2.3 Free-body diagrams: interactions, showing forces, and partial FBDs

A *free-body diagram* is a sketch of the system of interest and the forces that act on the system.

A free-body diagram precisely defines the system to which you are applying mechanics equations and the forces to be considered. Any reader of your calculations needs to see your free-body diagrams. To put it directly, if you want to be right and be seen as right, then ①

Draw a free-body diagram!

The concept of the free-body diagram is simple. In practice, however, drawing useful free-body diagrams takes some thought, even for those practiced at the art. Some basic tips are described below a few different ways.

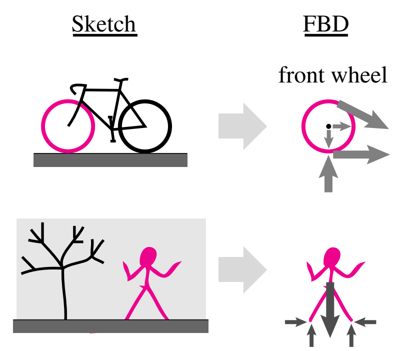


Figure 2.37: A sketch of a bicycle and a free-body diagram of the braked front wheel. A sketch of a person and a free-body diagram of the whole person.

①**Free-body Perkins.** In the 1950's Cornell professor Harold C. Perkins was nick-named 'Free-Body Perkins'. Perkins would stop students in the hall and say "You! Come in my office! Draw a free-body diagram of *this!*" Students drew free-body diagrams to please persnickety Perkins. And, so learned how to get mechanics problems right.

How to draw a free-body diagram

We suggest the following procedure for drawing a free-body diagram, as shown schematically in fig. 2.38

1. **Define the system.** In your own mind, define what system, or what collection of material, you would like to apply the laws of mechanics to. This 'body' may be just a part of your overall system of interest. Figure 2.39 on page 3 shows some possible systems when considering pliers.
2. **Sketch the system.** Your sketch may include various cut marks to show how the 'body' is isolated from its environment. Imagine cutting the system free from its environment with a sharp scalpel, or with a chain-saw.
3. **Stare at each cut.** Look at the picture at all of the places that the system interacts with material *not* shown in the picture, places where you made 'cuts'.
4. **Fool the body.** Use forces and torques to fool the system into thinking it has not been cut. For example, if, at a given contact point where you have cut the system free, the system is being pushed in a given direction, then show a force in that direction at that point. If a system is being prevented from rotating by a (cut) rod, then show a torque at that cut.
5. **Replace gravity with a force.** To show that you have cut the system free from the earth's gravity force, show the force of gravity on the system's center-of-mass or on the centers of mass of its parts.

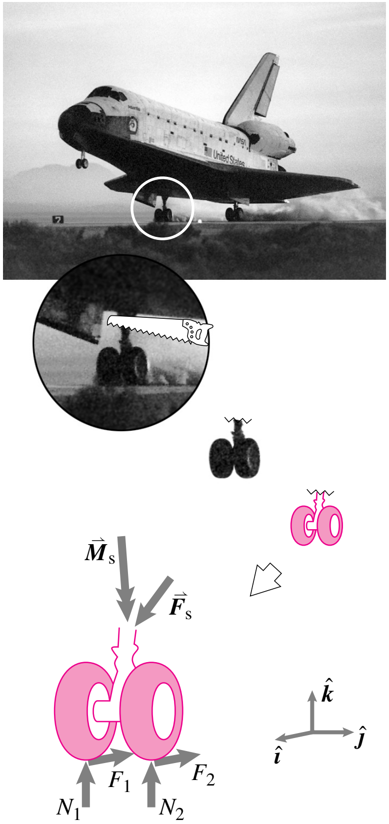


Figure 2.38: The process of drawing a FBD is illustrated by the sequence shown.

What shows on a free-body diagram? And what doesn't?

Here are some more details about the elements of a good free-body diagram. Even though some of these are stylistic issues, we think they help with problem solving. So, unless you are confident that you know better, take the comments here as truth.

- **The system.** A free-body diagram is a picture of the system for which you would like to apply the laws of mechanics (These are linear or angular momentum balance, force and moment balance being special cases, or power balance). The free-body diagram shows the system isolated ('free') from its environment. That is, the free-body diagram does *not* show things that are near to or touching the system of interest. The system must be *free* from its environment. See fig. 2.37.
- **The word 'body' means system.** A free-body diagram may show one or more particles, rigid objects, deformable objects, or parts thereof such as a machine, a component of a machine, or a part of a component of a machine. You can draw a free-body diagram of any collection of material that you can identify. The word *body* connotes a standard object in some people's minds. In the context of free-body diagrams, 'body' means system. The body in a free-body diagram may be a subsystem of the overall system of interest. For a system with n parts there are $2^n - 1$ sub-collections of parts. For the pliers of fig. 2.39 there are 4 parts and 15 possible FBDs (6 of which are shown).
- **Forces fool the system.** The free-body diagram of a system shows the forces and moments that the surroundings impose on the system. That is, since the only method of mechanical interaction that Nature has invented is force (and moment), the free-body diagram shows what it would take to mechanically fool the system if it were, literally, cut free. That is,

> The motion of the system would be totally unchanged if it were cut free and the forces shown on the free-body diagram were applied as a replacement for all of the actual external interactions②.

- **Each force has a source and a target.** Every force shown on a FBD acts *on* the system (the body) and *from* another object according to some *rule*. For each force you should be able to name the target (the 'free-body'), the source (*e.g.,* a contacting body) and the rule (*e.g.,* laws of gravity, a spring equation, the force sufficient to prevent interpenetration). Subscripts can help, such as F_{ED} indicating the force is from E and on D (See fig. 2.39).

- **Place forces at cuts.** The forces and moments are located on the free-body diagram at the points where they are applied. These are the places where you made ‘cuts’ to *free* the *body*.
- **Motion is caused by, or prevented by, forces.** At places where the outside environment causes, or restricts, translation of the isolated system, a contact force is drawn on the free-body diagram.
- **Rotation is caused by, or prevented by, torques.** At connections to the outside world that cause or restrict rotation of the system a contact torque (or couple or moment) is drawn. Draw this moment outside the system for viewing clarity. Look at fig. 2.40 and see how the moment on the block, due to the friction of the hinge, is best shown *outside* the block.
- **Draw contact forces outside the body.** Draw the contact force outside the sketch of the system for viewing clarity. A block supported by a hinge with friction in fig. 2.40 illustrates how the reaction force on the block due to the hinge is best shown outside the block.
- **Draw body forces (*e.g.,* gravity forces) inside the body.** The free-body diagram shows the system, cut free from the source of any *body forces* that are applied to the system. Body forces are forces that act on the inside of a body from objects outside the body. Draw the body forces on the interior of the body at their effective (average) point of application; this is the center of mass for near-earth gravity forces. Figure 2.40 shows the cleanest way to represent the gravity force on the uniform block acting at the center-of-mass. ③
- **Internal forces are not drawn.** The free-body diagram shows all external forces acting on the system but *no* internal forces — forces between objects within the body are not shown. See fig. 2.41 on page 8 for examples of what, despite temptation, *not* to do.
- **No velocity and no acceleration.** The free-body diagram shows nothing about the motion④. A free body diagram shows: no “centrifugal force”, no “acceleration force”, and no “inertial force” (Of course, for statics this is a non-issue because inertial terms are neglected for all purposes.), Repeating,

> *Velocities, accelerations and inertial forces do not show on a free-body diagram* ⑤

How to draw forces on free-body diagrams

How you draw a force on a free-body diagram depends on

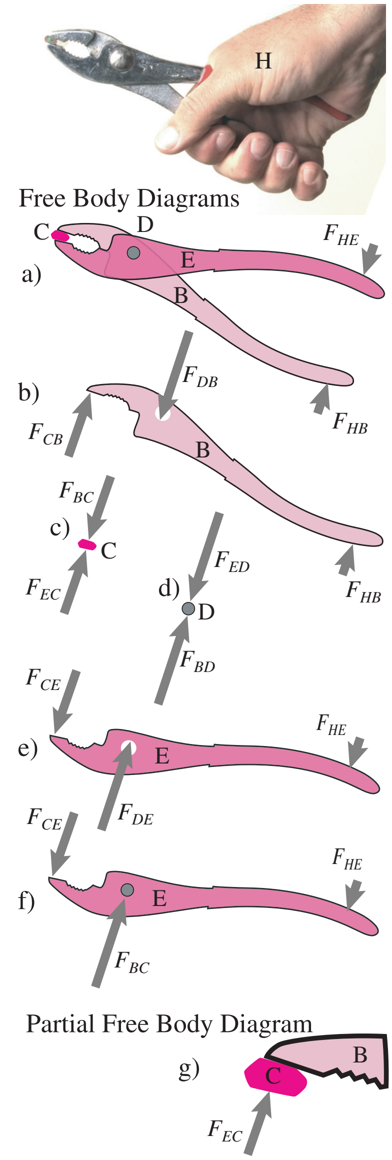


Figure 2.39: **Pliers crushing a pencil.** Some possible free-body diagrams (FBDs), neglecting gravity. With four major parts (upper jaw, lower jaw, pin & pencil) there are 15 possible subsystems: **a)** Whole system; **b)** Upper jaw; **c)** Pencil; **d)** Connecting pin; **e)** Lower jaw; **f)** Lower jaw with connecting pin; and nine others (*e.g.,* pliers without pencil, upper jaw plus pin, both jaws plus pencil, *etc.*). For each system, the external forces are shown. **g)** shows a *partial* free-body diagram, showing the force on the pencil and upper jaw but not all of the forces on the pencil and upper jaw. See fig. 2.41 on page 8 for some bad FBDs of this system.

- How much you know about the force before your analysis. Do you know its direction? its magnitude? and
- Your choice of notation (which may vary from vector to vector within one free-body diagram). See page 52 for a description of the 'symbolic' and 'graphical' vector notations.

Some of the possibilities are shown in fig. 2.42 when

(a) Any $\vec{F}$ possible,

(b) the direction of $\vec{F}$ is known, and

(c) Everything about $\vec{F}$ is known.

In each case three different notations are shown.

Using equivalent force systems to simplify

The concept of 'fooling' a system with forces is somewhat subtle. If the free-body diagram involves 'cutting' a rope what force should one show? A rope is made of many fibers so cutting the rope means cutting all of the rope fibers. Should one show hundreds of force vectors, one for each fiber that is cut? The answer is: yes and no. You would be correct to draw all of these hundreds of forces at the fiber cuts. But, since the equations that are used with any free-body diagram involve only the total force and total

② **Technical caveat.** What would be the same are the aspects of the motion determined by using the laws of mechanics on *that* free body diagram. To get all of the system's distortions right, all of the possible free body diagrams, of all possible subsystems, need also to be drawn correctly.

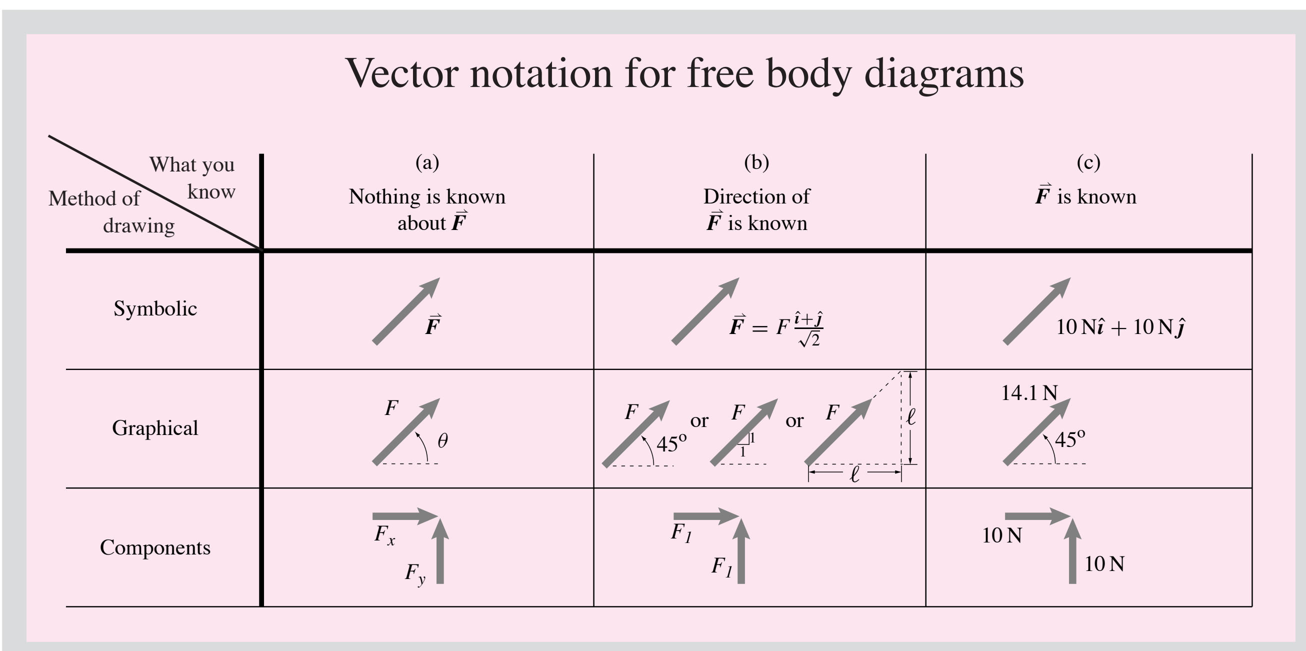


Figure 2.42: The various ways of notating a force on a free-body diagram. In column **(a)** nothing is known and everything is variable. In column **(b)** the direction is known and the magnitude isn't. In column **(c)** everything is known. In one free-body diagram different notations can be used for different forces, as needed or convenient. Other unusual cases can be extrapolated, *e.g.,* if the magnitude is known and the direction is unknown.

moment, you are also allowed to replace these forces with an equivalent force system (see section 2.1).

> *Any force system acting on a given free-body diagram can be replaced by an equivalent force and couple.*

In the case of a rope, a single force directed nearly parallel to the rope and acting at about the center of the rope's cross section is equivalent to the force system consisting of all the fiber forces. In the case of an ideal rope, the force is exactly parallel to the rope and acts exactly at its center.

Similarly the force of the net effect of the distributed ground forces on a shoe is often represented by a single force at "the center of pressure".

Action and reaction

For some systems you will want to draw free-body diagrams of subsystems. For example, to study a machine, you may need to draw free-body diagrams of several of its parts; for a building, you may draw free-body diagrams of various structural components; and, for a biomechanics analysis, you may 'cut up' a human body (with your imagined scalpel). When separating a system into parts, you must take account of how the subsystems interact. Call the two touching parts of a machine $\mathcal{A}$ and $\mathcal{B}$. Then,

If	$\mathcal{A}$	feels force	$\vec{F}$	and couple	$\vec{M}$	from $\mathcal{B}$,
then	$\mathcal{B}$	feels force	$-\vec{F}$	and couple	$-\vec{M}$	from $\mathcal{A}$.

To be precise, we must make clear that $\vec{F}$ and $-\vec{F}$ have the same line of action.

The principle of action and reaction doesn't say anything about what force or moment acts on one object. It only says that the actor of a force and moment gets back the opposite force and moment.

It is easy to make mistakes when drawing free-body diagrams involving action and reaction. Box 2.8 on page 14 shows some correct and incorrect partial FBD's of interacting bodies $\mathcal{A}$ and $\mathcal{B}$. Use notation consistent with fig. 2.42 on page 4 for the action and reaction vectors.

Is the principle of action and reaction an independent law? That's debatable⑥. But, no-one doubts the truth of it.

Which force is the action and which the reaction? Often one has a system in mind and the forces *on the system* are the actions and the forces *on the surroundings* by the system are the reactions⑦. For a complicated system, where one draws lots of free body diagrams of subsystems, just remember that forces come in pairs: A force on the system by the surroundings and a force on the surroundings by the system. It doesn't matter which one you call "the reaction".

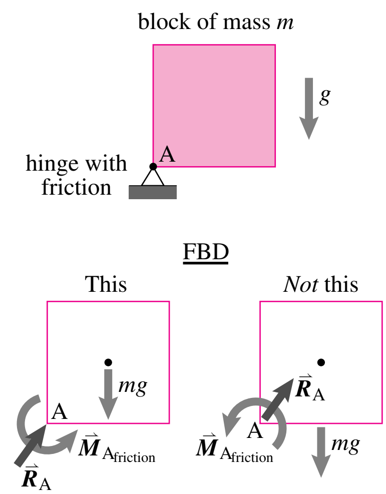


Figure 2.40: A uniform block of mass m supported by a hinge with friction in the presence of gravity. The free-body diagram on the right is correct, just less clear than the one on the left.

③**Gravity and Center of Gravity.** In this book, the only body force we consider is gravity. For near-earth gravity, gravity forces show on the free-body diagram as a single force at the center of gravity, or as a collection of forces each acting at the center of gravity of a system part. (For parts of electric motors and generators, not covered here in detail, electrostatic or electrodynamic body forces also need to be considered.)

④**Warning.** A common error made by beginning dynamics students is to put velocity and/or acceleration arrows on the free-body diagram.

⑤**A white lie.** The prescription that you *not show inertial forces* is a white lie. Actually, in the so-called d'Alembert approach to dynamics, a legitimate and intuitive approach for experts, one does show inertial forces on the free-body diagram. The d'Alembert approach is not followed in this book in any theory. Why not? Because of the frequent sign errors and mind-confusions it causes in beginners (translation for readers: "not allowed in homework or exams"). For those who are attracted to forbidden fruit, see the box titled "*D'Alembert's mechanics: beginners beware*" in the Dynamics book.

Interactions

The way objects interact mechanically is by the transmission of a force or a set of forces. If you want to show the effect of body $\mathcal{B}$ on $\mathcal{A}$, in the most general case you can expect a force and a moment. These two, together, are equivalent to the whole force set acting on $\mathcal{A}$ from $\mathcal{B}$, however complex.

That is, the most general interaction of two bodies requires knowing

- Three numbers in two dimensions (two force components and one moment), and
- Six numbers in three dimensions (three force components and three moment components)

Most things don't interact in this most general way so, usually, fewer numbers are required.

Some of the common ways in which mechanical things interact, at least ideally, are described in the following sections. As you read this, refer also to first three columns of the summary table on page 1219. You should look frequently at this table, until you have absorbed it. You will use the forces and moments of these connections again and again.

Constrained motion and free motion

One general principle of interaction forces and moments concerns 'geometric' constraints.

> Wherever a *motion* of $\mathcal{A}$ is either caused *or* prevented by $\mathcal{B}$ there is a corresponding *force* shown at the interaction point on the free-body diagram of $\mathcal{A}$.

Similarly,

> If $\mathcal{B}$ causes or prevents *rotation* there is a *moment* (or torque or couple) shown on the free-body diagram of $\mathcal{A}$ at the place of interaction.

The converse is also true. Many kinds of mechanical attachment gadgets are specifically designed to allow motion.

> If an attachment allows free motion in some direction, a so called *degree of freedom*, then the free-body diagram shows no force in that direction. If the attachment allows free rotation about an axis then the free-body diagram shows no moment (couple or torque) about that axis.

You can think of each attachment point as having a variety of jobs to do. For every possible direction of translation and rotation, the attachment has to either allow free motion or restrict the motion. In every way that motion is restricted (or caused) by the connection, a force or moment is required. In every way that motion is free, there is no force or couple. Motion of body $\mathcal{A}$ is caused and restricted by forces and couples which act on $\mathcal{A}$. Motion is freely allowed by the absence of such forces and couples.

Here, demonstrating the ideas above, are some of the common connections.

Cuts at 'rigid' connections

Sometimes the body you draw in a free-body diagram is firmly attached to another. Figure 2.43 shows a cantilever structure on a building. The free-body diagram of the cantilever has to show all possible force and load components. Since we have used vector notation for the force $\vec{\boldsymbol{F}}$ and the moment $\vec{\boldsymbol{M}}_{\mathrm{C}}$ we can be ambiguous about whether we are doing a two- or three-dimensional analysis.

Gravity is pointing down, so why do we show a horizontal reaction force at C? This is a reasonable question because a quick statics analysis shows that, for a stationary building and cantilever, that $\vec{\boldsymbol{F}}_C$ must be vertical. There are two reasons to show the horizontal force anyway

1. Mechanics includes both statics *and* dynamics. In dynamics *the forces on a body do not add to zero*. In fact, we forgot to tell you, the building shown in fig. 2.43 happens to be accelerating rapidly to the right due to the motions of a violent earthquake occurring at the instant pictured in the figure.
2. Whether or not there is an earthquake, the attachment of the cantilever to the building at C in fig. 2.43 is surely intended to be rigid and prevent the cantilever from moving up or down (falling), and from moving sideways (and drifting into another building) and from rotating about point C. In most of the building's life, the horizontal reaction at C is small. But, because the connection at C clearly prevents relative horizontal motion, it is probably best to draw a horizontal reaction force on the free-body diagram. Then, the same free-body diagram is good during earthquakes and during more boring times.

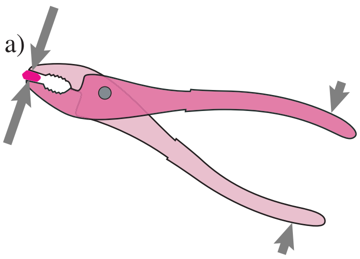
   When you know a force is going to turn out to be zero, as for the sideways force in this example if treated as a statics problem, it is a matter of taste whether or not you show the sideways force on the free-body diagram (Box 2.7 on page 12 discusses just this issue).

Our general advice is

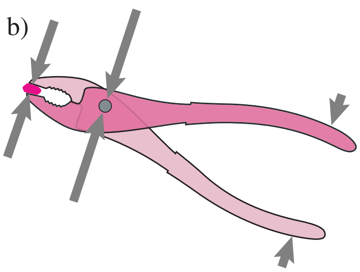
> 'Better safe than sorry'. If you don't *know* that a force or moment is going to turn out to be zero, leave it in the free-body diagram.

Wrong Free Body Diagrams

a)



b)



Wrong Partial Free Body Diagram

c)

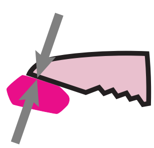


Figure 2.41: **The pencil is totally crushed by the pliers.** So it seems natural to show the crushing force on a sketch of the pliers crushing the pencil (a). And one might want to show the big force on the connecting pin between the pliers jaws (b). But **these sketches are not Free-Body Diagrams.** Free-body diagrams only show external forces on the system. For the systems shown in the sketches, (a) and (b) above, the forces on the pencil and on the pin and between the parts are internal forces and *should not* show on a free-body diagram. Similarly a partial free-body diagram (c) of part of the upper jaw and the pencil should *not* show the action and reaction pair (they are internal to the subsystem shown). See fig. 2.39 on page 3 for some good FBDs related to the pliers.

The situation with rigid connections, like the cantilever above, is shown more abstractly in both 3D and 2D in fig. 2.44.

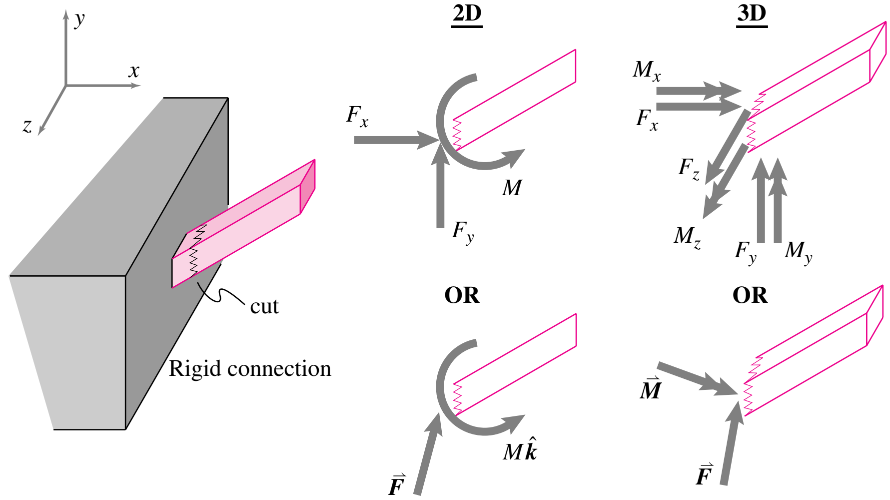


Figure 2.44: A rigid connection shown with partial free-body diagrams in two and three dimensions. One has a choice between showing the separate force components (top) or using the vector notation for forces and moments (bottom). On the moment vector, the double head is optional.

Cuts at hinges

A hinge, shown in fig. 2.46, allows rotation and prevents translation. Thus, the free-body diagram of an object cut at a hinge shows *no* torque about the hinge axis but does show the force or its components which prevent translation.

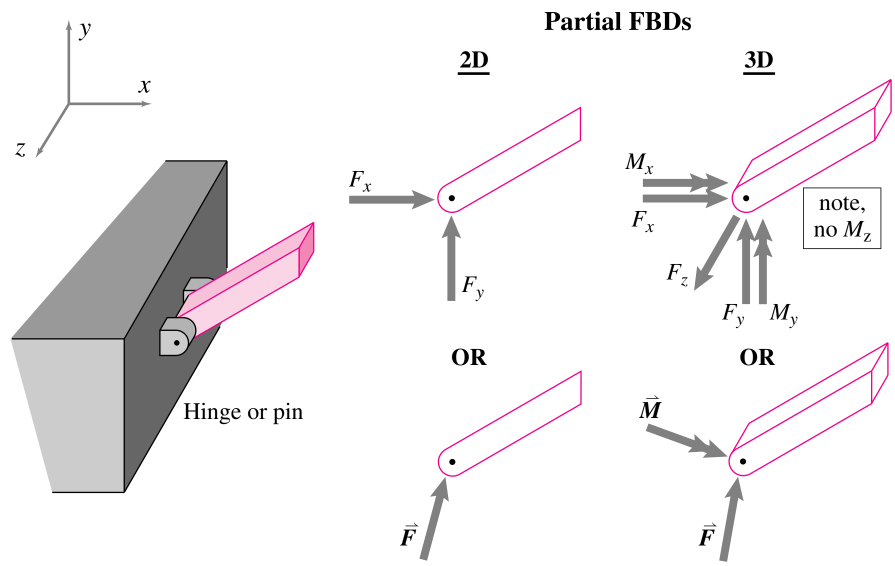


Figure 2.46: **A hinge with partial free-body diagrams in 2D and 3-D.** A hinge joint is also called a pin joint because it is sometimes built by drilling a hole and inserting a pin.

There is some ambiguity about how to model pin joints (hinges) in three dimensions. The ambiguity is shown with reference to a hinged door (fig. 2.45) and discussed in detail below. Clearly, one hinge, if the sole attachment, prevents rotation of the door about the x and y axes shown. So, it is natural to show a couple (torque or moment) in the x direction, M_x, and in the y direction, M_y. But, the hinge does not provide very stiff resistance to rotations in these directions compared to the resistance of the other hinge. That is, even if both hinges are modeled as ball-and-socket joints (see the next sub-section), offering no resistance to rotation, the door still cannot rotate about the x and y axes.

The stiffer constraint wins. If a connection between objects prevents relative translation or rotation that is already prevented by another stiffer connection, then the more compliant connection reaction is often neglected. Even without rotational constraints, the translational constraints at the hinges A and B restrict rotation of the door shown in fig. 2.45. Thus each of the two hinges are probably well-modeled — that is, they will lead to reasonably accurate calculations of forces and motions — by ball-and-socket joints at A and B.

In 2-D a ball-and-socket joint is equivalent to a hinge or pin joint (with the axis of the hinge orthogonal to the page).

Bearing alignment. If two connections both do the same job, for example the two door hinges above, they might not do it exactly the same way. And the incompatibility can be a structural problem. Thus, for example, door hinges need to be well aligned in order that the door opening is free and to prevent large forces and moments of the hinges fighting each other.

Ball-and-socket joint

Sometimes one wishes to attach two objects in a way that allows no relative translation but for which all rotation is free. The device that is used for this purpose is called a 'ball-and-socket' joint. It is constructed by rigidly attaching a sphere (the ball) to one of the objects and rigidly attaching a partial spherical cavity (the socket) to the other object.

⑥ The principle of action and reaction can be derived from the momentum balance laws by drawing free-body diagrams of little slivers of material. Nonetheless, in practice you can think of the principle of action and reaction as a basic law of mechanics. Newton did. The principle of action and reaction is "Newton's third law".

⑦ Another, less-useful, interpretation of "the principle of Action and Reaction" is that some forces on a system are constraints, to be found, and some forces are more active (hence the word 'action'). The 'reactions' are the constraint forces that hold the system in place. In this (not supported here) interpretation, the principle of Action and Reaction is really just a statement about equilibrium of the system.

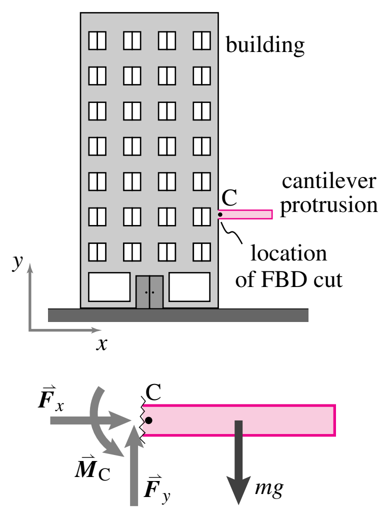


Figure 2.43: **A rigid connection: a cantilever structure on a building.** At the point C where the cantilever structure is connected to the building all motions are restricted so every possible force needs to be shown on the free-body diagram cut at C.

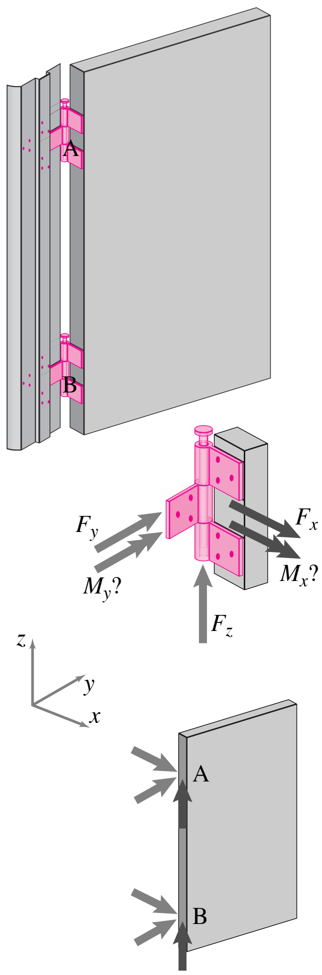


Figure 2.45: **A door held by hinges.** One must decide whether to model hinges as proper hinges or as ball-and-socket joints. The partial free-body diagram of the door at the lower right neglects the couples at the hinges, effectively idealizing the hinges as ball-and-socket joints. This idealization is generally quite accurate because the rotations that each hinge might resist are already resisted by there being two connection points.

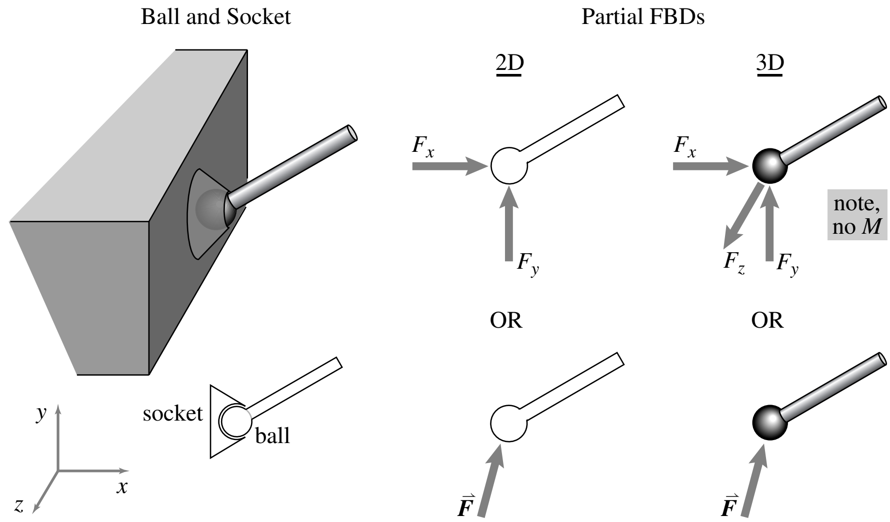


Figure 2.47: **A ball-and-socket joint** allows all relative rotations and no relative translations so reaction forces, but not moments, are shown on the partial free-body diagrams. In two dimensions a ball-and-socket joint is just like a pin joint. The top partial free-body diagrams show the reaction in component form. The bottom illustrations show the reaction in vector form.

The human hip joint is a ball-and-socket joint (See fig. 2.48). At the upper end of the femur bone is the femoral head, a sphere to within a few thousandths of an inch. The hip bone has a spherical cup that accurately fits the femoral head. The human hip joint is not so different from engineered ball and socket joints ⑧.

Car suspensions are constructed from a three-dimensional truss-like mechanism. Some of the parts need free relative rotation in three dimensions and thus use a joint called a ‘ball joint’ or ‘rod end’. A ‘rod-end is a ball-and-socket joint (Figure 2.49).

Since the ball-and-socket joint allows all rotations, no moment is shown at a cut ball-and-socket joint. Since a ball-and-socket joint prevents relative translation in all directions, the possibility of force in any direction is shown.

String, rope, wires, and light chain

One way to keep a radio tower from falling over is with wire, as shown in fig. 2.50. If the weight of the wires seems small, and the wind resistance is negligible, it is common to assume they can only transmit forces along the line connecting their end points. Moments are not shown because ropes, strings, and wires are generally assumed to be so compliant in bending that the bending moments are negligible. For wires

tension is the force pulling away from a free-body diagram cut⑨.

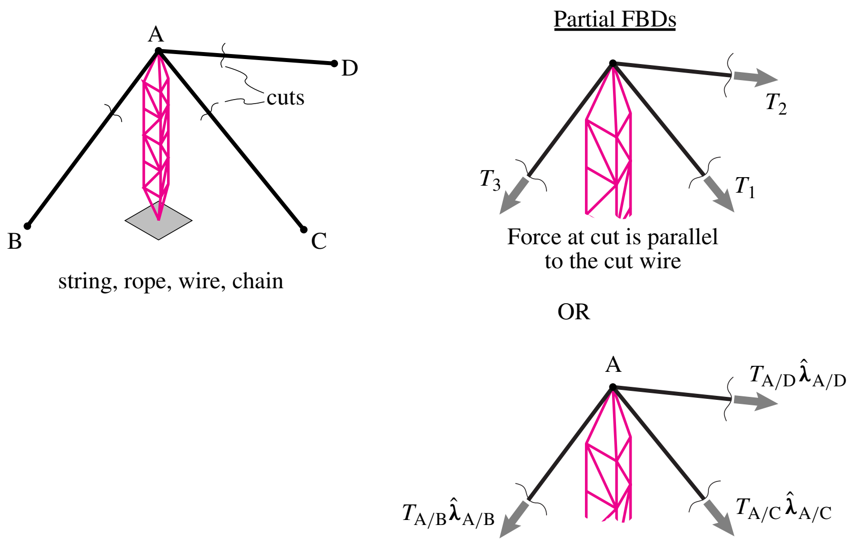


Figure 2.50: **A radio tower kept from falling with three wires.** A partial free-body diagram of the tower is drawn two different ways. The upper figure shows three tensions that are parallel to the three wires. The lower partial free-body diagram is more explicit, showing the forces to be in the directions of the $\hat{\lambda}$s, unit vectors parallel to the wires.

All this talk about force, what is force?

Force is the measure of mechanical interaction. It is a vector. It obeys the principle of action and reaction. Using forces on free-body diagrams, with

I. constitutive laws, like $F = kx$ and $F = mg$ (see Pillar 1 on page 28) and

II. mechanics laws, like $\sum \vec{F}_i = \vec{0}$ or $\vec{F} = m\vec{a}$ (see Pillar 3 on page 31)

we make accurate predictions. What is force? Its that quantity, that miraculously, has all these properties. What is force really? Beyond this constellation of relations, force is really ... never mind, that's just too deep.

Operationally, you can define force by how you can measure it. A force on a system can be measured by comparing its effect on the given system to

- A weight suspended by a string which goes over a pulley and is attached to the system of interest instead of the force.
- The effect of a calibrated spring on the system, or
- The effect, and would be hard to arrange in practice, of an accelerating mass connected by pulleys and strings to the system. Of course if you have some way of moving the force around without changing its magnitude, you can apply it to a mass and measure the acceleration it causes.

Figure 2.48: The ball part of an artificial hip joint. (Photo courtesy of Daniel Rutter, www.dansdata.com).

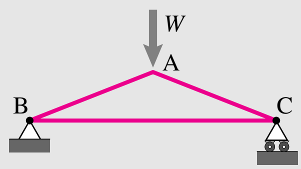
⑧**True story.** The Mann biomechanics lab at MIT put strain gauges in artificial hip joints, then surgically implanted these artificial joints in patients with bad bones to measure the hip forces (they measured contact pressure up to 18 Mpa$\approx$ 2600 lbf / $\in^2$). Dicky at the MIT boat house said he wanted a ball-and-socket joint for the base of the mast of the sailboat he was building. "Oh" said Crispin of the Mann lab, "we have a hip joint we don't need", and gave Dicky an uninstrumented metal hip to which Dicky welded this and that for use in one of his boat projects.

Figure 2.49: **A rod end.** Shown here in a jeep. A steel sphere slides in a truncated spherical cavity encased in the 'rod end'. Often the cavity is brass encased in the steel rod end. In this case, a lubrication grease-fitting nipple is also visible. (From fourwheeler.com)

- Other contraptions that somehow show the effect of the questionable force on a suspended weight, a stretched spring or an accelerated mass.

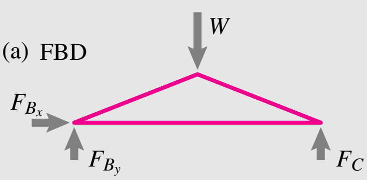
2.7 How much mechanics reasoning should you use when you draw a free-body diagram?

Consider the simple symmetric truss with a load W in the middle, a pin support at the left and a roller support at the right.



Here are various options for drawing a free-body-diagram of this truss.

(a) FBD

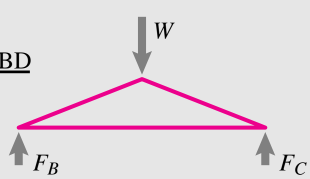


(a) The simple prescription is to draw an unknown force every place a motion is (caused or) prevented and an unknown torque where rotation is (caused or) prevented, as shown above. In particular, there is an unknown force restricting both horizontal and vertical motion at B.

Someone thinking ahead and noting that $F_{Bx} = 0$ might say that the free-body diagram in (a) is wrong. It is not. In FBD (a) force F_{Bx} is not specified because it is not known from just looking at the FBD cut at the pin. That $F_{Bx} = 0$ turns out to be zero is consistent with FBD (a) because $F_{Bx} = 0$ is not specified and thus could have any value, including zero.

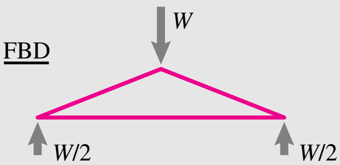
As a rule, we favor FBD (a).

(b) FBD



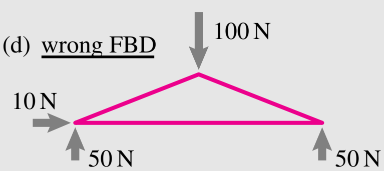
(b) A person who knows some statics will quickly deduce that the horizontal force at B is zero and thus draw the free-body diagram in figure (b). This is also correct, although somewhat violating the philosophy of drawing FBDs and later using mechanics reasoning.

(c) FBD



(c) Thinking ahead even more one could draw the free body diagram above. All three free-body diagrams above are correct. In particular diagram (a) is correct even though F_{Bx} turns out to be zero and (b) is correct even though F_B turns out to be equal to F_C. FBD (c) has the most information in it, but also most violates the problem solving approach: FBD first, mechanics later.

(d) wrong FBD



(d) In contrast, the free-body diagram above explicitly and incorrectly assigns a non-zero value to F_{Bx}, so it is wrong.

What to do? When in doubt we recommend following the naive rules yielding FBD (a). Then later use the force and momentum equations to find out more about the forces, *e.g.,* FBD (c). You might then never explicitly draw FBD (c), as it will be implicit in your assignments of values to the forces (*e.g.,* $F_{Bx} = 0$). If you are confident about the anticipated results, it might be a time saver to use diagrams analogous to (b) or (c) but

Beware of:

- making assumptions that are not reasonable, rather than just being more naive and correct, and
- wasting time trying to think ahead when the force and momentum balance equations will tell all in the end anyway.

A common error is to sloppily think through the mechanics laws and then incorrectly eliminate, or over-specify, forces on a FBD.

Summary of free-body diagrams.

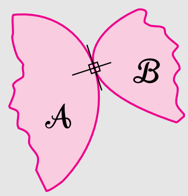
- Draw one or more clear free-body diagrams!
- Forces and moments on the free-body diagram show *all* mechanical interactions from outside the body.
- Every point on the boundary of a body has a force in every direction that motion is either being caused *or* prevented. Similarly with torques.
- If you do not know the direction of a force, use vector notation to show that the direction is yet to be determined.
- Leave off the free-body diagram forces that you think are negligible such as, possibly:
  - The force of air on small slowly moving bodies;
  - Forces that prevent motion that is already prevented by a much stiffer means (as for the torques at each of a pair of hinges);
  - See the table on page 1219 to see the forces at various connections.

9 **Caution:** Sometimes string-like things should *not* be treated as idealized strings. For example, *Short* wires can be stiff so bending moments may not be negligible. And the mass of chains can be significant so that the mass and weight may not be negligible. For a sagging chain, the direction of the tension force is not in the direction connecting the two chain endpoints.

2.8 Action and reaction on partial FBD’s

All students should master the notations here.

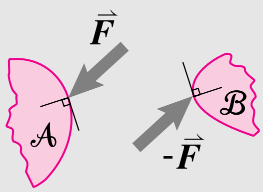
Objects $\mathcal{A}$ and $\mathcal{B}$ are interacting. You want to draw separate free-body diagrams (FBD’s) of each.



One force on the FBD of each shows the interaction force. The FBD of $\mathcal{A}$ shows the force of $\mathcal{B}$ on $\mathcal{A}$. And, the FBD of $\mathcal{B}$ shows the force of $\mathcal{A}$ on $\mathcal{B}$. There are various ways to show this, and many common errors for doing so. Below are several partial FBD’s of both $\mathcal{A}$ and $\mathcal{B}$ that show the principle of action and reaction. Figures (a - d) are correct and Figures (e - g) are not. See sample 1.1 on page 60 to resolve related subtleties about vector notation.

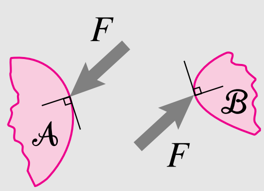
Correct partial FBD’s

(a)



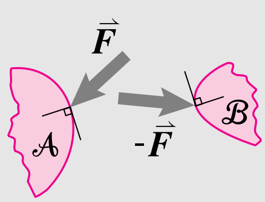
(a) These are good partial FBD’s. The action and re-action vectors ($\vec{F}$ and $-\vec{F}$) are equal in magnitude, opposite in sign, and applied on the same line of action. Because the symbolic notation takes precedence (see page 53), the direction and length of the drawn arrows, although drawn nicely here, are irrelevant.

(b)



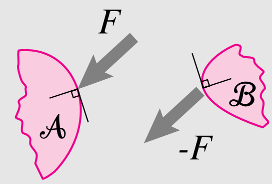
(b) These partial FBD’s are also good: the opposite arrows multiplied by equal magnitude F produce net vectors that are equal in magnitude and opposite in direction.

(c)



(c) The partial FBD’s may *look* wrong, and they are impractically misleading and not advised. But, technically, they are okay. Why? Because we take the vector notation to have precedence over the drawing inaccuracy.

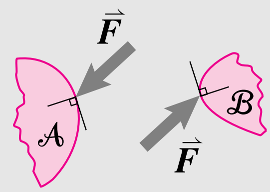
(d)



(d) The partial FBD’s may look wrong, but because no vector notation is used, the forces should be interpreted as in the direction of the drawn arrows and multiplied by the shown scalars. Since the same arrow is multiplied by F and $-F$, the net vectors are actually equal (in magnitude) and opposite (in direction).

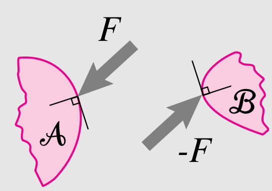
Wrong partial FBD’s

(e)



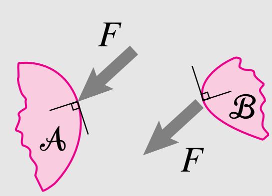
(e) These partial FBD’s are wrong because the vector notation $\vec{F}$ takes precedence over the drawn arrows. So, this shows the *same* force $\vec{F}$ acting on both $\mathcal{A}$ and $\mathcal{B}$, rather than the opposite force.

(f)



(f) Because the opposite arrow is multiplied by the negative scalars, the partial FBD’s here show the *same* force acting on both $\mathcal{A}$ and $\mathcal{B}$. Treating a double-negative as a negative is a common beginner’s error.

(g)



(g) These partial FBD’s are obviously wrong. They again show the same force acting on $\mathcal{A}$ and $\mathcal{B}$. These partial FBD’s would represent *the principle of double action* which applies to laundry detergents and not to mechanics.

SAMPLE 2.13 **A mass and a pulley.** A block of mass m is held up by applying a force F through a massless pulley as shown in the figure. Assume the string to be massless. Draw free-body diagrams of the mass and the pulley separately and as one system.

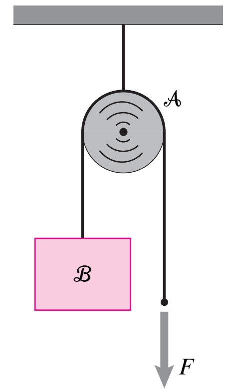


Figure 2.49

Solution The free-body diagrams of the block and the pulley are shown in Fig. 2.50. Since the string is massless and we assume an ideal massless pulley, the tension in the string is the same on both sides of the pulley. Therefore, the force applied by the string on the block is simply F. When the mass and the pulley are considered as one system, the force in the string on the left side of the pulley doesn't show because it is internal to the system.

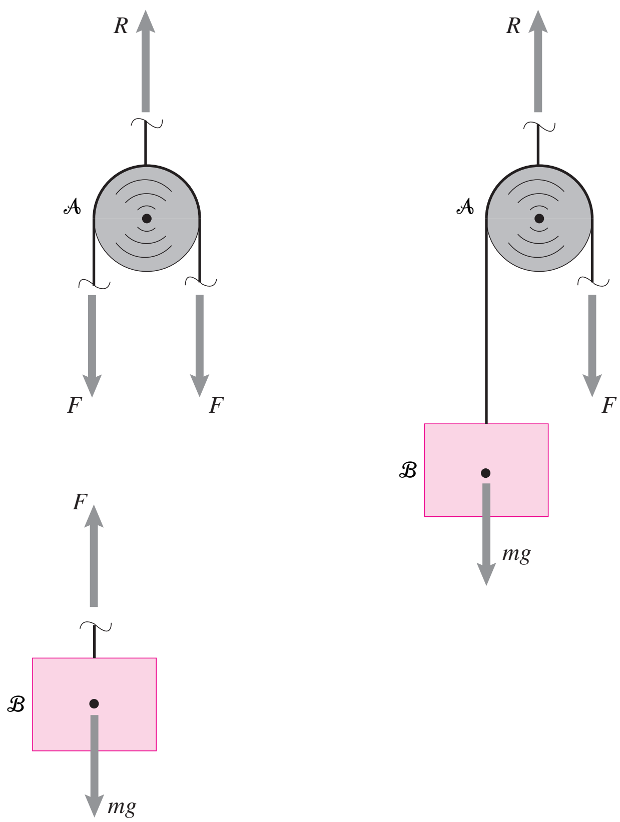


Figure 2.50: The free-body diagrams of the mass, the pulley, and the mass-pulley system. Note that for the purpose of drawing the free-body diagram we need not show that we know that $R = 2F$. Similarly, we could have chosen to show two different rope tensions on the sides of the pulley and reasoned that they are equal as is done in the text.

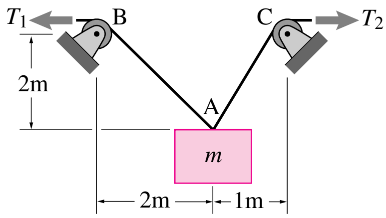


Figure 2.51

SAMPLE 2.14 Forces in strings. A block of mass m is held in position by strings AB and AC as shown in Fig. 2.51. Draw a free-body diagram of the block and write the vector sum of all the forces shown on the diagram. Use a suitable coordinate system.

Solution To draw a free-body diagram of the block, we first *free* the block. We cut strings AB and AC very close to point A and show the forces applied by the cut strings on the block. We also isolate the block from the earth and show the force due to gravity. (See Fig. 2.52.)

To write the vector sum of all the forces, we need to write the forces as vectors. To write these vectors, we first choose an xy coordinate system with basis vectors $\hat{\imath}$ and $\hat{\jmath}$ as shown in Fig. 2.52. Then, we express each force as a product of its magnitude and a unit vector in the direction of the force. So,

$$\vec{\boldsymbol{T}}_1 = T_1 \hat{\boldsymbol{\lambda}}_{AB} = T_1 \frac{\vec{\boldsymbol{r}}_{AB}}{|\vec{\boldsymbol{r}}_{AB}|},$$

where $\vec{\boldsymbol{r}}_{AB}$ is a vector from A to B and $|\vec{\boldsymbol{r}}_{AB}|$ is its magnitude. Neglecting the size of pulleys, we get, from the given geometry,

$$\begin{aligned} \vec{\boldsymbol{r}}_{AB} &= -2\,\mathrm{m}\hat{\imath} + 2\,\mathrm{m}\hat{\jmath} \\ \Rightarrow \qquad \hat{\boldsymbol{\lambda}}_{AB} &= \frac{2\cancel{\mathrm{m}}(-\hat{\imath} + \hat{\jmath})}{\sqrt{2^2 + 2^2}\,\cancel{\mathrm{m}}} = \frac{1}{\sqrt{2}}(-\hat{\imath} + \hat{\jmath}). \end{aligned}$$

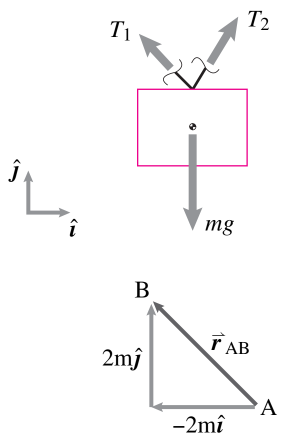


Figure 2.52: Free-body diagram of the block and a diagram of the vector $\vec{\boldsymbol{r}}_{AB}$.

Thus,

$$\vec{\boldsymbol{T}}_1 = T_1 \frac{1}{\sqrt{2}}(-\hat{\imath} + \hat{\jmath}).$$

Similarly,

$$\begin{aligned} \vec{\boldsymbol{T}}_2 &= T_2 \frac{1}{\sqrt{5}}(\hat{\imath} + 2\hat{\jmath}) \\ m\vec{\boldsymbol{g}} &= -mg\hat{\jmath}. \end{aligned}$$

Now, we write the sum of all the forces:

$$\begin{aligned} \sum \vec{\boldsymbol{F}} &= \vec{\boldsymbol{T}}_1 + \vec{\boldsymbol{T}}_2 + m\vec{\boldsymbol{g}} \\ &= \left(-\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{5}}\right)\hat{\imath} + \left(\frac{T_1}{\sqrt{2}} + \frac{2T_2}{\sqrt{5}} - mg\right)\hat{\jmath}. \end{aligned}$$

The $\hat{\imath}$ and $\hat{\jmath}$ components of the net force depend on the values of the scalars (magnitudes) T_1, T_2 and mg.

$$\boxed{\sum \vec{\boldsymbol{F}} = \left(-\frac{T_1}{\sqrt{2}} + \frac{T_2}{\sqrt{5}}\right)\hat{\imath} + \left(\frac{T_1}{\sqrt{2}} + \frac{2T_2}{\sqrt{5}} - mg\right)\hat{\jmath}}$$

SAMPLE 2.15 **Two bodies connected by a massless spring.** Two carts A and B are connected by a massless spring. The carts are pulled to the left with a force F and to the right with a force T as shown in Fig. 2.53. Assume the wheels of the carts to be massless and frictionless. Draw free-body diagrams of

- cart A,
- cart B, and
- carts A and B together.

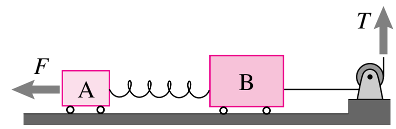


Figure 2.53: Two carts connected by a massless spring

Solution The three free-body diagrams are shown in Fig. 2.54 (a) and (b). In Fig. 2.54 (a) the force F_s is applied by the spring on the two carts. Why is this force the same on both carts? In Fig. 2.54(b) the spring is a part of the system. Therefore, the forces applied by the spring on the carts and the forces applied by the carts on the spring are internal to the system. Therefore these forces do not show on the free body diagram.

Note that the normal reaction of the ground can be shown either as separate forces on the two wheels of each cart or as a resultant reaction.

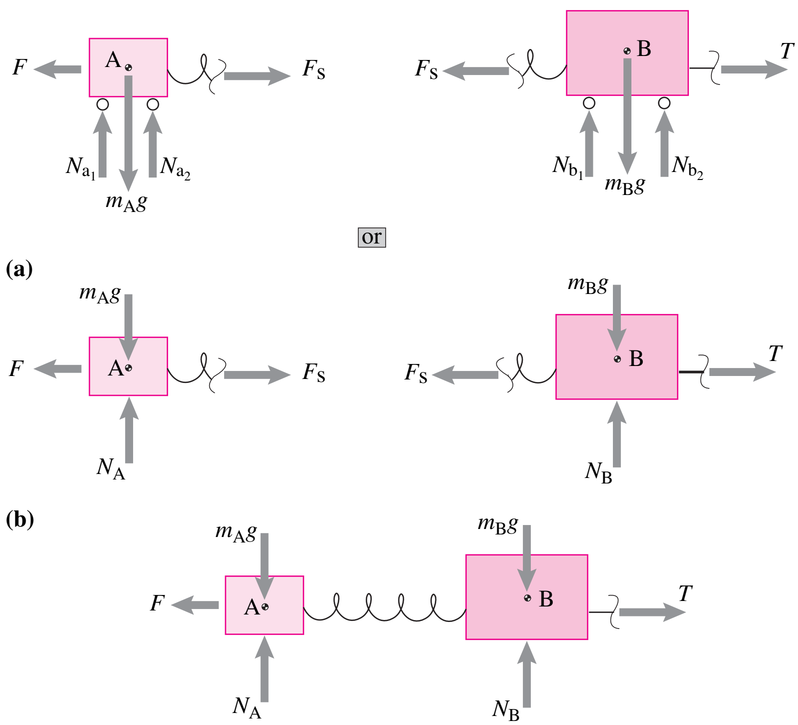


Figure 2.54: Free-body diagrams of (a) cart A and cart B separately and (b) cart A and B together

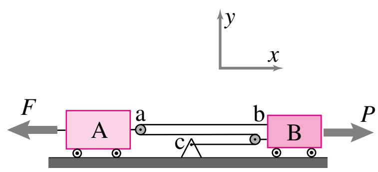


Figure 2.55: Two carts connected by massless pulleys.

SAMPLE 2.16 Two carts connected by pulleys. The two masses shown in Fig. 2.55 have frictionless bases and round frictionless pulleys. The inextensible massless cord connecting them is always taut. Mass A is pulled to the left by force F and mass B is pulled to the right by force P as shown in the figure. Draw free-body diagrams of each mass.

Solution Let the tension in the cord be T. Since the pulleys and the cord are massless, the tension is the same in each section of the cord. This equality is clearly shown in the Free-body diagrams of the two masses below.

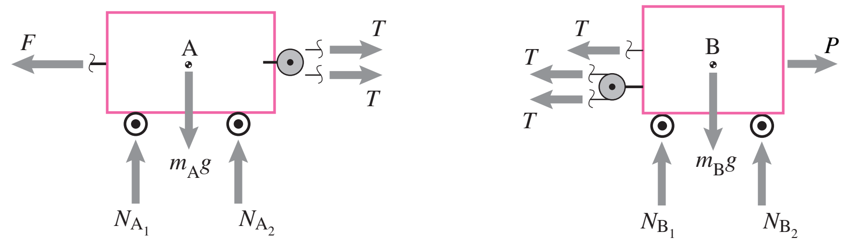


Figure 2.56: Free-body diagrams of the two masses.

Comments: We have shown unequal normal reactions on the wheels of mass B. In fact, the two reactions would be equal only if the forces applied by the cord on mass B satisfy a particular condition. Can you see what condition must be satisfied for, say, $N_{A_1} = N_{A_2}$.

[Hint: think about the moment balance about the center-of-mass A.]

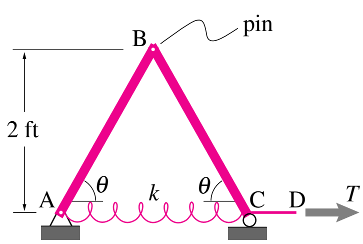


Figure 2.57

SAMPLE 2.17 Structures with pin connections. A horizontal force T is applied on the structure shown in the figure. The structure has pin connections at A and B and a roller support at C. Bars AB and BC are rigid. Draw free-body diagrams of each bar and the structure including the spring.

Solution The free-body diagrams are shown in figure 2.58. Note that there are both vertical and horizontal forces at the pin connections because pins restrict translation in any direction. At the roller support at point C there is only vertical force from the support (T is an externally applied force).

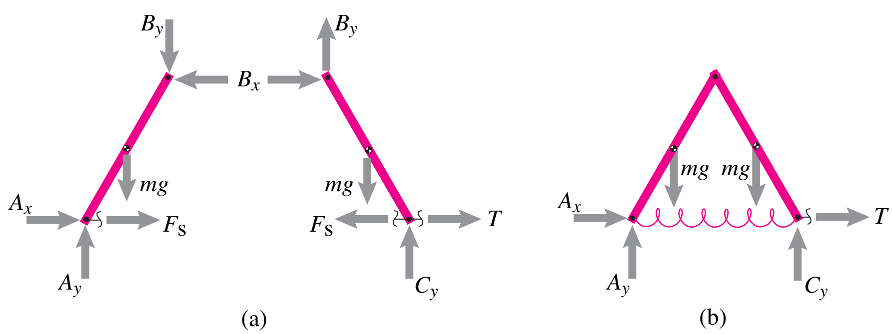


Figure 2.58: Free-body diagrams of (a) the individual bars and (b) the structure as a whole.

SAMPLE 2.18 **The four-bar linkage.** The four-bar linkage① shown in the figure is pushed to the right with a force F. Pins A, C & D have negligible friction but joint B is rusty and resists rotation (has non-negligible friction). Draw free-body diagrams of each of the bars separately and of the whole structure. Use consistent notation for the interaction forces and moments. Clearly mark the action-reaction pairs. Neglect gravity.

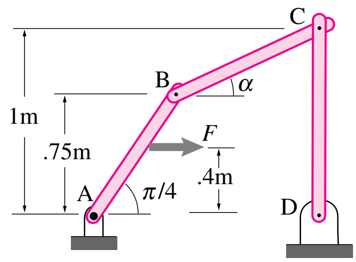


Figure 2.59: A four bar linkage. Force F is the only load. Pin B has non-negligible friction, the other pins have negligible friction.

①Although there are only three bars here, it is called a *four-bar linkage* because the rigid ground connection AD is also counted as another "bar" in the linkage.

Solution An ideal pin resists any relative translation of the pinned parts by exerting forces on them. We could draw three separate free-body diagrams of the pin, the first part, and the second part. Usually, however, we let the pin be a part of one of the objects and just draw two free-body diagrams.

Because rusty joint B resists relative rotation we also show a moment at point B acting on rods AB and BC.

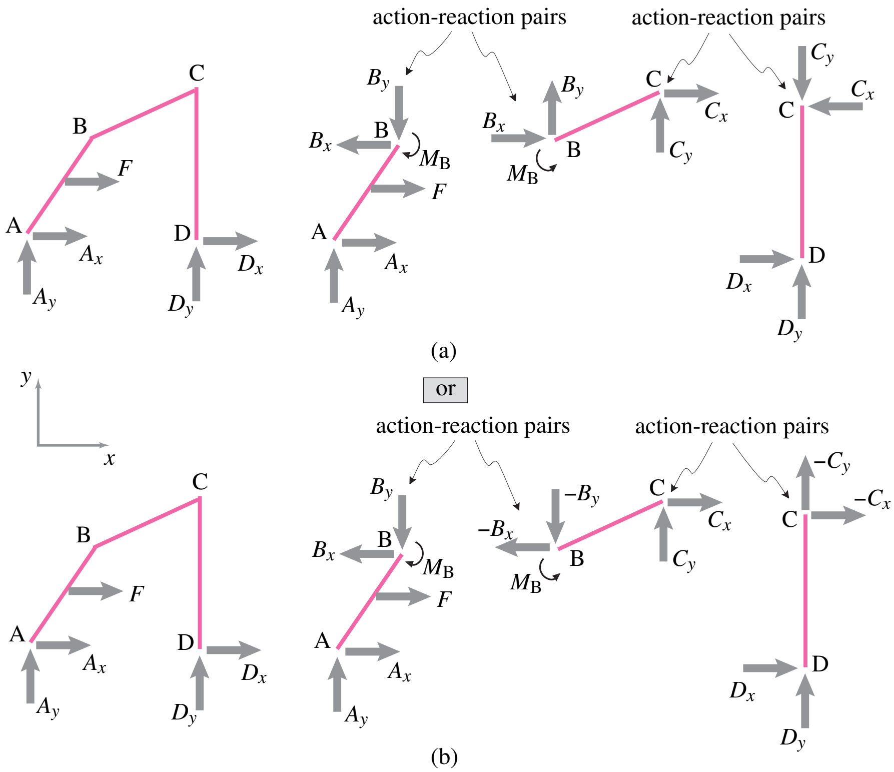


Figure 2.60: Style 1, components: Free-body diagrams of the structure and the individual bars. The forces shown in (a) and (b) are the same.

Figure 2.60 shows the forces in terms of their x- and y-components. The directions of the force components are shown by the arrows and the scalar multipliers are labeled as A_x, A_y, etc. Here the word scalar is sloppily called 'magnitude', even though it can be positive or negative. Therefore, a force, shown as an arrow in the positive x-direction with magnitude A_x, is the same as that shown as an arrow in the negative x-direction with 'magnitude' $-A_x$. Thus, the free-body diagrams in Fig. 2.60(a) show exactly the same forces as in Fig. 2.60(b).

In Fig. 2.61, we show the forces by an arrow in an arbitrary direction. The corresponding labels represent their magnitudes. The angles represent the unknown directions of the forces.

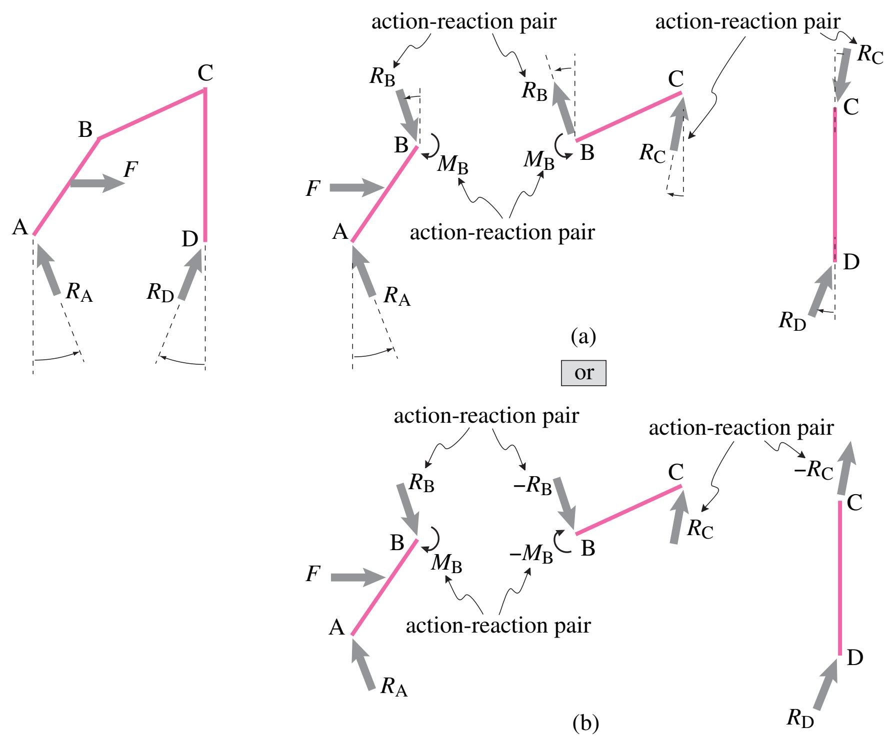


Figure 2.61: Style 2, magnitude times drawn unit vector: The forces in (a) have arcs indicating angles which would show the directions of the forces. The FBDs in (b) are more informal and not recommended. Why not? There is no visible notation showing the directions of the forces.

In Fig. 2.62, we show yet another way of drawing and labeling the free-body diagrams, where the forces are labeled as vectors.

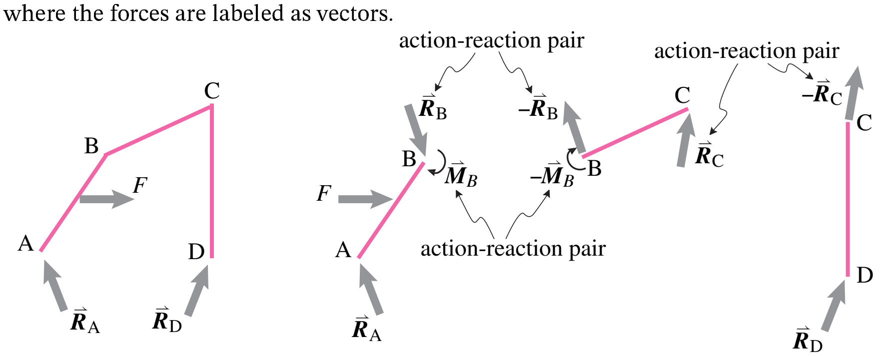


Figure 2.62: Style 3, vector notation: The labels of forces as vectors indicate both their magnitude and direction. The arrows are arbitrary and merely indicate that a force or a moment acts at those locations.

Note: Bar CD is a two-force member. We could have used that to simplify all of the free-body diagrams above by showing equal and opposite forces at C and D that were parallel to the line CD.

2.3 Interactions, Partial FBDs

Preparatory Problems

2.3.1 How does one know what forces and moments to use in

a) the statics force balance and moment-balance equations? *

b) the dynamics linear momentum balance and angular momentum balance equations? *

2.3.2 In a free-body diagram of a whole man standing with his right hand extended how do you show the force of his right arm on his body? *

2.3.3 Reproduce the first column of the table in fig. 2.42 on page 169 for the force acting on your right foot from the ground as you step on a stair.

2.3.4 Reproduce the second column of the table in fig. 2.42 on page 169 for a force in the direction of $3\hat{\imath} + 4\hat{\jmath}$ but with unknown magnitude.

2.3.5 Reproduce the third column of the table in fig. 2.42 on page 169 for a 50 N force in the direction of the vector $3\hat{\imath}+4\hat{\jmath}$.

More-Involved Problems

2.3.6 Simple massless pulleys. Draw free-body diagrams of

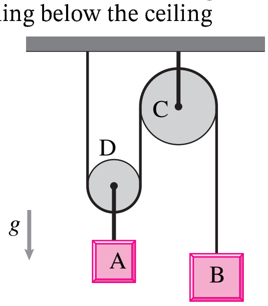
a) mass A with a little bit of rope,

b) mass B with a little bit of rope,

c) Pulley C with three bits of rope,

d) Pulley D with three bits of rope, and

e) The system consisting of everything below the ceiling



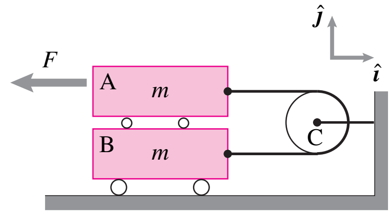
Problem 2.3.6

2.3.7 For the block and pulley arrangement shown in the figure, assume negligible friction at the wheels. Draw the free-body diagrams of

a) the upper mass A,

b) the lower mass B, and

c) the pulley C.

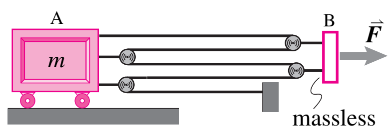


Problem 2.3.7

2.3.8 Multiple pulleys. A goods container of mass m is pulled to the right using a force $\vec{F}$ and the pulley arrangement shown in the figure. Draw the free-body diagram of

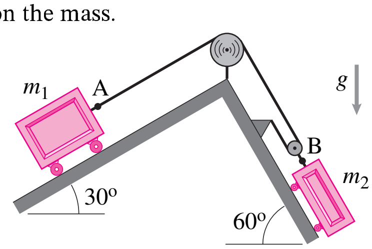
a) the massless block B, and

b) the container A along with the two pulleys attached to it.



Problem 2.3.8

2.3.9
Pulleys on inclined planes. Draw the free-body diagram of mass m_2 and write the expression for each force vector acting on the mass.



Problem 2.3.9

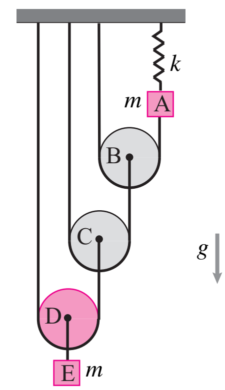
2.3.10 Nested pulleys. In the nested arrangement of pulleys shown in the figure, assume that all the pulleys are massless. Draw the free-body diagram of

a) mass A,

b) mass E, and

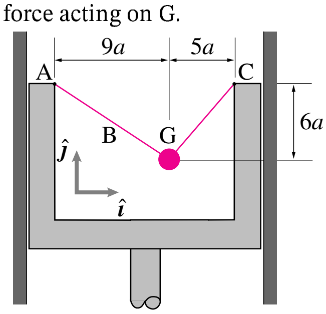
c) pulley D.

Write the expression for the net force on pulley D.



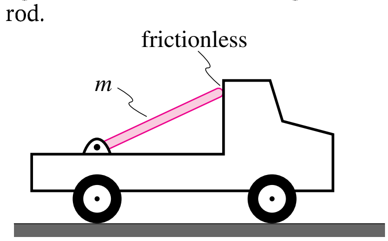
Problem 2.3.10

2.3.11 A point mass m at G is attached to a piston by two inextensible cables. There is gravity. Draw a free-body diagram of the mass with a little bit of the cables and write the vector expression for the net force acting on G.



Problem 2.3.11

2.3.12 A uniform rod of mass m rests in the back of a flatbed truck as shown in the figure. Draw a free-body diagram of the rod.



Problem 2.3.12

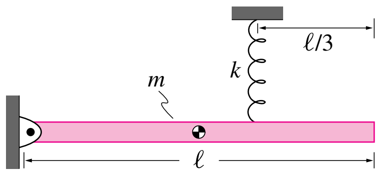
2.3.13 A spring mounted pinned rod. The uniform rigid rod shown in the figure hangs in the vertical plane with the support of the spring shown. In this position the spring is stretched by Δs from its rest length.

a) Draw a free body diagram of the spring.

b) Draw a free-body diagram of the rod.

c) Write an expression for the net force on the rod.

d) Write the expression for the net moment on the rod about its center of mass.

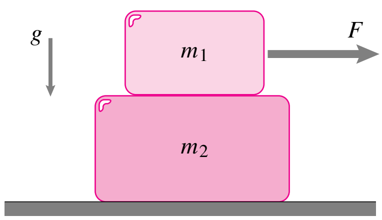


Problem 2.3.13

2.3.14 Two stacked blocks sliding without friction. Two frictionless blocks sit stacked on a frictionless surface. A force F is applied to the top block.

a) Draw free-body diagrams of each block separately.

b) Draw a free-body diagram of the two blocks together.



Problem 2.3.14

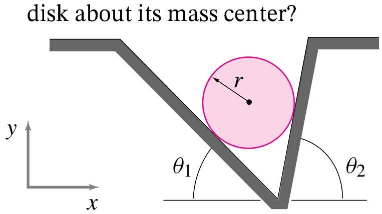
2.3.15 A disk in a frictionless groove. A disc of mass m sits in a wedge shaped groove. There is gravity but no friction.

a) Draw a free-body diagram of the disk.

b) Write vector expressions for the reaction forces from the two walls.

c) Write the expression for the net force on the disk.

d) What is the net moment on the disk about its mass center?



Problem 2.3.15

2.3.16 FBD of an arm throwing a ball. An arm throws a ball up. A crude model of an arm is that it is made of four rigid bodies (shoulder, upper arm, fore-arm and a hand) that are connected with hinges. At each hinge there are muscles that apply torques between the links. Draw a FBD of

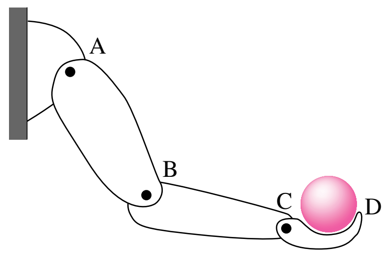
a) the system consisting of the whole arm (three parts, but not the shoulder) and the ball.

b) the ball,

c) the hand, and

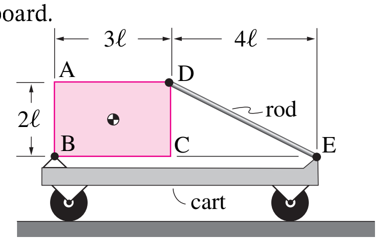
d) the fore-arm,

e) the upper arm,



Problem 2.3.16

2.3.17 A uniform rectangular board of mass m sits on a cart supported by a rod on one corner and a pin at the diagonally opposite corner as shown in the figure. Draw a free-body diagram of the board and write an appropriate vector expression for the force exerted by the rod on the board.



Problem 2.3.17

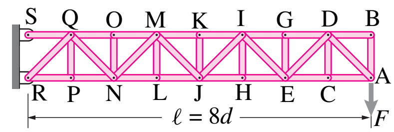
2.3.18 Cantilevered truss A cantilever truss, shown in the figure, is made up of identical horizontal and vertical bars of length d. A vertical force F is applied at point A. The truss is pinned to the wall at joints S and R.

a) Draw a free-body diagram of the entire truss.

b) Cut the truss to the right of bar GE by cutting rods GD, EC and ED, and draw a free-body diagram of that portion of the truss to the right of bar GE.

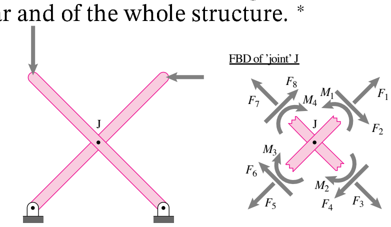
c) Draw a free-body diagram of bar IE.

d) Draw a free-body diagram of the joint at I with a small length of the bars protruding from I.



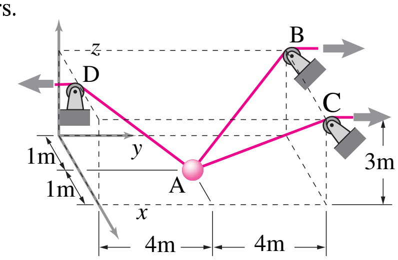
Problem 2.3.18

2.3.19 An X structure. Two rods are pinned together in the middle to form a structure in the shape of 'X' as shown in the figure. A freebody diagram of the joint J with a little bit of the bars near J is shown. Draw free-body diagrams of each bar and of the whole structure. *



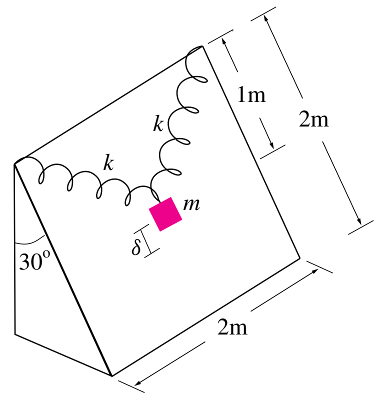
Problem 2.3.19

2.3.20 The strings connected to winches at B, C, and D hold up the mass $m = 3\,\text{kg}$ at A. The relevant dimensions are shown in the figure. There is gravity. Draw a free-body diagram of the mass and express the string forces as appropriate vectors.



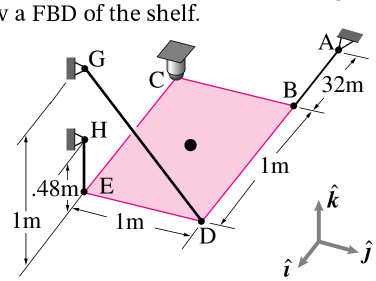
Problem 2.3.20

2.3.21 Mass on inclined plane. A block of mass m rests on a frictionless inclined plane. It is supported by two stretched springs. The mass is pulled down the plane by an amount δ and released. Draw a FBD of the mass just after it is released.



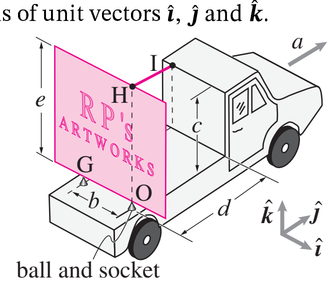
Problem 2.3.21

2.3.22 Hanging a shelf. A shelf with negligible mass supports a 0.5 kgmass at its center. The shelf is supported at one corner with a ball and socket joint and the other three corners with strings. Draw a FBD of the shelf.



Problem 2.3.22

2.3.23 Billboard in the back of a truck. Draw a free-body diagram of the billboard sitting in the back of the truck. There are ball and socket joints at G and O. IH is a rod. Write all forces shown on your diagram as appropriate vectors in terms of unit vectors $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$.



Problem 2.3.23