat{i} - (5\,in)\,\hat{j} + (10\,in)\,\hat{k}$$

$$|\vec{r}_{AB}| = \sqrt{(10\,in)^2 + (-5\,in)^2 + (10\,in)^2}$$

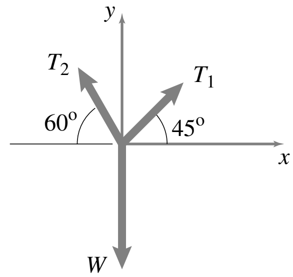
$$= 15\,in$$

Divide $\vec{r}_{AB}$ by its magnitude.

$$\hat{\lambda}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$= \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

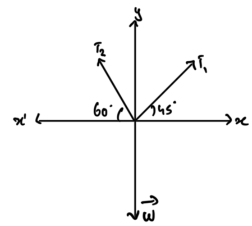
1.1.31 In the figure shown, $T_1 = 20\sqrt{2}\,\mathrm{N}, T_2 = 40\,\mathrm{N}$, and W is such that the sum of the three forces equals zero. If W is doubled, find α and β such that $\alpha\vec{\boldsymbol{T}}_1, \beta\vec{\boldsymbol{T}}_2$, and $2\vec{\boldsymbol{W}}$ still sum up to zero.



Problem 1.31

$|\vec{T}_1| = 20\sqrt{2}\,\mathrm{N}$

$|\vec{T}_2| = 40\,\mathrm{N}$



$\vec{T}_1 = (20\sqrt{2}\cos 45)\hat{i} + (20\sqrt{2}\sin 45)\hat{j}$

$= 20(\hat{i} + \hat{j})$

$\vec{T}_2 = (40\cos 120)\hat{i} + (40\sin 120)\hat{j}$

$= -20\hat{i} + 20\sqrt{3}\,\hat{j}$

$\vec{W} = k\hat{j}$

Equate sum of forces to zero,

$\vec{T}_1 + \vec{T}_2 + \vec{W} = 0$

$20(\hat{i} + \hat{j}) + 20(-\hat{i} + \sqrt{3}\hat{j}) + k\hat{j} = 0$

$20\hat{j}(1+\sqrt{3}) = -k\hat{j}$

$k = -20(1+\sqrt{3})$

$\vec{W} = -20(1+\sqrt{3})\hat{j}$

$\alpha\vec{T}_1 + \beta\vec{T}_2 + 2\vec{W} = 0$

$20\alpha(\hat{i}+\hat{j}) + 20\beta(-\hat{i}+\sqrt{3}\hat{j}) - 40(1+\sqrt{3})\hat{j} = 0$

Separate equation in terms of $\hat{i}$ and $\hat{j}$,

$20\alpha\hat{i} - 20\beta\hat{i} = 0$

$\Rightarrow \alpha = \beta$

$20\alpha\hat{j} + 20\sqrt{3}\beta\hat{j} - 40(1+\sqrt{3})\hat{j} = 0$

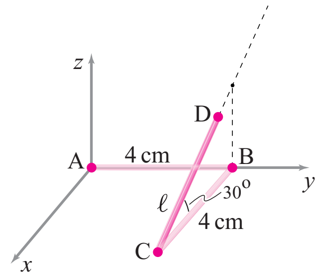
$20\alpha(1+\sqrt{3})\hat{j} = 40(1+\sqrt{3})\hat{j}$

$\Rightarrow \alpha = 2 = \beta$

1.1.32 In the figure shown, rods AB and BC are each 4 cm long and lie along y and x axes, respectively. Rod CD is in the xz plane and makes an angle $\theta = 30^\circ$ with the x-axis.

1. Find $\vec{r}_{AD}$ in terms of the variable length ℓ.
2. Find ℓ and α such that

$$\vec{r}_{AD} = \vec{r}_{AB} - \vec{r}_{BC} + \alpha\hat{k}.$$



Problem 1.32

1. $\vec{r}_{AD} = \vec{r}_{AB} + \vec{r}_{BC} + \vec{r}_{CD}$

$\vec{r}_{AB} = (4cm)\hat{j}$

$\vec{r}_{BC} = (4cm)\hat{i}$

$\vec{r}_{CD} = (\ell\cos 30)\hat{i} + (\ell\sin 30)\hat{k}$

$= \frac{\ell}{2}\hat{i} + \frac{\ell\sqrt{3}}{2}\hat{k}$

Add $\vec{r}_{AB}$, $\vec{r}_{BC}$, $\vec{r}_{CD}$,

$\vec{r}_{AD} = (4cm)\hat{j} + (4cm)\hat{i} + \frac{\ell}{2}\hat{i} + \frac{\sqrt{3}}{2}\ell\hat{k}$

$= \left(4cm + \frac{\ell}{2}\right)\hat{i} + (4cm)\hat{j} + \left(\frac{\sqrt{3}}{2}\ell\right)\hat{k}$

2. $\vec{r}_{AD} = \vec{r}_{AB} - \vec{r}_{BC} + \alpha\hat{k}$

$= (4cm)\hat{j} - (4cm)\hat{i} + \alpha\hat{k}$

$= \left(4cm + \frac{\ell}{2}\right)\hat{i} + (4cm)\hat{j} + \left(\frac{\sqrt{3}}{2}\ell\right)\hat{k}$

Use the equation for $\hat{i}$,

$-(4cm)\hat{i} = \left(4cm + \frac{\ell}{2}\right)\hat{i}$

$\frac{\ell}{2} = -8cm$

$\ell = -16cm$

Use the equation for $\hat{k}$,

$\alpha\hat{k} = \left(\frac{\sqrt{3}}{2}\ell\right)\hat{k}$

$\alpha = -8\sqrt{3}$

1.1.33 In Problem 1.1.32, find ℓ such that the length of the position vector $\vec{r}_{AD}$ is 6 cm.

$$\vec{r}_{AD} = \left(4\text{cm} + \frac{\ell}{2}\right)\hat{\imath} + (4\text{cm})\hat{\jmath} + \left(\frac{\sqrt{3}}{2}\ell\right)\hat{k}$$

Equate the magnitude of $\vec{r}_{AD}$ to its length,

$$|\vec{r}_{AD}| = \sqrt{\left(4\text{cm} + \frac{\ell}{2}\right)^2 + (4\text{cm})^2 + \left(\frac{\sqrt{3}}{2}\ell\right)^2}$$

$$36\,\text{cm}^2 = \left(4\,\text{cm} + \frac{\ell}{2}\right)^2 + 16\,\text{cm}^2 + \frac{3}{4}\ell^2$$

$$20\,\text{cm}^2 = 16\,\text{cm}^2 + \frac{\ell^2}{4} + 4\ell + \frac{3^2}{4}$$

$$\Rightarrow \ell^2 + 4\ell - 4\,\text{cm}^2 = 0$$

Use the Quadratic Formula,

$$\ell = \frac{-4 \pm \sqrt{16+16}}{2}\text{cm}$$

$$= \frac{-4 \pm 4\sqrt{2}}{2}\text{cm}$$

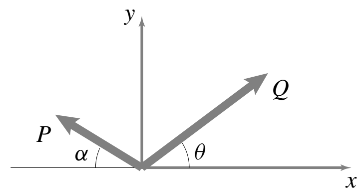
$$= (-2 \pm 2\sqrt{2})\,\text{cm}$$

Since ℓ cannot be negative

$$\ell = (-2 + 2\sqrt{2})\,\text{cm}$$
$$= 0.82\,\text{cm}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.34 Let two forces $\vec{P}$ and $\vec{Q}$ act in the directions shown in the figure. You are allowed to change the direction of the forces by changing the angles α and θ while keeping the magnitudes fixed. What are the values of α and θ that maximize the magnitude of $\vec{P}+\vec{Q}$?



Problem 1.34

$$\vec{P} = (|\vec{P}|\cos(180-\alpha))\hat{i} + (|\vec{P}|\sin(180-\alpha))\hat{j}$$

$$\vec{Q} = (|\vec{Q}|\cos\theta)\hat{i} + (|\vec{Q}|\sin\theta)\hat{j}$$

$$\vec{P}+\vec{Q} = (|\vec{P}|\cos(180-\alpha))\hat{i} + (|\vec{P}|\sin(180-\alpha))\hat{j} + (|\vec{Q}|\cos\theta)\hat{i} + (|\vec{Q}|\sin\theta)\hat{j}$$

$$= (|\vec{P}|\cos(180-\alpha)+|\vec{Q}|\cos\theta)\hat{i} + (|\vec{P}|\sin(180-\alpha)+|\vec{Q}|\sin\theta)\hat{j}$$

For maximum magnitude $\alpha = \theta$.

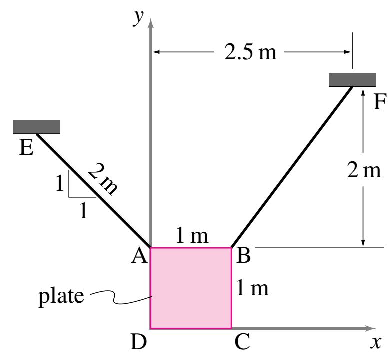
$$\vec{P}+\vec{Q} = (|\vec{P}|+|\vec{Q}|)\cos\theta\,\hat{i} + (|\vec{P}|+|\vec{Q}|)\sin\theta\,\hat{j}$$

$$= (|\vec{P}|+|\vec{Q}|)(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

In the above equation, $(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$ has to be maximised. This happens at $\theta = n\pi$, where $|\cos\theta\,\hat{i} + \sin\theta\,\hat{j}| = 1$. Hence for maximum value of $\vec{P}+\vec{Q}$,

$\alpha = \theta = n\pi$

1.1.35 A 1 m × 1 m square board is supported by two strings AE and BF. The tension in the string BF is 20 N. Express this tension as a vector.



Problem 1.35

$B \equiv (1m, 1m)$

$F \equiv (2.5m, 3m)$

$\vec{T} = (20N) \times \hat{\lambda}_{BF}$

Find unit vector in the direction of BF,

$\vec{BF} = \vec{F} - \vec{B}$

$= (2.5m)\hat{i} + (3m)\hat{j} - (1m)\hat{i} - (1m)\hat{j}$

$= (1.5m)\hat{i} + (2m)\hat{j}$

$|\vec{BF}| = \sqrt{(1.5m)^2 + (2m)^2}$

$= 2.5m$

$\hat{\lambda}_{BF} = \dfrac{\vec{BF}}{|\vec{BF}|}$

$= \dfrac{(1.5m)\hat{i} + (2m)\hat{j}}{2.5m}$

$= \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$

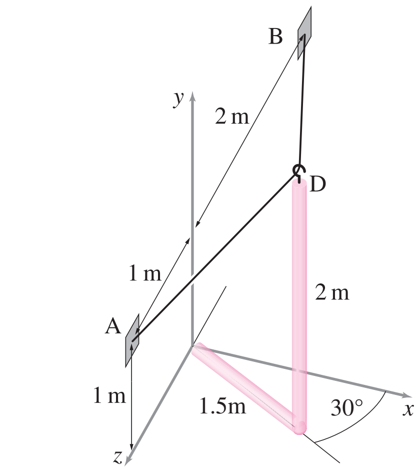
Multiply magnitude of Tension with $\hat{\lambda}_{BF}$,

$\vec{T} = (20N)\hat{\lambda}_{BF}$

$= (12N)\hat{i} + (16N)\hat{j}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.36 The top of an L-shaped bar, shown in the figure, is to be tied by strings AD and BD to the points A and B in the *yz* plane. Find the length of the strings AD and BD using vectors $\vec{r}_{AD}$ and $\vec{r}_{BD}$.



Problem 1.36

$$\vec{r}_A = (1m)\hat{j} + (1m)\hat{k}$$

Find Position Vectors of B and D,

$$\vec{r}_B = (2\ m \cos 30)\hat{i} + (1m)\hat{j} + (2\ m \sin 30)\hat{k}$$

$$= (\sqrt{3}\ m)\ \hat{i} + (1m)\hat{j} + (1\ m)\hat{k}$$

$$\vec{r}_D = (1.5m \cos 30)\hat{i} + (2m)\hat{j} + (1.5m \sin 30)\hat{k}$$

$$= \left(\frac{3\sqrt{3}}{4} m\right)\hat{i} + (2m)\hat{j} + \left(\frac{3}{4} m\right)\hat{k}$$

Subtract position vector of A from D,

$$\vec{r}_{AD} = \vec{r}_D - \vec{r}_A$$

$$= \left(\frac{3\sqrt{3}}{4} m\right)\hat{i} + (1\ m)\hat{j} + \left(\frac{-1}{4} m\right)\hat{k}$$

$$|\vec{AD}| = \sqrt{\left(\frac{3\sqrt{3}}{4} m\right)^2 + (1m)^2 + \left(\frac{-1}{4} m\right)^2}$$

$$= \frac{\sqrt{11}}{2} m = 1.65\ m$$

Subtract position vector of B from D,

$$\vec{r}_{BD} = \vec{r}_D - \vec{r}_B$$

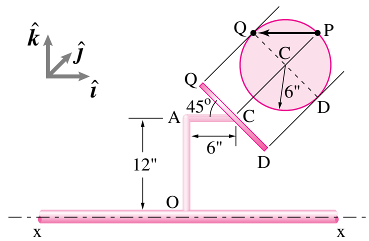
$$= \left(\frac{-\sqrt{3}}{4} m\right)\hat{i} + (1m)\hat{j} + \left(\frac{-1}{4} m\right)\hat{k}$$

$$|\vec{BD}| = \sqrt{\left(\frac{-\sqrt{3}}{4} m\right)^2 + (1m)^2 + \left(\frac{-1}{4} m\right)^2}$$

$$= \frac{\sqrt{5}}{2} m = 1.12m$$

1.1.37 A circular disk of radius 6∈ is mounted on axle x-x at the end of an L-shaped bar as shown in the figure. The disk is tipped 45° with respect to the horizontal bar AC. Two points, D and Q, are marked on the rim of the disk; with CP directly into the page, and Q at the highest point above the center C. Taking the base vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ as shown in the figure ($\hat{\jmath}$ into the page), find

1. the relative position vector $\vec{r}_{Q/P}$,
2. the magnitude $|\vec{r}_{Q/P}|$.



Problem 1.37

1. Find Position Vector of P,

$\vec{r}_P = \vec{r}_{OA} + \vec{r}_{AC} + \vec{r}_{CP}$

$= (12\,in)\hat{k} + (6\,in)\hat{\imath} + [(12\,in\cos 45)\hat{\imath} + (12\,in\sin 45)\hat{k}]$

$= (6(1+\sqrt{2})\,in)\hat{\imath} + (6(2+\sqrt{2})\,in)\hat{k}$

Find Position Vector of Q,

$\vec{r}_Q = \vec{r}_{OA} + \vec{r}_{AC} + \vec{r}_{CQ}$

$= (12\,in)\hat{k} + (6\,in)\hat{\imath} + [(6\,in\cos 135)\hat{\imath} + (6\,in\sin 135)\hat{k}]$

$= (3(2-\sqrt{2})\,in)\hat{\imath} + (3(4+\sqrt{2})\,in)\hat{k}$

Subtract position vector of P from Q,

$\vec{r}_{Q/P} = \vec{r}_Q - \vec{r}_P$

$= (9\sqrt{2}\,in)\hat{\imath} + (3\sqrt{2}\,in)\hat{k}$

2. $|\vec{r}_{Q/P}| = \sqrt{(9\sqrt{2}\,in)^2 + (3\sqrt{2}\,in)^2}$

$= 6\sqrt{5}\,in$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.1.38 Write the vectors $\vec{F}_1 = 30\,\mathrm{N}\hat{\imath} + 40\,\mathrm{N}\hat{\jmath} - 10\,\mathrm{N}\hat{k}$, $\vec{F}_2 = -20\,\mathrm{N}\hat{\jmath} + 2\,\mathrm{N}\hat{k}$, and $\vec{F}_3 = -10\,\mathrm{N}\hat{\imath} - 100\,\mathrm{N}\hat{k}$ as a list of numbers (rows or columns). Find the sum of the forces using a computer.

Code :

```
F1=[30;40;-10];
F2=[0;-20;2];
F3=[-10;0;-100];

F1+F2+F3
```

The solution to the sum of forces is : $20\hat{i} + 20\hat{j} - 108\hat{k}$

1.1.39 Let $\alpha\vec{\boldsymbol{F}}_1 + \beta\vec{\boldsymbol{F}}_2 + \gamma\vec{\boldsymbol{F}}_3 = \vec{\boldsymbol{0}}$, where $\vec{\boldsymbol{F}}_1, \vec{\boldsymbol{F}}_2,$ and $\vec{\boldsymbol{F}}_3$ are as given in Problem 1.1.38. Solve for $\alpha, \beta,$ and γ using a computer.

Code:

```
F1=[30;40;-10];
F2=[0;-20;2];
F3=[-10;0;-100];

syms a b g

solve((a*F1 + b*F2 + g*F3) == [0;0;0])
```

The solution to the unkowns are: $\alpha = 0$, $\beta = 0$, $\gamma = 0$

1.1.40 Let $\vec{\boldsymbol{r}}_{\mathrm{n}} = 1\,\mathrm{m}(\cos\theta_n\hat{\boldsymbol{\imath}} + \sin\theta_n\hat{\boldsymbol{\jmath}})$, where $\theta_n = \theta_0 - n\Delta\theta$. Using a computer generate the required vectors and find the sum

$$\sum_{n=0}^{44}\vec{\boldsymbol{r}}_{\mathrm{i}},\quad \text{with } \Delta\theta = 1^\circ \qquad \text{and} \qquad \theta_0 = 45^\circ.$$

Code:

```
F1=[30;40;-10];
F2=[0;-20;2];
F3=[-10;0;-100];

syms a b g

solve((a*F1 + b*F2 + g*F3) == [0;0;0])
```

The solution to the sum is : $40.3668\hat{\mathrm{i}} + 17.1347\hat{\mathrm{j}}$

1.1.41 Find two non-zero and non-parallel vectors $\vec{A}$ and $\vec{B}$ so that $|\vec{A}+2\vec{B}| = 2|\vec{A}+\vec{B}|$.

Answer:
One solution, given in sample ??, is $\vec{A} = -16\hat{\imath} + 8\hat{\jmath}$ and $\vec{B} = 15\hat{\imath}$

$$\vec{A} = x_a\hat{i} + y_a\hat{j} + z_a\hat{k}$$

$$\vec{B} = x_B\hat{i} + y_B\hat{j} + z_B\hat{k}$$

$$|A + 2\vec{B}| = \sqrt{(x_A+2x_B)^2 + (y_a+2y_B)^2 + (z_a+2z_B)^2} \quad -(i)$$

$$2|\vec{A}+\vec{B}| = 2\sqrt{(x_A+x_B)^2 + (y_A+2y_B)^2 + (z_a+z_B)^2} \quad -(ii)$$

Equate (i) to (ii) and Square

$$(x_A+2x_B)^2 + (y_A+2y_B)^2 + (z_A+2z_B)^2$$

$$= 4\left[(x_A+x_B)^2 + (y_A+y_B)^2 + (z_A+z_B)^2\right]$$

$$x_A^2 + \cancel{4x_B^2} + 4x_Ax_B + y_A^2 + \cancel{4y_B^2} + 4y_Ay_B + z_A^2 + \cancel{4z_B^2} + 4z_Az_B$$

$$= 4x_A^2 + \cancel{4x_B^2} + 8x_Ax_B + 4y_A^2 + \cancel{4y_B^2} + 8y_Ay_B + 4z_A^2 + \cancel{4z_B^2}$$

$$+ 8z_Az_B$$

$$3x_A^2 + 3y_A^2 + 3z_A^2 + 4x_Ax_B + 4y_Ay_B + 4z_Az_B = 0$$

Take $\vec{A} = \hat{i}$, $\vec{B} = \hat{j}$

$$3 = 0$$

Since RHS $\neq$ LHS, the vectors are non-parallel and non-zeros.

$$\vec{A} = \hat{i}, \quad \vec{B} = \hat{j}$$

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