9.1 Net force and moments in hydrostatics

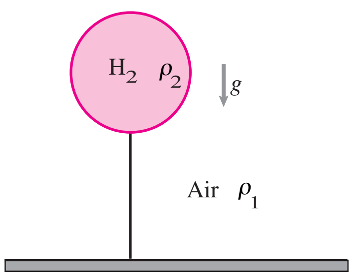
Preparatory Problems

More-Involved Problems

9.1.1 A balloon with volume V, whose membrane has negligible mass, holds a gas with density ρ_2. It is surrounded by a gas with density ρ_1.

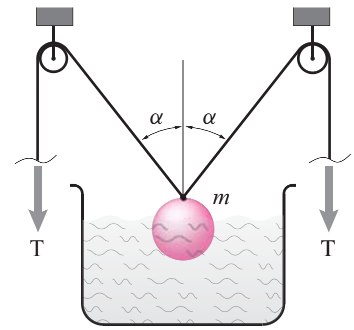
a) In terms of ρ_1, ρ_2, g, and V, find the tension in the string.

b) By some means look up the density of Helium and air at atmospheric temperature and pressure and calculate the volume, in cubic feet and in cubic meters, of a helium balloon that could lift 75 kg.



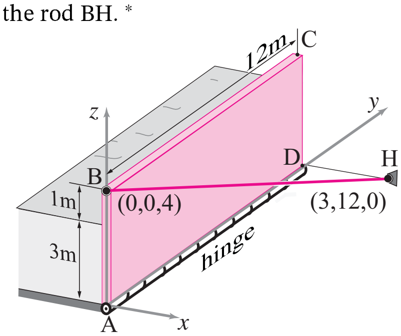
Problem 9.1.1: Balloon

9.1.2 A spherical body of mass $m = 10\,\mathrm{kg}$ and radius $R = 100\,\mathrm{mm}$ hangs from a continuous string as shown in the figure. The body is partially submerged in water and angle $\alpha = 45^\circ$ (fixed). If the force of buoyancy is $\rho V g$ where $\rho = 1000\,\mathrm{kg/m^3}$ = density of water, V is the submerged volume of the body, and g is the usual g; find the tension in the string as a function of the submerged volume V. Find the maximum and the minimum tension corresponding to fully and zero submerged volume of the body respectively.



Problem 9.1.2

9.1.3 A 4-meter-high ‘door’ holds back a stream ($\gamma = 10{,}000\,\mathrm{N/m^3}$) that is 3m deep and 12m wide. The door is hinged along its bottom and is propped up by a thin rod B that goes from a ball joint at H at (3,12,0) to a ball joint at the upper left corner B of the door at (0,0,4). Neglect the mass of the door. Find the axial-force in the rod BH. *

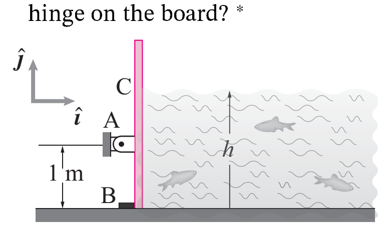


Problem 9.1.3

9.1.4 Water is held in a reservoir by a board with negligible weight that is 5 meters long. It is hinged 1 meter off the bottom at A and kept from leaking by a seal at B. Assume $\rho = 1000\,\mathrm{kg/m^3}$, $g = 10\,\mathrm{N/kg}$.

a) What is h when the board starts to pull away from the stop at B? *

b) At that h what is the force of the hinge on the board? *

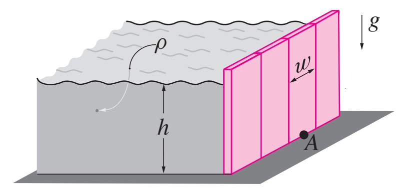


Problem 9.1.4

9.1.5 The side of a pool is made of vertical boards which are stuck in the ground. Assuming that the boards, on average, get no support from their neighbors, and neglect the weight of the board itself,

a) calculate the force and moment from the ground on one board (answer in terms of some or all of w, h, ρ, and g.

b) For a one foot board and 8 foot deep pool, find the size of a force, and its location, so the force is equivalent to the water pressure on the board (answer in lbf).

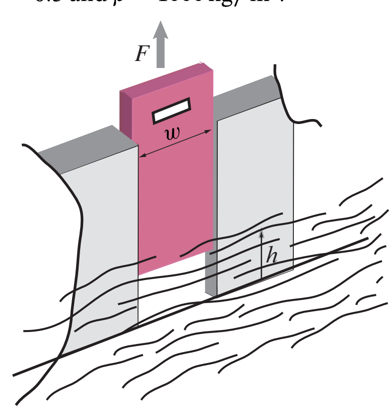


Problem 9.1.5

9.1.6 A sluice gate is a dam that can be opened. Sometimes it is just a board in a slot that is opened by pulling up the board. For water with density ρ and depth h pressing against a board with width w pressing against one face of the slot (the face away from the water) with coefficient of friction μ.

a) find the force F needed to pull up the board in terms of g, ρ, μ, h, and w.

b) Find the force assuming $g = 10\,\mathrm{m/s^2}$, $h = 1\,\mathrm{m}$, $w = 1\,\mathrm{m}$, $\mu = 0.5$ and $\rho = 1000\,\mathrm{kg/m^3}$.



Problem 9.1.6

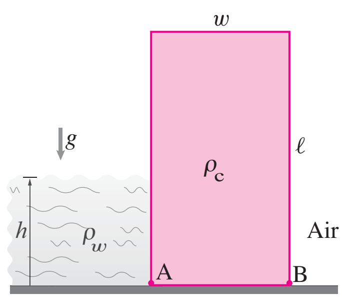
9.1.7 A concrete (density = ρ_c) wall with height ℓ, width w and length (into the paper) d rests on a flat rigid floor and serves as a dam for water with depth h and density ρ_w. Assume the wall only makes contact at edges A and B.

a) Assume there is a seal at A, so no water gets under the dam. What is the coefficient of friction needed to keep the block from sliding?

b) What is the maximum depth of water before the block tips?

c) Assume that there is a seal at B and that water gets under the block. What is the coefficient of friction needed to keep the block from sliding?

d) What is the maximum depth of water before the block tips?

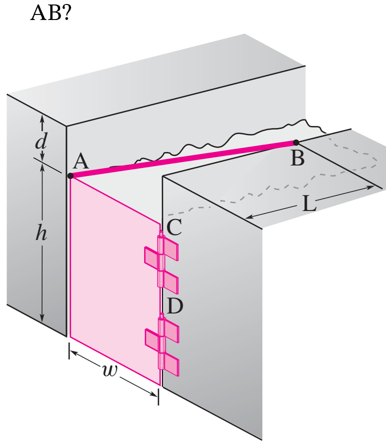


Problem 9.1.7

9.1.8 A door holds back the water at a lock on a canal. The water surface is at the top of the door. The rope AB keeps it from swinging open. The door has hinges at C and D. The height of the door is h, the width w. The point B is a distance d above the top of the door and is set back a distance L. The weight density of the water is γ.

a) What is the total force of the water on the door?

b) What is the tension in the rope AB?

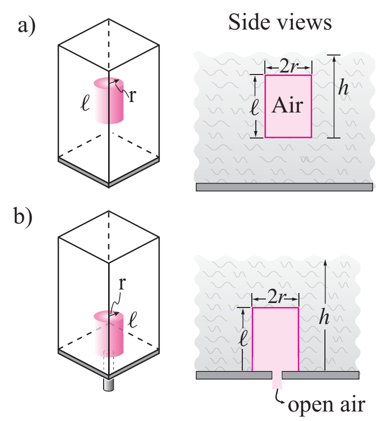


Problem 9.1.8

9.1.9 This problem somewhat explains the workings of some toilet valves. Open the tank of a toilet and look at the rubber piece at the bottom that sits on the bottom but then floats after initially lifted by the turning of the flush lever. The puzzle this problem solves is this: Why does the valve stick to the bottom, but then float when lifted.

a) A hollow cylinder with an open bottom (like an upside down but open can) is filled with air but is under water. What force is required to hold it under water (in terms of ρ, r, h, ℓ and g? *

b) The same can is on the bottom of a tank of water and its edges are sealed. The bottom is open to atmospheric air. How much force is needed to hold the can down now (so there is no force from the bottom of the tank onto the edges of the cylinder)? *



Problem 9.1.9

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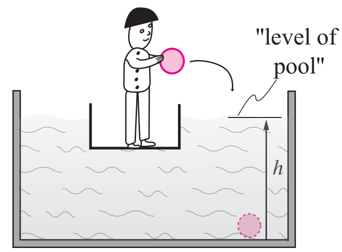
Some puzzles. The following puzzles are sometimes presented as brain teasers. You should be able to reason the answers carefully and irrefutably. **9.1.10** A person is in a boat in a pool with surface area A. She is holding a ball with volume V and mass m in a still pool. The ball is then thrown into the pool, no water is splashed out and the pool comes to rest again.

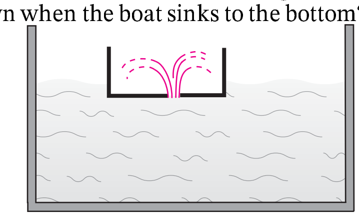
a) Assuming the ball floats, by how much does the pool level go up or down?

b) Assuming the ball sinks to the bottom by how much does the pool level go up or down?



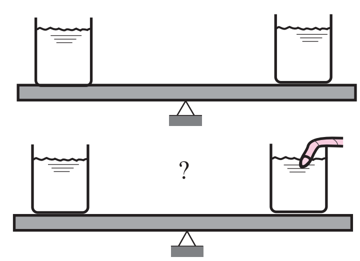
Problem 9.1.10

9.1.11 A steel boat with mass m and density ρ_s is floating in a pool of water with density ρ_w and cross sectional area A. By how much does the pool level go up or down when the boat sinks to the bottom?



Problem 9.1.11

9.1.12 Two cups of water are balanced. You then gently stick your finger into one of them. Does this upset the balance? This experiment can be set up with two cups and a hexagonal-cross-section pencil. The cups need not be identical, they just need to be balanced at the start.

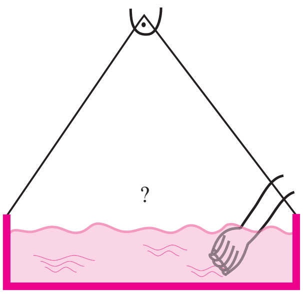


Problem 9.1.12

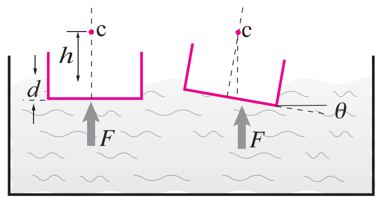
9.1.13 A tray of water is suspended and level.

a) A hand is gently placed in the tray but does not touch the edges or bottom. Is the level of the tray upset.

b) *Challenge:* Assuming the tray is massless with width w and water depth h, how high must be the hinge so the equilibrium is stable. That is, imagine the tray is rotated slightly about the hinge, the water pressure should cause a torque which tends to restore the vertical orientation shown.



Problem 9.1.13



Problem 9.1.14: Ship stability.

9.1.14 *Challenge:* This challenge problem is closely related to the challenge problem above, but is much more famous. It seems to have been first solved by Leonard Euler and Pierre Bouguer in about 1735. This solution seems to be the first mechanics problem in which the significance of the area moment of inertia was appreciated [this is a hint].

For simplicity assume that a boat is shaped like a box with width h and and length into the paper of b. Assume that the boat floats with its bottom a depth d under water. Now rotate the boat about an axis at the surface of the water and along its length (into the paper). Imagine that giant hands hold the boat in this position. In this rotated position the effect of the water pressure on the boat is a buoyant force and moment. This is equivalent to a force that is displaced slightly sideways.

Your goal is to find the height of the point that the line of action of this force intersects a mast of the boat. For small angles of boat tip the location is independent of the amount of tip.

This point is called the *metacenter* of the hull, and its distance up from the centroid of the boat's submerged volume is the hull's *metacentric height*. The condition of boat stability is that the metacenter be above the center of mass of the boat (thus the moment of the buoyant forces about the center of mass will tend to restore the boat to level).

Euler and Bouguer did the calculation you are asked to do here *after*, *e.g.*, , the launching of the 'great' Swedish ship Vasa, which capsized in the harbor on day 1. This was unfortunate for Sweden at the time, but fortunate now, because the brand-new 375 year old ship is a seeworthy tourist attraction in Stockholm.