SAMPLE 1.1 **Various ways of representing a vector:** The examples here contradict many people's conceptions.

> The examples in this Sample Problem box should be mastered before proceeding to other samples.

A vector $\vec{F} = 3\,\text{N}\hat{\imath} + 3\,\text{N}\hat{\jmath}$ is represented in various ways below, some incorrect. For each representation, determine whether it is correct or incorrect, and why. The base vectors used are shown first.

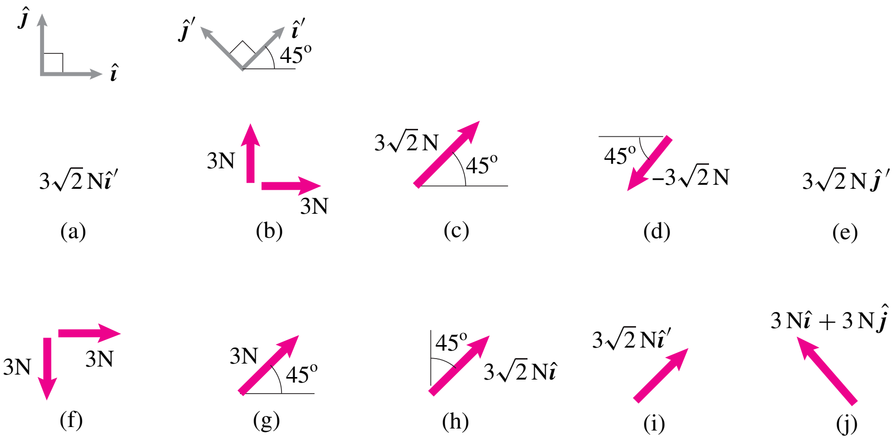

Figure 1.11:

Solution First note that the unit vectors $\hat{\imath}'$ and $\hat{\jmath}'$ can be expressed in terms of their components along $\hat{\imath}$ and $\hat{\jmath}$ as follows:

$$\hat{\imath}' = |\hat{\imath}'|\cos 45°\,\hat{\imath} + |\hat{\imath}'|\sin 45°\,\hat{\jmath} = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath}). \tag{1.2}$$

Similarly,

$$\hat{\jmath}' = -|\hat{\jmath}'|\cos 45°\,\hat{\imath} + |\hat{\jmath}'|\sin 45°\,\hat{\jmath} = \frac{1}{\sqrt{2}}(-\hat{\imath} + \hat{\jmath}).$$

a) Correct: $3\sqrt{2}\,\text{N}\hat{\imath}'$. From the picture defining $\hat{\imath}'$, you can see that $\hat{\imath}'$ is a unit vector with equal components in the $\hat{\imath}$ and $\hat{\jmath}$ directions; *i.e.,* it is parallel to $\vec{F}$. So $\vec{F}$ is given by its magnitude $\sqrt{(3\,\text{N})^2 + (3\,\text{N})^2}$ times a unit vector in its direction, in this case $\hat{\imath}'$. Algebraically,

$$3\sqrt{2}\,\text{N}\hat{\imath}' = 3\sqrt{2}\,\text{N}\cdot\frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath}) = 3\,\text{N}\hat{\imath} + 3\,\text{N}\hat{\jmath} = \vec{F}.$$

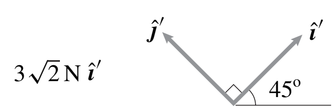

Figure 1.12: Case (a): Correct representation of $\vec{F}$.

b) Correct: Here two vectors are shown — one with magnitude 3 N in the direction of the horizontal arrow $\hat{\imath}$, and one with magnitude 3 N in the direction of the vertical arrow $\hat{\jmath}$. When two forces act on an object at a point, their effect is additive. So the net vector is the sum of the vectors shown, that is, $3\,\text{N}\hat{\imath} + 3\,\text{N}\hat{\jmath}$. It is a correct representation.

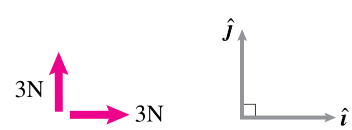

Figure 1.13: Case (b): Correct representation of $\vec{F}$.

c) Correct: Here we have a scalar $3\sqrt{2}\,\text{N}$ next to an arrow. The vector described is the scalar multiplied by a unit vector in the direction of the arrow. Since the arrow's direction is the same as that of $\hat{\imath}'$, which we already know is parallel to $\vec{F}$, this vector represents the given $\vec{F}$. Using the standard base vectors we can write,

$$3\sqrt{2}\,\mathrm{N}(\cos 45°\hat{\imath} + \sin 45°\hat{\jmath}) \quad = \quad = 3\,\mathrm{N}\hat{\imath} + 3\,\mathrm{N}\hat{\jmath} = \vec{F}.$$

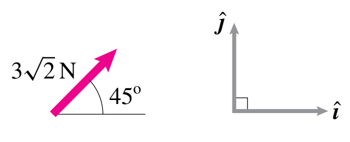

Figure 1.14: Case (c): Correct representation of $\vec{F}$.

d) Correct: The scalar $-3\sqrt{2}\,\mathrm{N}$ is multiplied by a unit vector in the direction indicated, $-\hat{\imath}'$. So we get $(-3\sqrt{2}\,\mathrm{N})(-\hat{\imath}')$ which is $3\sqrt{2}\,\mathrm{N}\hat{\imath}'$ as before. Thus, it is the same vector $\vec{F}$.

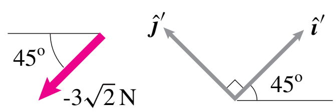

Figure 1.15: Case (d): Correct representation of $\vec{F}$.

e) Incorrect: $3\sqrt{2}\,\mathrm{N}\hat{\jmath}'$. The magnitude is correct, but the direction is off by 90 degrees. It is a different vector. Algebraically,

$$\begin{aligned} 3\sqrt{2}\,\mathrm{N}\hat{\jmath}' &= 3\sqrt{2}\,\mathrm{N}\cdot\frac{1}{\sqrt{2}}(-\hat{\imath}+\hat{\jmath}) \\ &= -3\,\mathrm{N}\hat{\imath} + 3\,\mathrm{N}\hat{\jmath} \\ &\neq \vec{F}. \end{aligned}$$

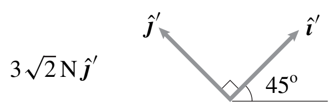

Figure 1.16: Case (e): Incorrect representation of $\vec{F}$.

f) Incorrect: $3\,\mathrm{N}\hat{\imath} - 3\,\mathrm{N}\hat{\jmath}$. The $\hat{\imath}$ component of the vector is correct but the $\hat{\jmath}$ component is in the opposite direction. The vector is in the wrong direction by 90 degrees. It is a different vector.

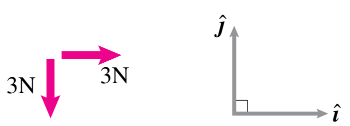

Figure 1.17: Case (f): Incorrect representation of $\vec{F}$.

g) Incorrect: Right direction but the magnitude is off by a factor of $\sqrt{2}$.

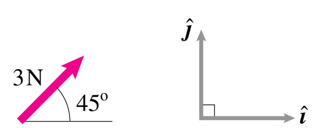

Figure 1.18: Case (g): Incorrect representation of $\vec{F}$.

h) Incorrect: The magnitude is right. The direction indicated is right. But, the algebraic symbol $3\sqrt{2}\,\mathrm{N}\hat{\imath}$ takes precedence and it is in the wrong direction ($\hat{\imath}$ instead of $\hat{\imath}'$). It is a different vector.

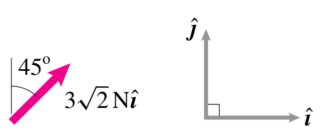

Figure 1.19: Case (h): Incorrect representation of $\vec{F}$.

i) Correct: A labeled arrow. The arrow is only schematic. The algebraic symbol $3\sqrt{2}\,\mathrm{N}\hat{\imath}'$ defines the vector. We draw the arrow to remind us that there is a vector to represent. The tip or tail of the arrow would be drawn at the point of the force application. In this case, the arrow is drawn in the direction of $\vec{F}$, but strictly speaking, it need not be, because all the information needed is already in the algebraic expression.

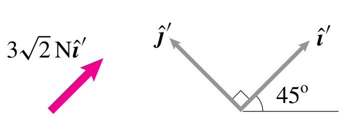

Figure 1.20: Case (i): Correct representation of $\vec{F}$.

j) Correct: Like (i) above, the directional and magnitude information are embedded in the algebraic symbol $3\,\mathrm{N}\hat{\boldsymbol{\imath}} + 3\,\mathrm{N}\hat{\boldsymbol{\jmath}}$. The arrow is there to indicate a vector. In this case, it points in the wrong direction, so it is not ideally communicative. In fact, it is confusing and therefore, not recommended. But it still correctly represents the given vector because the algebraic symbol takes precedence over the graphical symbol.

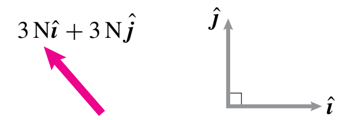

Figure 1.21: Case (j): Correct but misleading representation of $\vec{\boldsymbol{F}}$.

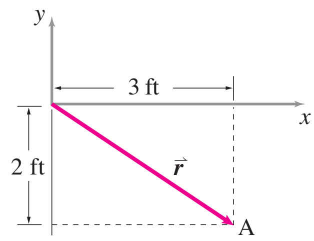

Figure 1.22: Drawing vector $\vec{\boldsymbol{r}} = 3\,\mathrm{ft}\hat{\boldsymbol{\imath}} - 2\,\mathrm{ft}\hat{\boldsymbol{\jmath}}$ using its components.

SAMPLE 1.2 Drawing a vector from its components: Draw the vector $\vec{\boldsymbol{r}} = 3\,\mathrm{ft}\hat{\boldsymbol{\imath}} - 2\,\mathrm{ft}\hat{\boldsymbol{\jmath}}$ using its components.

Solution To draw $\vec{\boldsymbol{r}}$ using its components, we first draw the axes and measure 3 units (any units that we choose on the ruler) along the x-axis and 2 units along the negative y-axis. We mark this point as A (say) on the paper and draw a line from the origin to the point A. We write the dimensions '3 ft' and '2 ft' on the figure. Finally, we put an arrowhead on this line pointing towards A.

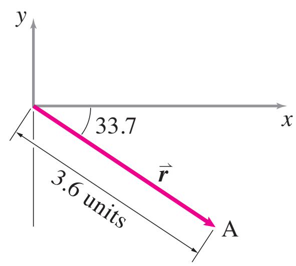

Figure 1.23: Drawing vector $\vec{\boldsymbol{r}}$ given its length (3.6 ft) and direction (slope angle $\theta = -33.7^{\circ}$).

SAMPLE 1.3 Drawing a vector from its length and direction: A vector $\vec{\boldsymbol{r}}$ is 3.6 ft long and is directed 33.7° clockwise (CW) from the positive x-axis. Draw $\vec{\boldsymbol{r}}$.

Solution We first draw the x and y axes and then draw $\vec{\boldsymbol{r}}$ as a line from the origin at an angle -33.7° from the x-axis (minus sign means measuring clockwise), measure 3.6 units (magnitude of $\vec{\boldsymbol{r}}$) along this line and finally put an arrowhead pointing away from the origin.

Comments Note that this is about (at least to 2 digit accuracy) the same vector as in Sample 1.2. In fact, you can easily verify that $r_x = r\cos\theta = 3.6\,\mathrm{ft}\cdot\cos(-33.7^{\circ}) = 3\,\mathrm{ft}$, $r_y = r\sin\theta = 3.6\,\mathrm{ft}\cdot\sin(-33.7^{\circ}) = -2\,\mathrm{ft}$.

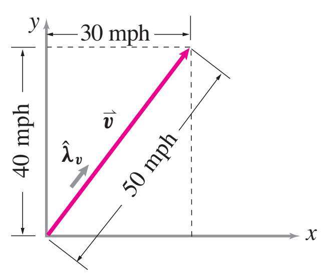

Figure 1.24: Speed and direction (indicated by the unit vector $\hat{\boldsymbol{\lambda}}_v$) from the given velocity vector $\vec{\boldsymbol{v}} = 30\,\mathrm{mph}\hat{\boldsymbol{\imath}} + 40\,\mathrm{mph}\hat{\boldsymbol{\jmath}}$.

SAMPLE 1.4 Magnitude and direction of a vector: The velocity of a car is given as $\vec{\boldsymbol{v}} = (30\hat{\boldsymbol{\imath}} + 40\hat{\boldsymbol{\jmath}})\,\mathrm{mph}$. Find the speed (magnitude of $\vec{\boldsymbol{v}}$) of the car, its direction as a unit vector, and write the velocity in terms of its magnitude and the unit vector.

Solution

1. **Speed of the car $v = |\vec{\boldsymbol{v}}|$:**

$$\begin{aligned} \vec{\boldsymbol{v}} &= 30\,\mathrm{mph}\hat{\boldsymbol{\imath}} + 40\,\mathrm{mph}\hat{\boldsymbol{\jmath}}, \\ v = |\vec{\boldsymbol{v}}| &= \sqrt{v_x^2 + v_y^2} = \sqrt{(30\,\mathrm{mph})^2 + (40\,\mathrm{mph})^2} \\ &= 50\,\mathrm{mph} \end{aligned}$$

2. **Direction of $\vec{\boldsymbol{v}}$ as a unit vector along $\vec{\boldsymbol{v}}$:** The unit vector along a given vector is found by dividing the given vector with its magnitude. Let $\hat{\boldsymbol{\lambda}}_v$ be the unit vector along $\vec{\boldsymbol{v}}$. Then,

$$\hat{\boldsymbol{\lambda}}_v = \frac{\vec{\boldsymbol{v}}}{|\vec{\boldsymbol{v}}|} = \frac{30\,\mathrm{mph}\hat{\boldsymbol{\imath}} + 40\,\mathrm{mph}\hat{\boldsymbol{\jmath}}}{50\,\mathrm{mph}} = 0.6\hat{\boldsymbol{\imath}} + 0.8\hat{\boldsymbol{\jmath}}.$$

3. **$\vec{\boldsymbol{v}}$ as a product of its magnitude and the unit vector $\hat{\boldsymbol{\lambda}}_v$:**

$$\vec{\boldsymbol{v}} = |\vec{\boldsymbol{v}}|\hat{\boldsymbol{\lambda}}_v = 50\,\mathrm{mph}(0.6\hat{\boldsymbol{\imath}} + 0.8\hat{\boldsymbol{\jmath}})$$

which, of course, is the same vector as given in the problem.

speed $v = 50$ mph, $\hat{\boldsymbol{\lambda}}_v = 0.6\hat{\boldsymbol{\imath}} + 0.8\hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{v}} = 50(0.6\hat{\boldsymbol{\imath}} + 0.8\hat{\boldsymbol{\jmath}})\,\mathrm{mph}$

SAMPLE 1.5 **Adding vectors:** Three forces, $\vec{F}_1 = 2\,\mathrm{N}\hat{\imath} + 3\,\mathrm{N}\hat{\jmath}, \vec{F}_2 = -10\,\mathrm{N}\hat{\jmath}$, and $\vec{F}_3 = 3\,\mathrm{N}\hat{\imath} + 1\,\mathrm{N}\hat{\jmath} - 5\,\mathrm{N}\hat{k}$, act on a particle. Find the net force on the particle.

Solution The net force on the particle is the vector sum of all the forces, *i.e.*,

$$\begin{aligned}
\vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \\
&= (2\,\mathrm{N}\hat{\imath} + 3\,\mathrm{N}\hat{\jmath}) + (-10\,\mathrm{N}\hat{\jmath}) + (3\,\mathrm{N}\hat{\imath} + 1\,\mathrm{N}\hat{\jmath} - 5\,\mathrm{N}\hat{k}) \\
&= \quad 2\,\mathrm{N}\hat{\imath} + 3\,\mathrm{N}\hat{\jmath} + 0\hat{k} \\
&\quad + 0\hat{\imath} - 10\,\mathrm{N}\hat{\jmath} + 0\hat{k} \\
&\quad + 3\,\mathrm{N}\hat{\imath} + 1\,\mathrm{N}\hat{\jmath} - 5\hat{k} \\
&= (2\,\mathrm{N} + 3\,\mathrm{N})\hat{\imath} + (3\,\mathrm{N} - 10\,\mathrm{N} + 1\,\mathrm{N})\hat{\jmath} + (-5\,\mathrm{N})\hat{k} \\
&= 5\,\mathrm{N}\hat{\imath} - 6\,\mathrm{N}\hat{\jmath} - 5\,\mathrm{N}\hat{k}.
\end{aligned}$$

$$\boxed{\vec{F}_{\text{net}} = 5\,\mathrm{N}\hat{\imath} - 6\,\mathrm{N}\hat{\jmath} - 5\,\mathrm{N}\hat{k}}$$

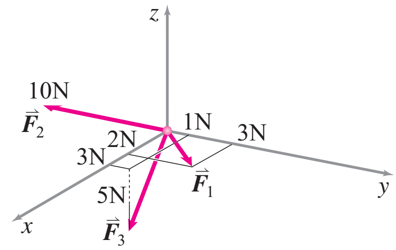

Figure 1.25: Force vectors in 3D space.

Comments: In general, we do not need to write the summation so elaborately. Once you feel comfortable with the idea of summing only similar components in a vector sum, you can do the calculation in two lines.

SAMPLE 1.6 **Subtracting vectors:** Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a body. The net force on the body is $\vec{F}_{\text{net}} = 2\,\mathrm{N}\hat{\imath}$. If $\vec{F}_1 = 10\,\mathrm{N}\hat{\imath} - 10\,\mathrm{N}\hat{\jmath}$, find the other force $\vec{F}_2$.

Solution

$$\begin{aligned}
\vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 \\
\Rightarrow \quad \vec{F}_2 &= \vec{F}_{\text{net}} - \vec{F}_1 \\
&= 2\,\mathrm{N}\hat{\imath} - (10\,\mathrm{N}\hat{\imath} - 10\,\mathrm{N}\hat{\jmath}) \\
&= -8\,\mathrm{N}\hat{\imath} + 10\,\mathrm{N}\hat{\jmath}.
\end{aligned}$$

$$\boxed{\vec{F}_2 = -8\,\mathrm{N}\hat{\imath} + 10\,\mathrm{N}\hat{\jmath}}$$

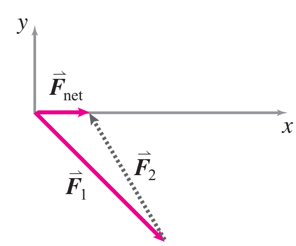

Figure 1.26: $\vec{F}_2 = \vec{F}_{\text{net}} - \vec{F}_1$.

SAMPLE 1.7 **Vector sums and their magnitudes:** If $|\vec{A} + 2\vec{B}| = 2|\vec{A} + \vec{B}|$, does it imply that $\vec{A} + 2\vec{B} = 2(\vec{A} + \vec{B})$? Let $\vec{A} = -16\hat{\imath} + 8\hat{\jmath}, \vec{B} = 15\hat{\imath}$ and $\vec{C} = \vec{A} + \vec{B}$. Show that if $\vec{D} = \vec{A} + 2\vec{B}$, then $|\vec{D}| = 2|\vec{C}|$ but $\vec{D} \neq 2\vec{C}$?

Solution

$$\begin{aligned}
\vec{C} &= \vec{A} + \vec{B} = (-16\hat{\imath} + 8\hat{\jmath}) + (15\hat{\imath}) = -1\hat{\imath} + 8\hat{\jmath} \\
|\vec{C}| &= \sqrt{1^2 + 8^2} = \sqrt{65}.
\end{aligned}$$

If $\vec{B}$ is doubled, then the new vector sum $\vec{D}$ is

$$\begin{aligned}
\vec{D} &= \vec{A} + 2\vec{B} = (-16\hat{\imath} + 8\hat{\jmath}) + 2(15\hat{\imath}) = 14\hat{\imath} + 8\hat{\jmath} \\
|\vec{D}| &= \sqrt{14^2 + 8^2} = \sqrt{260} = \sqrt{4 \times 65} = 2\sqrt{65} = 2|\vec{C}|.
\end{aligned}$$

Thus $|\vec{D}| = 2|\vec{C}|$. But $2\vec{C} = 2(-1\hat{\imath} + 8\hat{\jmath}) = -2\hat{\imath} + 16\hat{\jmath} \neq \underbrace{14\hat{\imath} + 8\hat{\jmath}}_{\vec{D}}$.

$$\boxed{\text{If } |\vec{A} + 2\vec{B}| = 2|\vec{A} + \vec{B}|, \text{ it does not imply that } \vec{A} + 2\vec{B} = 2(\vec{A} + \vec{B})}$$

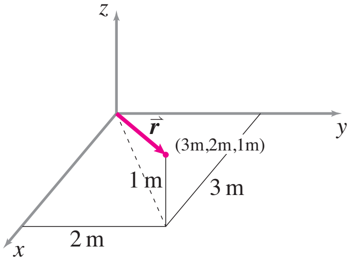

Figure 1.27: The position vector of the particle is a vector drawn from the origin of the coordinate system to the position of the particle.

SAMPLE 1.8 Position vector from the origin: In the xyz coordinate system, a particle is located at the coordinate (3m, 2m, 1m). Find the position vector of the particle.

Solution The position vector of the particle at P is a vector drawn from the origin of the coordinate system to the position P of the particle. See Fig. 1.27. We can write this vector as

$$\begin{aligned} \vec{r}_P &= (3\,\text{m})\hat{\imath} + (2\,\text{m})\hat{\jmath} + (1\,\text{m})\hat{k} \\ \text{or} \quad \vec{r}_P &= (3\hat{\imath} + 2\hat{\jmath} + \hat{k})\,\text{m}. \end{aligned}$$

$$\boxed{\vec{r}_P = 3\,\text{m}\hat{\imath} + 2\,\text{m}\hat{\jmath} + 1\,\text{m}\hat{k}}$$

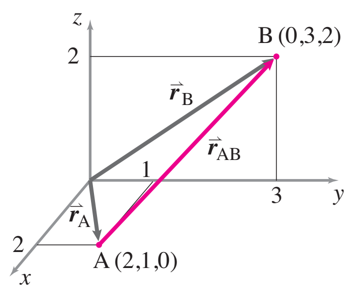

Figure 1.28: The position vector of B with respect to A is found from $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$.

SAMPLE 1.9 Relative position vector: Let A (2m, 1m, 0) and B (0, 3m, 2m) be two points in the xyz coordinate system. Find the position vector of point B with respect to point A, *i.e.*, find $\vec{r}_{AB}$ (or $\vec{r}_{B/A}$).

Solution From the geometry of the position vectors shown in Fig. 1.28 and the rules of vector sums, we can write,

$$\begin{aligned} \vec{r}_B &= \vec{r}_A + \vec{r}_{AB} \\ \Rightarrow \quad \vec{r}_{AB} &= \vec{r}_B - \vec{r}_A = (0\hat{\imath} + 3\,\text{m}\hat{\jmath} + 2\,\text{m}\hat{k}) - (2\,\text{m}\hat{\imath} + 1\,\text{m}\hat{\jmath} + 0\hat{k}) \\ &= -2\,\text{m}\hat{\imath} + 2\,\text{m}\hat{\jmath} + 2\,\text{m}\hat{k}. \end{aligned}$$

$$\boxed{\vec{r}_{AB} \equiv \vec{r}_{B/A} = -2\,\text{m}\hat{\imath} + 2\,\text{m}\hat{\jmath} + 2\,\text{m}\hat{k}}$$

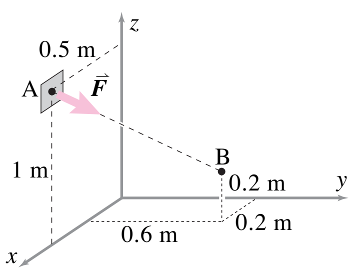

Figure 1.29

SAMPLE 1.10 Finding a force vector given its magnitude and line of action: A string AB is pulled with a force $F = 100$ N as shown in fig. 1.29. Write F as a vector.

Solution A vector can be written as the product of a scalar and a unit vector along its direction. Here, the magnitude of the force is given and we know it acts along AB. Therefore, we may write $\vec{F} = F\hat{\lambda}_{AB}$, where $\hat{\lambda}_{AB}$ is a unit vector along AB. So now we need to find $\hat{\lambda}_{AB}$. We can easily find $\hat{\lambda}_{AB}$ if we know vector AB. Let us denote vector AB by $\vec{r}_{AB}$ (same as $\vec{r}_{B/A}$). To find $\vec{r}_{AB}$, we note that (see Fig. 1.30)

$$\vec{r}_A + \vec{r}_{AB} = \vec{r}_B$$

where $\vec{r}_A$ and $\vec{r}_B$ are the position vectors of point A and point B respectively. Hence,

$$\begin{aligned} \vec{r}_{B/A} &\equiv \vec{r}_{AB} = \vec{r}_B - \vec{r}_A \\ &= (0.2\,\text{m}\hat{\imath} + 0.6\,\text{m}\hat{\jmath} + 0.2\,\text{m}\hat{k}) - (0.5\,\text{m}\hat{\imath} + 1.0\,\text{m}\hat{k}) \\ &= -0.3\,\text{m}\hat{\imath} + 0.6\,\text{m}\hat{\jmath} - 0.8\,\text{m}\hat{k}. \end{aligned}$$

Therefore,

$$\hat{\lambda}_{AB} = \frac{-0.3\,\text{m}\hat{\imath} + 0.6\,\text{m}\hat{\jmath} - 0.8\,\text{m}\hat{k}}{\sqrt{(-0.3)^2 + (0.6)^2 + (-0.8)^2}\ \ \text{m}} = -0.29\hat{\imath} + 0.57\hat{\jmath} - 0.77\hat{k},$$

and, finally,

$$\vec{F} = \overbrace{(100\,\text{N})}^{F}\,\hat{\lambda}_{AB} = -29\,\text{N}\hat{\imath} + 57\,\text{N}\hat{\jmath} - 77\,\text{N}\hat{k}.$$

$$\boxed{\vec{F} = -29\,\text{N}\hat{\imath} + 57\,\text{N}\hat{\jmath} - 77\,\text{N}\hat{k}}$$

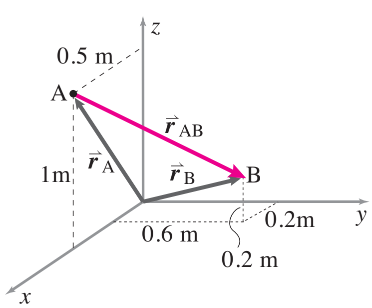

Figure 1.30: $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$.

SAMPLE 1.11 Adding vectors on computers: The following six forces act at different points of a structure. $\vec{F}_1 = -3\,\mathrm{N}\hat{\jmath}$, $\vec{F}_2 = 20\,\mathrm{N}\hat{\imath} - 10\,\mathrm{N}\hat{\jmath}$, $\vec{F}_3 = 1\,\mathrm{N}\hat{\imath} + 20\,\mathrm{N}\hat{\jmath} - 5\,\mathrm{N}\hat{k}$, $\vec{F}_4 = 10\,\mathrm{N}\hat{\imath}$, $\vec{F}_5 = 5\,\mathrm{N}(\hat{\imath} + \hat{\jmath} + \hat{k})$, $\vec{F}_6 = -10\,\mathrm{N}\hat{\imath} - 10\,\mathrm{N}\hat{\jmath} + 2\,\mathrm{N}\hat{k}$.

1. Write all the force vectors in column form.
2. Find the net force by hand calculation.
3. Write a computer program to sum n vectors, each with three components. Use your program to compute the net force.

Solution

1. The 3-D vector $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$ is represented as a column (or a row) as follows:

$$[\vec{F}]_{xyz} = \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}$$

   Following this convention, we write the given forces as

$$[\vec{F}_1]_{xyz} = \begin{pmatrix} 0 \\ -3\,\mathrm{N} \\ 0 \end{pmatrix}, [\vec{F}_2]_{xyz} = \begin{pmatrix} 20\,\mathrm{N} \\ -10\,\mathrm{N} \\ 0 \end{pmatrix}, \cdots, [\vec{F}_6]_{xyz} = \begin{pmatrix} -10\,\mathrm{N} \\ -10\,\mathrm{N} \\ 2\,\mathrm{N} \end{pmatrix}$$

2. The net force $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6$, or

$$\begin{aligned} [\vec{F}_{\text{net}}]_{xyz} &= \begin{pmatrix} 0 & & 20 & & 1 & & 10 & & 5 & & -10 \\ -3 & + & -10 & + & 20 & + & 0 & + & 5 & + & -10 \\ 0 & & 0 & & -5 & & 0 & & 5 & & 2 \end{pmatrix} \mathrm{N} \\ &= \begin{pmatrix} 26 \\ 2 \\ 2 \end{pmatrix} \mathrm{N} \end{aligned}$$

3. The steps to do this addition on computers are as follows.
   - Enter the vectors as rows or columns:

```
F1 = [0  -3  0]
F2 = [20  -10  0]
F3 = [1  20  -5]
F4 = [10  0  0]
F5 = [5  5  5]
F6 = [-10  -10  2]
```

   - Sum the vectors, using a summing operation that automatically does element by element addition of vectors:

```
Fnet = F1 + F2 + F3 + F4 + F5 + F6
```

   - The computer generated answer is:

```
Fnet = [26  2  2].
```

$$\boxed{\vec{F}_{\text{net}} = 26\,\mathrm{N}\hat{\imath} + 2\,\mathrm{N}\hat{\jmath} + 2\,\mathrm{N}\hat{k}}$$