7.3 Mechanisms

We would now like to analyze ways to connect pieces so that a force or moment is amplified, attenuated or redirected.

For completeness, we present the statics recipe for machines, although it is an exact repeat of the recipe used for frames.

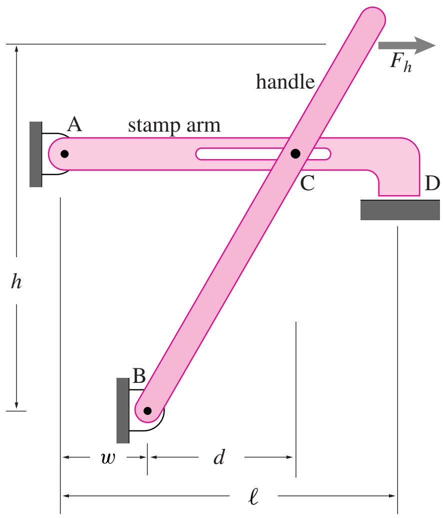
- Draw free-body diagrams of
  - the whole machine; and
  - the separate parts of the machine; and
  - collections of parts of the machine, if such seem likely to be fruitful;
  - Use the principle of action and reaction in the free-body diagrams so that there is only one unknown force at a point where two bodies contact;
- for each free-body diagram write equilibrium conditions. These should yield three independent scalar equations for each non-point part (in 2D)
- solve some or all of the equilibrium equations for the desired unknowns

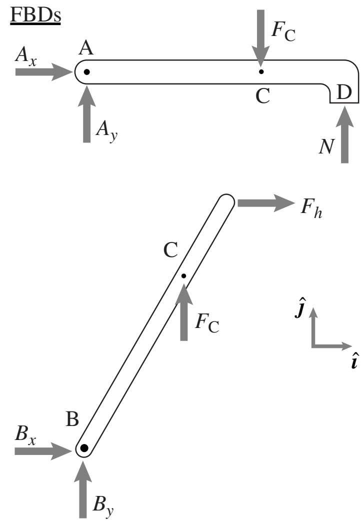
Some useful tricks and shortcuts include:

- for any two-force bodies, assign an equal-valued tension to each end (thus eliminating any need or use for equilibrium equations for that object)
- To minimize calculation, look for a subset of the equilibrium equations that
  - contains your unknowns of interest, and
  - has as many unknowns as scalar equations, and
  - contains as few equations as possible.

Example: Stamp machine

Pulling on the handle (below) causes the stamp arm to press down with a force N at D. We can find N in terms of F_h by drawing free-body diagrams of the handle and stamp arm, writing three equilibrium equations for each piece and then solving these 6 equations for the 6 unknowns (A_x, A_y, F_C, N, B_x, and B_y).





For this problem, the answer can be found more quickly with a judicious choice of equilibrium equations.

$$\begin{aligned}
&\text{For the handle,} \left\{\sum \vec{M}_{/B} = \vec{0}\right\} \cdot \hat{k} && \Rightarrow && -hF_h + dF_c &&= 0\\
&\text{For the stamp arm,} \left\{\sum \vec{M}_{/A} = \vec{0}\right\} \cdot \hat{k} && \Rightarrow && -(d+w)F_c + \ell N &&= 0.
\end{aligned}$$

$$\text{Eliminating } F_c \quad \Rightarrow \quad N = \frac{h(d+w)}{d\ell} F_h.$$

Note that the stamp force N can be made very large by making d small and thus the handle nearly vertical. Often in structural or machine design, one or another force gets extremely large or small as the design is changed to put pieces in near alignment.

Example: Improved stamp machine
Figure 7.53 shows a stamp machine with all the same components. The method of analysis is identical. However, the design represents an improvement in two ways:

- The lever in the stamp arm amplifies rather than attenuates the stamp force.
- In the previous design, it is harder and harder to generate a given stamp force as the stamped object compresses. In this design, the toggle mechanism associated with the lever arm and sliding pin is in compression. Thus, as the stamping progresses and the handle becomes more vertical, for a constant hand-force, the stamping force increases as the motion progresses.

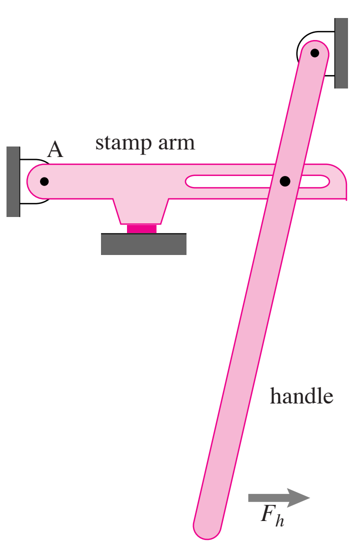


Figure 7.53: An improved stamp machine.

Non-rigid structures are mechanisms

A non-rigid structure cannot carry all loads and, if not also redundant, has more equilibrium equations than unknown reaction or interaction force components. Such a structure is also called a mechanism. The stamp machine above is a mechanism if we assume there is no contact at D. In particular, the equilibrium equations cannot be satisfied unless $F_h = 0$. Mechanisms have variable configurations. That is, the constraints still allow relative motion.

An attempt to design a rigid structure that turns out to be a mechanism is a design failure. But for machine design, the mechanism aspect of a structure is essential. Even though mechanisms are called 'statically indeterminate' because they cannot carry all possible loads, the desired forces can often be determined using statics. For the stamp machine above, the equilibrium equations are made solvable by treating one of the applied forces, say N, as an unknown, and the other, F_h in this case, as a known. This is a common situation in machine design where you want to determine the loads at one part of a mechanism in terms of loads at another part. For the purposes of analysis, a trick is to make a mechanism determinate by putting a pin on the roller's connection to the ground at the location of any forces with unknown magnitudes but known directions.

Example: Stamp machine with roller
Putting a roller at D, the location of the unknown stamp force, turns the stamp machine into a determinate structure.

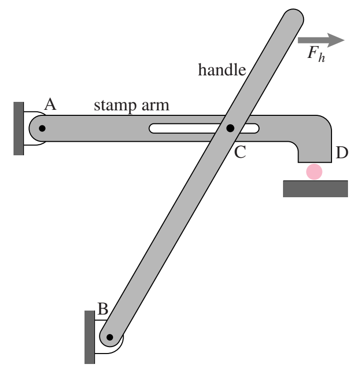


Figure 7.54: Stamp machine with roller.

Pulley and chain drives

Chain and pulley drives may be thought of as spread-out gears (fig. 7.55). The rotation of two shafts is coupled not by the contact of gear teeth but by a belt around a pulley or a chain around a sprocket. For a simple analysis, one draws free-body diagrams for each sprocket or pulley with a little bit of chain as in fig. 7.55b. Note that $T_1 \neq T_2$, unlike the case of an ideal undriven pulley. Applying moment balance we find,

$$\begin{aligned}
&\text{For gear A,} \left\{\sum \vec{\boldsymbol{M}}_{\mathrm{i/A}} = \vec{\boldsymbol{0}}\right\} \cdot \hat{\boldsymbol{k}} \quad \Rightarrow \quad -R_\mathrm{A}(T_2 - T_1) + M_\mathrm{A} = 0 \\
&\text{For gear B,} \left\{\sum \vec{\boldsymbol{M}}_{\mathrm{i/B}} = \vec{\boldsymbol{0}}\right\} \cdot \hat{\boldsymbol{k}} \quad \Rightarrow \quad R_\mathrm{B}(T_1 - T_2) - M_\mathrm{B} = 0.
\end{aligned}$$

$$\text{Eliminating } (T_2 - T_1) \quad \Rightarrow \quad M_B = \frac{R_B}{R_A} M_A \quad \text{or} \quad M_A = \frac{R_A}{R_B} M_B,$$

exactly as for a pair of gears. Note that we cannot find T_2 or T_1 but only their difference. Typically in design if, say, M_A is positive, one would try to keep T_1 as small as possible without the belt slipping or the chain jumping teeth. If T_1 grows, then so must T_2, to preserve their difference. This increase in tension increases the loads on the bearings as well as on the chain or belt itself.

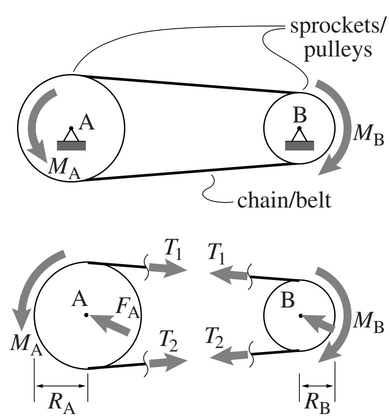


Figure 7.55: a) A chain or pulley drive involving two sprockets or pulleys and one chain or belt, b) free-body diagrams of each of the sprockets/pulleys.

Four-bar linkages

Four-bar linkages often, confusingly, have 3 bars, the fourth piece is something bigger that the linkage is attached to. A planar mechanism with four pieces connected in a loop by hinges is a four-bar linkage. Four-bar linkages are remarkably common. After a single object connected at a hinge (like a gear or lever), a four-bar linkage is one of the simplest

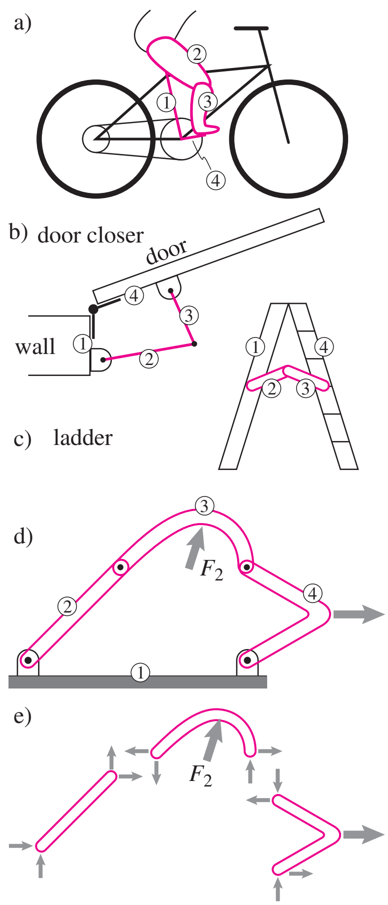


Figure 7.56: Four bar linkages. a) A bicycle, thigh, calf, and crank, b) a door closer, c) a folding ladder, d) a generic mechanism, e) free-body diagrams of the parts of a generic mechanism.

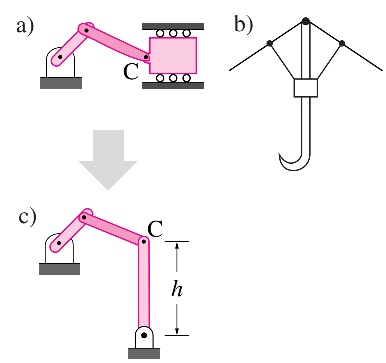


Figure 7.57: a) a slider crank, b) an umbrella has a slider-crank mechanism, c) the equivalent four-bar linkage, at least when $h \to \infty$.

mechanisms that can move in just one way (*i.e.,* have just one degree of freedom).

A reasonable model of a person seated on a bicycle uses a 4-bar linkage (fig. 7.56a). The whole bicycle frame is one bar, the human thigh is the second, the calf is the third, and the bicycle crank is the fourth. The four hinges are the hip joint, the knee joint, the pedal axle, and the bearing at the bicycle crank axle. A more sophisticated model of the system would include the ankle joint and the foot would make up a fifth bar.

A standard door-closing mechanism is part of a 4-bar linkage (fig. 7.56b). The door jamb and door are two bars and the mechanism pieces make up the other two.

A standard folding ladder design is, until locked open, a 4-bar linkage (fig. 7.56c).

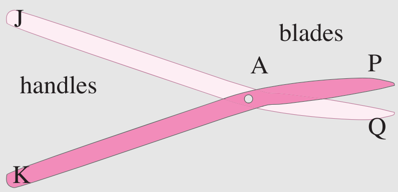
An abstracted 4-bar linkage with two loads is shown in fig. 7.56d with free-body diagrams in fig. 7.56e. If one of the applied loads is given, then the other applied load along with interaction and reaction forces compose nine unknown components (after using the principle of action and reaction). With three equilibrium equations for each of the three bars, all these unknowns can be found.

Slider crank

A mechanism closely related to a four-bar linkage is a slider crank (fig. 7.57a). An umbrella is one example (rotated 90° in fig. 7.57b). If the sliding part is replaced by a bar, as in fig. 7.57c, the point C moves in a circle instead of a straight line. If the height h is very large, then the arc traversed by C is nearly a straight line so the motion of the four-bar linkage is almost the same as the slider crank. For this reason, slider cranks are sometimes regarded as a special case of a four-bar linkage in the limit as one of the bars gets infinitely long.

7.6 Shears with gears

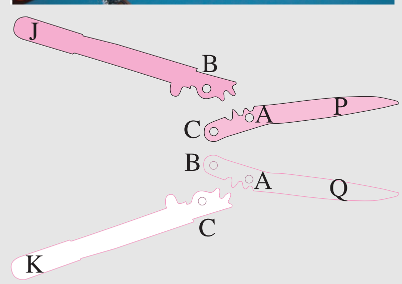
Many cutters, pliers and shears are essentially two levers pivoting against each other. For example these shears

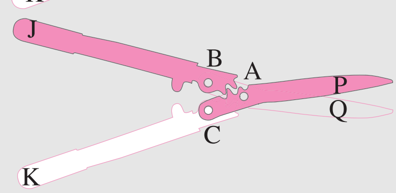


consist of two levers, JAQ and KAP, pivoted at A. The hands squeeze the handles at J and K, causing a cutting force on an object between the blades at P and Q. The force at P, say, is $|KA|/|AP|$ times the force at K (from moment balance about A using a free-body diagram of KAP). Two possible deficiencies of this bi-lever design are that

- One may want more mechanical advantage but not longer handles, and
- For a given hand strength (available force at J and K) the force at the cutting edge gets less and less as the location of the cut force at P and Q moves farther out on the blade, away from A.

The Fiskars company, known mostly for scissors using the basic design above, has some designs that address these deficiencies. The loppers in problem 7.3.9 use a 3-piece mechanism to address these issues. Here, even more elaborately, are Fiskars shears using 4 moving parts.





The two identical blades AP and AQ are hinged at A. The two identical handles JB and KC are hinged to the blades at B and C. Each handle also has gear teeth at the end that engage gear teeth on the opposite blade. Let's take P and Q to be the point of contact of the object being cut.

Let's try to understand the mechanism without detailed analysis (see homework problem 7.3.10). To start, forget handle KC and assume that blade BAQ is held firmly by something outside.

(continued...)

7.6 Shears with gears (continued)

Blade CAP is attached at A about which it is free to spin. Handle JB is attached at B about which it is free to spin. But JB and CAP roll against each other with engaged gear teeth. So if handle JB rotates counter-clockwise about B, then CAP rotates clockwise about A.

Although the gear teeth are complex-looking, there is always an *effective* contact point G between handle JB and blade CAP on the line segment AB. G is effectively a hinge between JB and CAP. You can think of the handle as a lever with force points at J, B and G. Thus blade CAP is closed by the force on the gear teeth at G. The shorter BG, the bigger the forces at B and G.

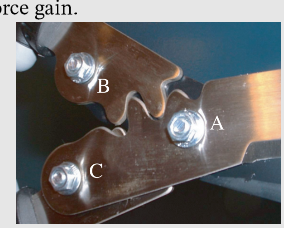
Simultaneously you could think of blade CAP as fixed with blade BAQ and handle KC hinged to it and geared to each other at G' (not shown). Thus blade CAP is also closed by an upwards force at C from handle KC. Similarly blade BAQ is closed by a downwards force at B from handle JB and a downwards force at G' from handle KC.

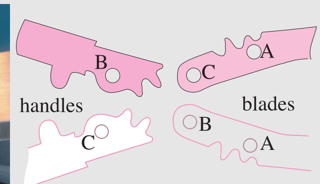
The effective hinges G and G' have locations which change as the blades close. When the blades are wide open, G and G' are near A. When the blades are closed, G and G' have moved to about the midpoint between B and A and C and A, respectively.

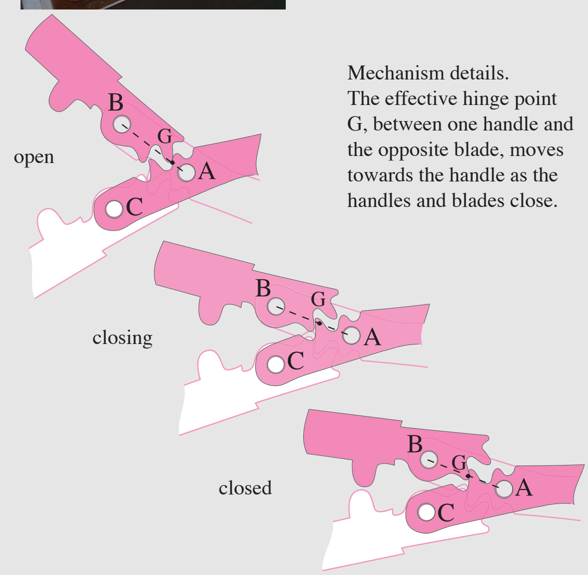
If G was *at* A, then this 4-piece design would be equivalent to a standard 2-piece cutter. Because BG is shorter than BA, this design gives a bigger downwards force at B.

The shape of the geared curves makes the distance BG decrease, and the distance AG increase, as the blades close. Thus for given forces acting at J and Q, as the blades close the force at B increases, the force at G increases, and the lever-arm AG increases. These three effects partially compensate for the standard scissors problem, the decreasing mechanical advantage from the distance AP increasing as the blades close.

Another way to see the mechanical advantage of this design compared to the 2-piece design is to see that during a cut the handle angle decrease is greater than the blade angle decrease. Following the general rule for mechanisms, a motion attenuation is a force gain.







Mechanism details.
The effective hinge point G, between one handle and the opposite blade, moves towards the handle as the handles and blades close.

SAMPLE 7.13 A slider crank: A torque $M = 20\,\text{N·m}$ is applied at the bearing end A of the crank AD of length $\ell = 0.2\,\text{m}$. If the mechanism is in static equilibrium in the configuration shown, find the load F on the piston.

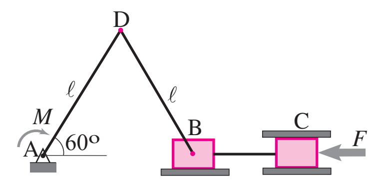


Figure 7.58

Solution The free-body diagram of the whole mechanism is shown in Fig. 7.59. From the moment equilibrium about point A, $\sum \vec{\boldsymbol{M}}_{\text{A}} = \vec{\boldsymbol{0}}$, we get

$$\begin{aligned}
\vec{\boldsymbol{M}} + \vec{\boldsymbol{r}}_{\text{B/A}} \times (\vec{\boldsymbol{B}} + \vec{\boldsymbol{F}}) &= \vec{\boldsymbol{0}} \\
-M\hat{\boldsymbol{k}} + 2\ell\cos\theta\hat{\boldsymbol{\imath}} \times (B_y\hat{\boldsymbol{\jmath}} - F\hat{\boldsymbol{\imath}}) &= \vec{\boldsymbol{0}} \\
(-M + 2B_y\ell\cos\theta)\hat{\boldsymbol{k}} &= \vec{\boldsymbol{0}} \\
\Rightarrow \quad B_y &= \frac{M}{2\ell\cos\theta}.
\end{aligned}$$

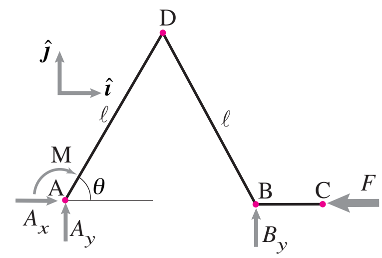


Figure 7.59

The force equilibrium, $\sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}}$, gives

$$\begin{aligned}
(A_x - F)\hat{\boldsymbol{\imath}} + (A_y + B_y)\hat{\boldsymbol{\jmath}} &= 0 \\
A_x &= F \\
A_y &= -B_y
\end{aligned}$$

Note that we still need to find F or A_x. So far, we have had only three equations in four unknowns (A_x, A_y, B_y, F). To solve for the unknowns, we need one more equation. We now consider the free-body diagram of the mechanism without the crank, that is, the connecting rod DB and the piston BC together. See Fig. 7.60. Unfortunately, we introduce two more unknowns (the reactions) at D. However, we do not care about them. Therefore, we can write the moment equilibrium equation about point D, $\sum \vec{\boldsymbol{M}}_{\text{D}} = \vec{\boldsymbol{0}}$ and get the required equation without involving D_x and D_y.

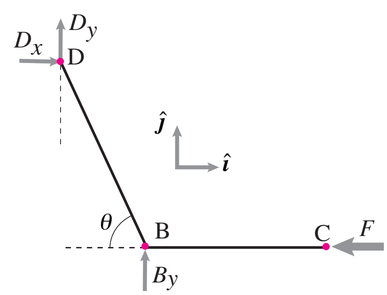


Figure 7.60

$$\begin{aligned}
\vec{\boldsymbol{r}}_{\text{B/D}} \times (-F\hat{\boldsymbol{\imath}} + B_y\hat{\boldsymbol{\jmath}}) &= \vec{\boldsymbol{0}} \\
\ell(\cos\theta\hat{\boldsymbol{\imath}} - \sin\theta\hat{\boldsymbol{\jmath}}) \times (-F\hat{\boldsymbol{\imath}} + B_y\hat{\boldsymbol{\jmath}}) &= \vec{\boldsymbol{0}} \\
B_y\ell\cos\theta\hat{\boldsymbol{k}} - F\ell\sin\theta\hat{\boldsymbol{k}} &= \vec{\boldsymbol{0}}.
\end{aligned}$$

Dotting the last equation with $\hat{\boldsymbol{k}}$ we get

$$\begin{aligned}
F &= B_y\frac{\cos\theta}{\sin\theta} \\
&= \frac{M}{2\ell\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} \\
&= \frac{M}{2\ell\sin\theta} \\
&= \frac{20\,\text{N·m}}{2 \cdot 0.2\,\text{m} \cdot \sqrt{3}/2} \\
&= 57.74\,\text{N}.
\end{aligned}$$

$\boxed{F = 57.74\,\text{N}}$

Note that the force equilibrium carried out above is not really useful since we are not interested in finding the reactions at A. We did it above to show that just one free-body diagram of the whole mechanism was not sufficient to find F. On the other hand, writing moment equations about A for the whole mechanism and about D for the connecting rod plus the piston is enough to determine F.

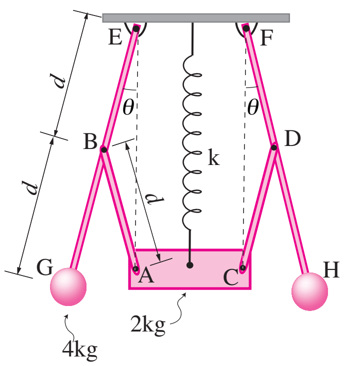


Figure 7.61

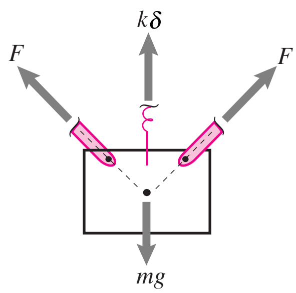


Figure 7.62

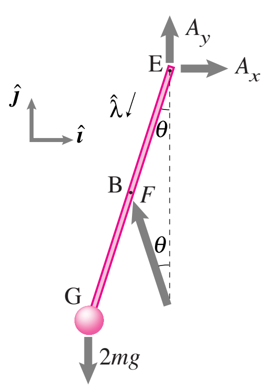


Figure 7.63

SAMPLE 7.14 A flyball governor: A flyball governor, usually considered in a dynamics context, is shown in static equilibrium. The relaxed length of the spring is 0.15 m and its stiffness is 500 N/m.

1. Find the static equilibrium position of the center collar.
2. Find the force in the strut AB.
3. How does the spring force required to hold the collar depend on θ?

Solution Let $\ell_0 (= 0.15\,\text{m})$ denote the relaxed length of the spring and let ℓ be the stretched length in the static equilibrium configuration of the flyball, *i.e.*, the collar is at a distance ℓ from the fixed support EF. Then the net stretch in the spring is $\delta \equiv \Delta\ell = \ell - \ell_0$. We need to determine ℓ, the spring force $k\delta$, and its dependence on the angle θ of the ball-arm.

The free-body diagram of the collar is shown in fig. 7.62. Note that the struts AB and CD are two-force bodies (forces act only at the two end points on each strut). Therefore, the force at each end must act along the strut. From geometry (AB = BE = d), then, the strut force F on the collar must act at angle θ from the vertical. Now, the force balance in the vertical direction, *i.e.*, $[\sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}}] \cdot \hat{\boldsymbol{\jmath}}$, gives

$$-2F\cos\theta + k\delta = mg. \tag{7.15}$$

Thus to find δ we need to find F and θ. Now we draw the free-body diagram of arm EBG as shown in fig. 7.63. From the moment balance about point E, we get

$$\begin{aligned}
\vec{\boldsymbol{r}}_{G/E} \times (-2mg\hat{\boldsymbol{\jmath}}) + \vec{\boldsymbol{r}}_{B/E} \times \vec{\boldsymbol{F}} &= \vec{\boldsymbol{0}} \\
2d\hat{\boldsymbol{\lambda}} \times (-2mg\hat{\boldsymbol{\jmath}}) + d\hat{\boldsymbol{\lambda}} \times F(-sin\theta\hat{\boldsymbol{\imath}} + \cos\theta\,\hat{\boldsymbol{\jmath}}) &= \vec{\boldsymbol{0}} \\
-4mgd\underbrace{(\hat{\boldsymbol{\lambda}} \times \hat{\boldsymbol{\jmath}})}_{-\sin\theta\hat{\boldsymbol{k}}} + Fd[-\sin\theta\underbrace{(\hat{\boldsymbol{\lambda}} \times \hat{\boldsymbol{\imath}})}_{\cos\theta\hat{\boldsymbol{k}}} + \cos\theta\underbrace{(\hat{\boldsymbol{\lambda}} \times \hat{\boldsymbol{\jmath}})}_{-\sin\theta\hat{\boldsymbol{k}}}] &= \vec{\boldsymbol{0}} \\
4mgd\sin\theta\hat{\boldsymbol{k}} + Fd(-\sin\theta\cos\theta\hat{\boldsymbol{k}} - \cos\theta\sin\theta\hat{\boldsymbol{k}}) &= \vec{\boldsymbol{0}} \\
(4mgd\sin\theta - 2Fd\sin\theta\cos\theta)\hat{\boldsymbol{k}} &= \vec{\boldsymbol{0}}.
\end{aligned}$$

Dotting this equation with $\hat{\boldsymbol{k}}$ and assuming that $\theta \neq 0$, we get

$$2F\cos\theta = 4mg. \tag{7.16}$$

Substituting eqn. (7.16) in eqn. (7.15) we get

$$\begin{aligned}
k\delta &= mg + 2F\cos\theta = mg + 4mg = 5mg \\
\Rightarrow \quad \delta &= \frac{5mg}{k} = \frac{5 \cdot 2\,\text{kg} \cdot 9.81\,\text{m/s}^2}{500\,\text{N/m}} = 0.196\,\text{m}.
\end{aligned}$$

1. The equilibrium configuration is specified by the stretched length ℓ of the spring (which specifies θ). Thus,

$$\ell = \ell_0 + \delta = 0.15\,\text{m} + 0.196\,\text{m} = 0.346\,\text{m}.$$

   Now, from $\ell = 2d\cos\theta$, we find that $\theta = 30.12°$.

2. The force in strut AB is

$$F = 2mg/\cos\theta = 45.36\,\text{N}.$$

3. The force in the spring $k\delta = 5mg$ as shown above and thus, it does not depend on θ! In fact, the angle θ is determined by the relaxed length of the spring.

(a) $\ell = 0.346\,\text{m}$, (b) $F = 45.36\,\text{N}$, (c) $k\delta \neq f(\theta)$

SAMPLE 7.15 : A motor housing support: A slotted arm mechanism is used to support a motor housing that has a belt drive as shown in the figure. The motor housing is bolted to the arm at B and the arm is bolted to a solid support at A. The two bolts are tightened enough to be modeled as welded joints (*i.e.,* they can also take some torque). Find the support reactions at A.

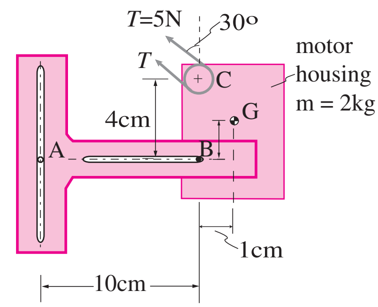


Figure 7.64

Solution Although the mechanism looks complicated, the problem is straightforward. We cut the bolt at A and draw the free-body diagram of the motor housing plus the slotted arm. Since the bolt, modeled as a welded joint, can take some torque, the unknowns at A are $\vec{\boldsymbol{A}}(=A_x\hat{\boldsymbol{\imath}}+A_y\hat{\boldsymbol{\jmath}})$ and $\vec{\boldsymbol{M}}_{\mathrm{A}}$. The free-body diagram is shown in fig. 7.65. Note that we have replaced the tension at the two belt ends by a single equivalent tension $2T$ acting at the center of the axle. Now taking moments about point A, we get

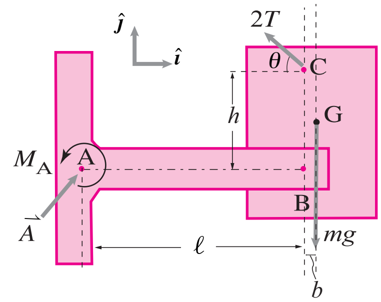


Figure 7.65

$$\vec{\boldsymbol{M}}_{\mathrm{A}}+\vec{\boldsymbol{r}}_{\mathrm{C/A}}\times 2\vec{\boldsymbol{T}}+\vec{\boldsymbol{r}}_{\mathrm{G/A}}\times m\vec{\boldsymbol{g}}=\vec{\boldsymbol{0}}$$

where

$$\begin{aligned}
\vec{\boldsymbol{r}}_{\mathrm{C/A}}\times 2\vec{\boldsymbol{T}} &= (\ell\hat{\boldsymbol{\imath}}+h\hat{\boldsymbol{\jmath}})\times 2T(-\cos\theta\hat{\boldsymbol{\imath}}+\sin\theta\hat{\boldsymbol{\jmath}})\\
&= 2T(\ell\sin\theta+h\cos\theta)\hat{\boldsymbol{k}}\\
\vec{\boldsymbol{r}}_{\mathrm{G/A}}\times m\vec{\boldsymbol{g}} &= [(\ell+d)\hat{\boldsymbol{\imath}}+(\text{anything})\hat{\boldsymbol{\jmath}}]\times(-mg\hat{\boldsymbol{\jmath}})\\
&= -mg(\ell+d)\hat{\boldsymbol{k}}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\vec{\boldsymbol{M}}_{\mathrm{A}} &= -\vec{\boldsymbol{r}}_{\mathrm{C/A}}\times 2\vec{\boldsymbol{T}}-\vec{\boldsymbol{r}}_{\mathrm{G/A}}\times m\vec{\boldsymbol{g}}\\
&= -2T(\ell\sin\theta+h\cos\theta)\hat{\boldsymbol{k}}+mg(\ell+d)\hat{\boldsymbol{k}}\\
&= -2(5\,\mathrm{N})(0.1\,\mathrm{m}\cdot\sin 60^\circ+0.04\,\mathrm{m}\cdot\cos 60^\circ)\hat{\boldsymbol{k}}\\
&\quad +2\,\mathrm{kg}\cdot 9.81\,\mathrm{m/s^2}\cdot(0.1+0.01)\,\mathrm{m}\hat{\boldsymbol{k}}\\
&= 1.092\,\mathrm{N{\cdot}m}\hat{\boldsymbol{k}}.
\end{aligned}$$

The reaction force $\vec{\boldsymbol{A}}$ can be determined from the force balance, $\sum\vec{\boldsymbol{F}}=\vec{\boldsymbol{0}}$ as follows.

$$\begin{aligned}
\vec{\boldsymbol{A}}+2\vec{\boldsymbol{T}}+m\vec{\boldsymbol{g}} &= \vec{\boldsymbol{0}}\\
\Rightarrow\quad \vec{\boldsymbol{A}} &= -2\vec{\boldsymbol{T}}-m\vec{\boldsymbol{g}}\\
&= -10\,\mathrm{N}(-\frac{1}{2}\hat{\boldsymbol{\imath}}+\frac{\sqrt{3}}{2}\hat{\boldsymbol{\jmath}})-(-19.62\,\mathrm{N}\hat{\boldsymbol{\jmath}})\\
&= 5\,\mathrm{N}\hat{\boldsymbol{\imath}}+10.96\,\mathrm{N}\hat{\boldsymbol{\jmath}}.
\end{aligned}$$

$$\boxed{\vec{\boldsymbol{M}}_{\mathrm{A}}=1.092\,\mathrm{N{\cdot}m}\hat{\boldsymbol{k}}\qquad\text{and}\qquad\vec{\boldsymbol{A}}=5\,\mathrm{N}\hat{\boldsymbol{\imath}}+10.96\,\mathrm{N}\hat{\boldsymbol{\jmath}}}$$

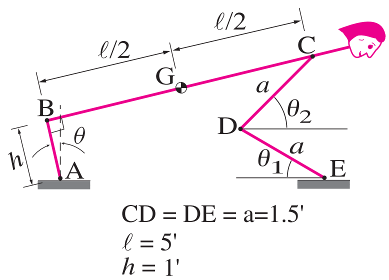


Figure 7.66

SAMPLE 7.16 Push-up mechanics: During push-ups, the body including the legs, usually moves as a single rigid unit; the ankle is almost locked, and the push-up is powered by the shoulder and the elbow muscles. A simple model of the body during push-ups is a four-bar linkage ABCDE shown in the figure. In this model, each link is a rigid rod, joint B is rigid (thus ABC can be taken as a single rigid rod), joints C, D, and E are hinges, but there is a motor at D that can supply torque. The weight of the person, $W = 150\,\text{lbf}$, acts through G. Find the torque at D for $\theta_1 = 30°$ and $\theta_2 = 45°$.

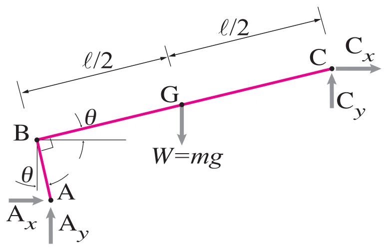


Figure 7.67

Solution The free-body diagram of part ABC of the mechanism is shown in fig. 7.67. Writing the moment-balance equation about point A, $\sum \vec{\boldsymbol{M}}_{\text{A}} = \vec{\boldsymbol{0}}$, we get

$$\vec{\boldsymbol{r}}_{\text{C}} \times \vec{\boldsymbol{C}} + \vec{\boldsymbol{r}}_{\text{G}} \times \vec{\boldsymbol{W}} = \vec{\boldsymbol{0}}.$$

Let $\vec{\boldsymbol{r}}_{\text{C}} = r_{C_x}\hat{\boldsymbol{\imath}} + r_{C_y}\hat{\boldsymbol{\jmath}}$ and $\vec{\boldsymbol{r}}_{\text{G}} = r_{G_x}\hat{\boldsymbol{\imath}} + r_{G_y}\hat{\boldsymbol{\jmath}}$ for now (we can figure it out later). Then, the moment equation becomes

$$(r_{C_x}\hat{\boldsymbol{\imath}} + r_{C_y}\hat{\boldsymbol{\jmath}}) \times (C_x\hat{\boldsymbol{\imath}} + C_y\hat{\boldsymbol{\jmath}}) + (r_{G_x}\hat{\boldsymbol{\imath}} + r_{G_y}\hat{\boldsymbol{\jmath}}) \times (-W\hat{\boldsymbol{\jmath}}) = \vec{\boldsymbol{0}}$$

$$[(C_y r_{C_x} - C_x r_{C_y})\hat{\boldsymbol{k}} - W r_{G_x}\hat{\boldsymbol{k}} = \vec{\boldsymbol{0}}]$$

$$[\,]\cdot\hat{\boldsymbol{k}} \quad \Rightarrow \quad C_y r_{C_x} - C_x r_{C_y} = W r_{G_x}. \tag{7.17}$$

We now draw free-body diagrams of the links CD and DE separately (fig. 7.68) and write the moment- and force-balance equations for them.

For link CD, the force equilibrium $\sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}}$ gives

$$(-C_x + D_x)\hat{\boldsymbol{\imath}} + (D_y - C_y)\hat{\boldsymbol{\jmath}} = \vec{\boldsymbol{0}}.$$

Dotting with $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ gives

$$\begin{aligned} D_x &= C_x \\ D_y &= C_y \end{aligned} \tag{7.18}$$

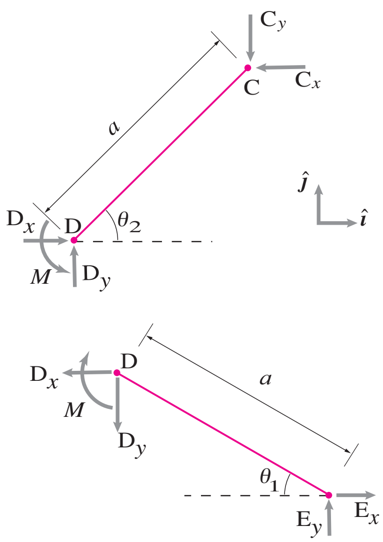


Figure 7.68

and the moment equilibrium about point D, gives

$$\begin{aligned} M\hat{\boldsymbol{k}} - a(\cos\theta_2\hat{\boldsymbol{\imath}} + \sin\theta_2\hat{\boldsymbol{\jmath}}) \times (-C_x\hat{\boldsymbol{\imath}} - C_y\hat{\boldsymbol{\jmath}}) &= \vec{\boldsymbol{0}} \\ M\hat{\boldsymbol{k}} + (C_y a\cos\theta_2 - C_x a\sin\theta_2)\hat{\boldsymbol{k}} &= \vec{\boldsymbol{0}}. \end{aligned} \tag{7.19}$$

Similarly, the force equilibrium for link DE requires that

$$\begin{aligned} E_x &= D_x \\ E_y &= D_y \end{aligned} \tag{7.20}$$

and the moment equilibrium of link DE about point E gives

$$-M + D_x a\sin\theta_1 + D_y a\cos\theta_1 = 0. \tag{7.21}$$

Now, from eqns. (7.18) and (7.21)

$$-M + C_x a\sin\theta_1 + C_y a\cos\theta_1 = 0. \tag{7.22}$$

Adding eqns. (7.19) and (7.22) and solving for C_x, we get

$$C_x = \frac{\cos\theta_1 + \cos\theta_1}{\sin\theta_2 - \sin\theta_1} C_y.$$

For simplicity, let

$$f(\theta_1, \theta_2) = \frac{\cos\theta_1 + \cos\theta_1}{\sin\theta_2 - \sin\theta_1}$$

so that

$$C_x = f(\theta_1, \theta_2) C_y. \tag{7.23}$$

Now substituting eqn. (7.23) in (7.17), we get

$$C_y = \frac{r_{G_x}}{r_{C_x} - r_{C_y} f} W.$$

Now substituting C_y and C_x into eqn. (7.22) we get

$$M = \frac{r_{G_x} a(\cos\theta_1 + f\sin\theta_1)}{r_{C_x} - r_{C_y} f} W$$

where

$$\begin{aligned} r_{G_x} &= (\ell/2)\cos\theta - h\sin\theta \\ r_{C_x} &= \ell\cos\theta - h\sin\theta \\ r_{C_y} &= \ell\sin\theta + h\cos\theta. \end{aligned}$$

Now plugging in all the given values: $W = 160\,\text{lbf}, \theta_1 = 30°, \theta_2 = 45°, \ell = 5\,\text{ft}, h = 1\,\text{ft}, a = 1.5\,\text{ft}$, and, from simple geometry, $\theta = 9.49°$,

$$\begin{aligned} f &= 7.60 \\ r_{C_x} &= 4.77\,\text{ft}, \\ r_{C_y} &= 1.81\,\text{ft}, \\ r_{G_x} &= 2.30\,\text{ft} \\ \Rightarrow \quad M &= -269\,\text{lb·ft}. \end{aligned}$$

$$\boxed{M = -269\,\text{lb·ft}}$$

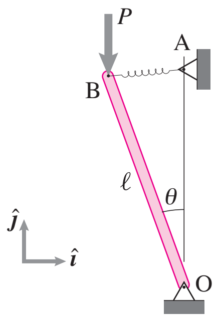


Figure 7.69

SAMPLE 7.17 A spring and rod buckling model: A simple model of sideways buckling of a flexible (elastic) rod can be constructed with a spring and a *rigid* rod as shown in the figure. The solution comes out most beautifully if we assume that the rod is held in place with a zero-rest-length spring that has no tension when B is on top of A. If $P = 0$ the system is in equilibrium with the rod vertical, $\theta = 0$. We find the so-called 'buckling' load by finding a load P at which there are solutions with $\theta \neq 0$. So, we assume the rod to be in static equilibrium at some angle $\theta \neq 0$ from the vertical. The vertical load is P, spring stiffness k, and bar length ℓ. For this problem, with a zero-rest-length spring we need not assume small angles.

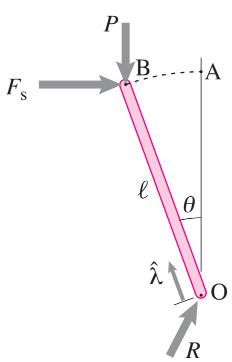


Figure 7.70

Solution When the rod is displaced from its vertical position, the zero-rest-length spring gets stretched, *i.e.,* , it is now under tension. The spring then exerts a force on the rod in the opposite direction of the tilt. The free-body diagram of the rod with a counterclockwise tilt θ is shown in Fig. 7.70. From the moment balance $\sum \vec{M}_O = \vec{0}$ (about the bottom support point O of the rod, which is also used as the origin for finding position vectors in the calculations below), we have

$$\vec{r}_B \times \vec{P} + \vec{r}_B \times \vec{F}_s = \vec{0}.$$

Noting that

$$\begin{aligned} \vec{r}_B &= \ell\hat{\boldsymbol{\lambda}}, \\ \vec{P} &= -P\hat{\boldsymbol{\jmath}}, \\ \text{and} \quad \vec{F}_s &= k(\vec{r}_A - \vec{r}_B) \quad \text{(For a zero-rest-length spring)} \\ &= k(\ell\hat{\boldsymbol{\jmath}} - \ell\hat{\boldsymbol{\lambda}}), \end{aligned}$$

we get

$$\begin{aligned} \ell\hat{\boldsymbol{\lambda}} \times (P\hat{\boldsymbol{\jmath}}) + \ell\hat{\boldsymbol{\lambda}} \times k\ell(\hat{\boldsymbol{\jmath}} - \hat{\boldsymbol{\lambda}}) &= \vec{0} \\ -P\ell(\hat{\boldsymbol{\lambda}} \times \hat{\boldsymbol{\jmath}}) + k\ell^2(\hat{\boldsymbol{\lambda}} \times \hat{\boldsymbol{\jmath}}) &= \vec{0}. \end{aligned}$$

Dotting this equation with $(\hat{\boldsymbol{\lambda}} \times \hat{\boldsymbol{\jmath}})$ we get

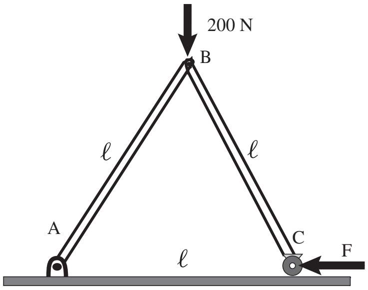
$$\begin{aligned} -P\ell + k\ell^2 &= 0 \\ \Rightarrow \quad P &= k\ell. \end{aligned}$$

Thus the equilibrium only requires that P be equal to $k\ell$ and it is independent of θ! That is, the system will be in static equilibrium at any θ as long as $P = k\ell$.

If $P = k\ell$, any θ is an equilibrium position. This is the 'buckling' load.

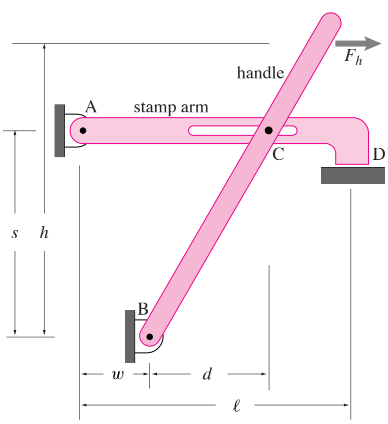
7.3 Machines

7.3.1 A simply supported two bar mechanism supports a load of 200 N at joint B with the help of a horizontal force F applied at joint C. Find F.



Problem 7.3.1

7.3.2 Pulling on the handle causes the stamp arm to press down at D. Neglect gravity and assume that the hinges at A and B, as well as the roller at C, are frictionless. Find the force N that the stamp machine causes on the support at D in terms of some or all of F_h, w, d, ℓ, h, and s. *



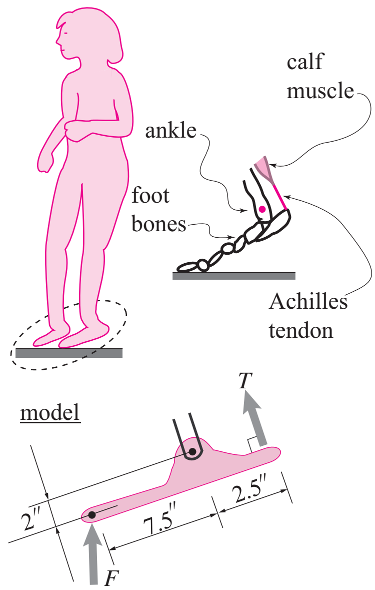
Problem 7.3.2

More-Involved Problems

7.3.3 See Problem 5.2.17 on page 305. A person who weighs W stands on tiptoes on one foot. Assume the weight of the foot is negligible.

a) Draw a free-body diagram of the whole person and find the force of the ground on the foot front.

b) Draw a free-body diagram of the foot and find the force of the calf on the foot at the ankle and the tension in the Achilles tendon.

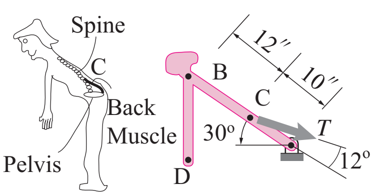


Problem 7.3.3

7.3.4 See Problem 5.2.17 on page 305. A person with weight $W = 140\,\text{lbf}$ has an upper body with weight $0.7W$ with center of mass at C. The back muscles are idealized as a single muscle with one end (the muscle *origin* also at C. Use the idealization and geometry shown.

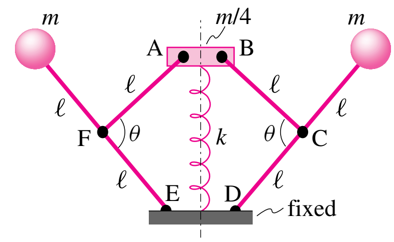
a) Find the back-muscle tension and the force of the lower body on the upper body at the hips.

b) Repeat the problem but assume that the person is lifting a 30 lbf load at D.



Problem 7.3.4

7.3.5 In the flyball governor shown, the mass of each ball is $m = 5\,\text{kg}$, and the length of each link is $\ell = 0.25$ m. There are frictionless hinges at points A, B, C, D, E, F where the links are connected. The central collar has mass $m/4$. Assuming that the spring of constant $k = 500$ N/m is uncompressed when $\theta = \pi$ radians, what is the compression of the spring?

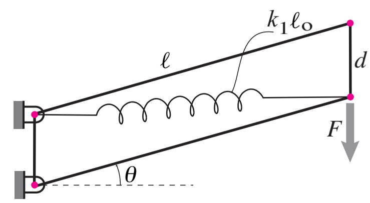


Problem 7.3.5

7.3.6

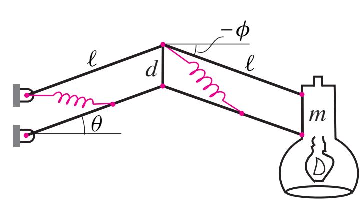
a) Find F for equilibrium for the parallelogram structure shown assuming the rest length of the spring is zero.

b) Comment on how your answer above depends on θ.



Problem 7.3.6

7.3.7 A common lamp design is shown. In principle, the lamp should be in equilibrium in all positions. According to the original patent from the 1930s, it can be in equilibrium, even with no friction in the joints. Unfortunately, the recent manufacturers of this lamp seem to have lost the wisdom of the original patent. Show how to place the springs so this lamp is in equilibrium for all $\theta < \pi/2$ and $\phi < \pi/2$. [Hint: use springs with zero rest length.]

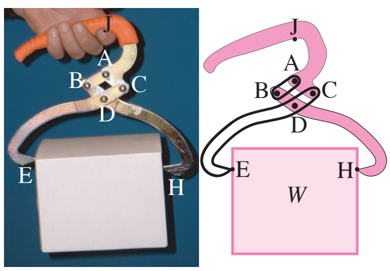


Problem 7.3.7: Lamp.

7.3.8 Log carrier. This self-locking scissors-mechanism gadget is used to pick up logs and blocks of ice. The 5 cm wide and 3 cm high diamond arrangement of hinges ABCD makes up a 4-bar linkage. The grips E and H are 16 cm apart and 9 cm below D. The block weighs 250 N. Neglect the weight of the mechanism.

a) What is the horizontal component of the force on the block at E?

b) What is the minimum coefficient of friction μ for which this device self locks?



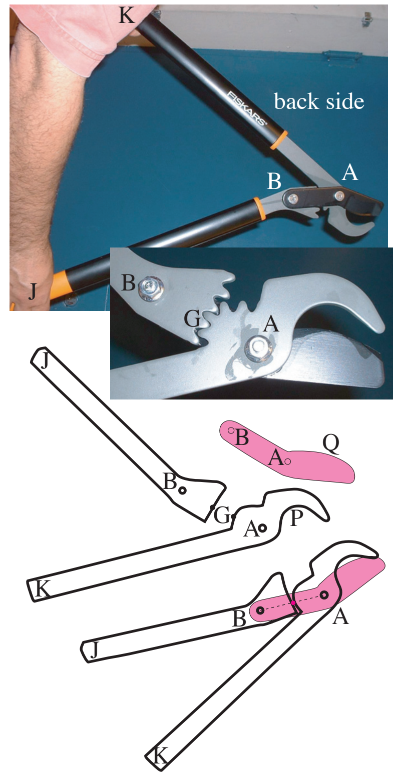
Problem 7.3.8

7.3.9 Gear teeth on handle JB mesh with teeth on handle-and-blade KAP at point G midway between hinges A and B. Assume that in the configuration of interest J, B and A are co-linear, that K, A and Q are co-linear and that the cutting contact points Q and P are effectively coincident, that angle JAK = 20°, $JB = 40$ cm, $BA = 6$ cm, $KA = 46$ cm, $AP = AQ = 3$ cm, and that the co-linear squeezing forces at J and K are 100 N.

a) Find the cutting force at Q and P.

b) Replace this design with one that has no tooth engagement at G. But instead handle JB and blade BAQ are welded together as one piece. Assuming the same geometry as before, what then is the cutting force at Q and P?

c) Without detailed calculations, explain the ratio of the two answers above. *



Problem 7.3.9

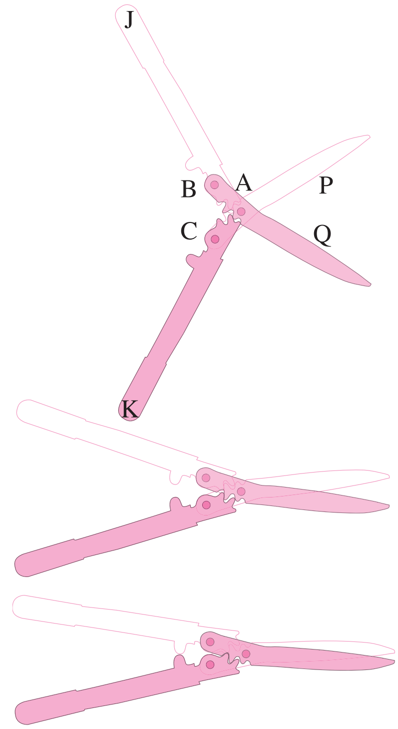
7.3.10 These 4-piece shears use the mechanism in problem 7.3.9 twice over. Co-linear hand forces F_{JK} are applied to handles JB and KC at J and K. Handle JB is hinged to blade BAQ at B. Handle KC is hinged to blade CAP at C. The blades are hinged to each other at A. Handle JB is effectively hinged to CAP, by means of gear teeth, at G, a point on the line segment AB. Similarly KC is effectively hinged to BAQ at point G' on the segment AC. The cut object presses with colinear forces F_{PQ} on the blades at P and Q. See box 7.6 on page 419 for more pictures of these shears.

Assume $F_{JK} = 100$ N, $JB = KC = 30$ cm, $AB = AC = 6$ cm, $AG = AG' = 3$ cm, and $AP = AQ = 20$ cm. Assume AB, BJ, AC,and CK all make angles of ±20° with a horizontal line. Assume P and Q are coincident and on a horizontal line extending from A.

a) Find F_{PQ}.

b) Replace this design with one where JB is welded to BAQ at B, KC is welded to CAP at C, and there are no contacting gears. In this same geometry what is F_{PQ}?

c) Give a quantitative estimate, but not a detailed calculation that tells you the ratio of the forces in the above two problems? *



Problem 7.3.10

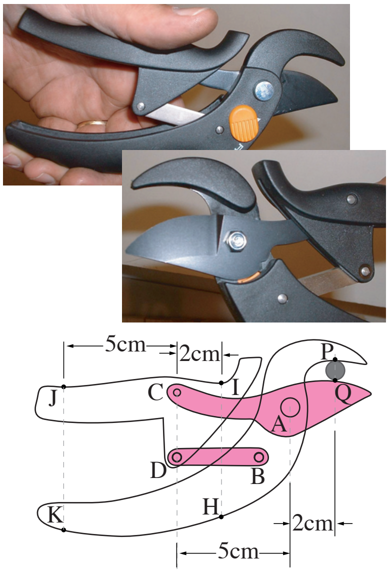
7.3.11 The garden cutters shown are a 4-bar linkage. Estimate the locations of points, as needed, using the given dimensions as a scale (the drawn clippers are shrunk slightly from reality to simplify the numbers).

a) If the handle is squeezed with a pair of 50 N forces at J and K what is the cutting force at P and Q? *

b) If the handle is squeezed with a pair of 50 N forces at I and H what is the cutting force at P and Q? *

c) If this design was changed by eliminating link DB and welding handle JCDI to the blade CAQ, what would be the answers to the two questions above. *

d) Describe in words, the reasons for the similarities and differences between the answers above. *



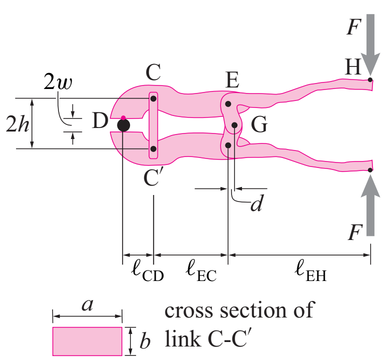
Problem 7.3.11

7.3.12 The pliers shown are made of five pieces modeled as rigid: HEG and its mirror image, DCE and its mirror image, and link CC'. You may assume that the geometry is symmetric about a horizontal line (the top is a mirror image of the bottom). The load F and dimensions shown are given. (See also similar problem 7.2.6)

a) Find the force squeezing the piece at D; *

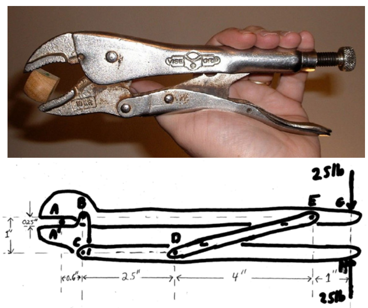
b) Find the tension in CC'; *

c) What happens to the squeezing force if d is made smaller, approaching zero? Why can't this work in practice? *



Problem 7.3.12

7.3.13 For simplicity the vice grips shown in the photo are approximated as in the drawing. Round piece AA' is gripped between the upper handle/jaw ABEG and the lower jaw A'BC. The upper handle ABEG is pinned to the lower jaw A'BC at B. Handle CDH is pinned to the lower jaw at C and to the bar DE at D. Bar DE is pinned to the upper handle ABEG at E. The 25 lbf forces act at G and H as shown. Dimensions are as shown. What is the magnitude of the force at A? *



Problem 7.3.13

7.3.14 Pipe wrench. A wrench is used to turn a pipe as shown in the figure. Neglecting the weight of the pipe, find

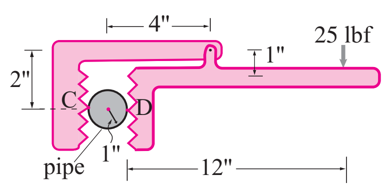
a) the torque of the pipe wrench forces about the center of the pipe

b) the forces on the pipe at C and D

c) the needed friction coefficient between the wrench and pipe for the wrench not to slip.

d) what design change would reduce this needed coefficient of friction (what change of dimensions)? *

e) given that the design change above is possible, why isn't it used? [hint: implement the design change and calculate the forces on the pipe.] *

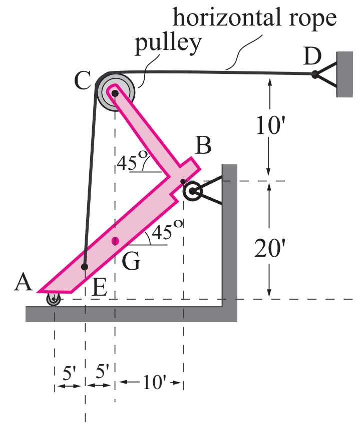


Problem 7.3.14

7.3.15 The center of mass of 200 pound structure AEGBC is at G. It is held by rollers at A and B as well as with the rope which starts at E, wraps around the pulley at C, and ends at D. You can assume the pulley at C has negligible size.

a) Find the force of the ground on the structure at A. *

b) Find the tension in the rope. *



Problem 7.3.15

7.3.16 Consider a bike on level ground that is held from falling sideways with forces that don't push it forward or back. Assume that all the bearings are ideal and that the wheels don't slip.

R_r = radius of rear wheel,
R_s = radius of rear sprocket,
R_p = crank length from crank-axle to pedal, and
R_c = radius of chain wheel (front sprocket).

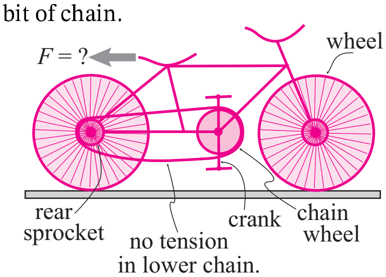
What backwards force F on the seat is required to keep the bike from going forward (*i.e.*, to maintain static equilibrium) if

a) A person sits on the bike and pushes back on the bottom pedal with a force F_p? (is $F > 0$?)

b) A person standing next to the bike pushes back on the pedal with force F_p? (is $F > 0$?)

Your answer should be in terms of some or all of R_r, R_s, R_p, R_c, and F_p. Of great interest is whether F is bigger or less than zero. So pay close attention to signs.

To solve this problem you have to draw several free-body diagrams: 1) of the whole bike and rider (if the rider is on the bike), 2) of the crank-pedal-chain-wheel system, with a little bit of chain, 3) The rear wheel and rear sprocket, with a little bit of chain.



Problem 7.3.16

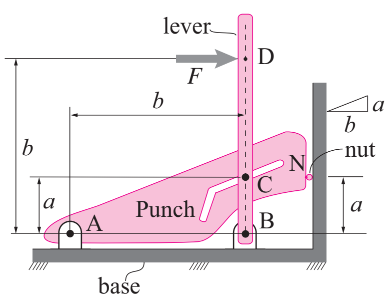
7.3.17 The proposed nutcracker design consists of two moving parts: a lever hinged to the fixed base at B and a punch hinged to the fixed base at A. All joints and slots are assumed to have negligible friction.

Mechanism and geometry clarifications: The vertical lever has a pin at C and a horizontal force F applied at D. The punch has a slot in which the lever pin slides at C. The slot is parallel to the line AC. The spherical nut is cracked by being squeezed between the vertical surface of the punch at N and the vertical surface attached to the base. Point N at the left edge of the nut is level with the sliding pin at C. The horizontal distance from C to N does not enter the solution, but assume it is c if you need it for an intermediate calculation.

Quantities: $F = 10\,\text{lbf}$, $a = 2\,\text{in}$, $b = 10\,\text{in}$.

a) Find the force acting on the nut at N. A number is desired (i.e., so many lbf force). [Hint: Only substitute in numbers when you have a formula for your answer in terms of a, b and F.] *

b) The answer to (a) is conspicuous in its being either much smaller than F, very similar to F, or much bigger than F. Which is it? Explain, in words, why. The best possible answer will generate an approximate formula for the force at N using next-to-no equations. *



Problem 7.3.17