6.5 Advanced truss concepts: determinacy

Your first concern when studying trusses is to develop the ability to solve a truss using free-body diagrams and equilibrium equations. You can do this with the method of joints. For some trusses you can use the method of sections as a short cut. However, not all trusses give a unique solution. In algebra there are equations with non-unique solutions (*e.g.,* $x + y = 4$) and sets of equations with no solutions ($x + y = 4$ and $2x + 2y = 9$). We have seen this issue before in the context of static equilibrium of a particle (see box 5.1 on page 247). With trusses the issues of existence and uniqueness remain.

Determinate, rigid, and redundant trusses

A truss that yields a solution, and only one solution, to such an analysis for all possible loadings is called *statically determinate* or just *determinate*. The braced box supported with one pin joint and one pin on rollers (see fig. 6.71a) is a classic statically determinate truss. A statically determinate truss is *rigid* and does not have *redundant* bars.

You should be warned, however: there are other possibilities.

Some trusses are *non-rigid*, like the one shown in fig. 6.71b, and can not carry arbitrary loads at the joints.

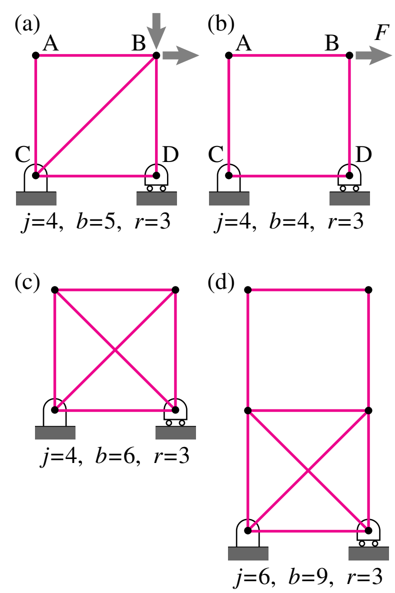


Figure 6.71: a) a statically determinate truss, b) a non-rigid truss, c) a redundant truss, and d) a non-rigid and redundant truss.

Example: Joint equations and non-rigid structures

Free-body diagrams of joints A and B of fig. 6.71b are shown in fig. 6.72.

$$\begin{aligned} \text{jointB}: \quad & \{\textstyle\sum \vec{F}_i = \vec{0}\} \cdot \hat{\imath} \quad \Rightarrow \quad T_{AB} = F \\ \text{jointA}: \quad & \{\textstyle\sum \vec{F}_i = \vec{0}\} \cdot \hat{\imath} \quad \Rightarrow \quad T_{AB} = 0 \end{aligned}$$

The contradiction that T_{AB} is both F and 0 implies that the equations of statics have no solution for a horizontal load at joint B.

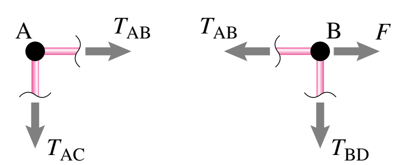


Figure 6.72: Free-body diagrams of joints A and B from 6.71b. The solutions to the equilibrium equations are in conflict. That the equilibrium equations can't be satisfied means the structure cannot carry the given load. It is not rigid.

A non-rigid truss can carry some loads, and you can find the bar tensions using the joint equilibrium equations when these loads are applied. For example, the structure of fig. 6.71b can carry a vertical load at joint B. Engineers sometimes choose to design trusses that are not rigid, the simplest example being a single piece of cable hanging a weight. A more elaborate example is a suspension bridge which, when analyzed as a truss, is not rigid.

A *redundant* truss has more bars than needed for rigidity. As you can tell from inspection or analysis, the braced square of fig. 6.71a is rigid. Nonetheless engineers will often choose to add extra redundant bracing as in fig. 6.71c for a variety of reasons.

- Redundancy is a safety feature. If one member breaks the whole structure holds up.
- Redundancy can increase a structure's strength.

- Redundancy can allow tensile bracing. In the structure of fig. 6.71a if the top load was to the left it would put bar BC in compression. Thus bar BC can't be, say, a cable. But in structure fig. 6.71c both diagonals can be cables and neither need carry compression for any load①.

① **Tensegrity structures.** Notice that you could make the diagonals in fig. 6.71c both sticks and all of the outside square from cables and the truss would still carry all loads. This is the simplest 'tensegrity' structure. In a tensegrity structure no more than one bar in compression is connected to any one joint. (See fig. 6.11 for a more elegant example.). The label 'Tensegrity structure' was coined by the truss-pre-occupied designer Buckminster Fuller. Fuller is also responsible for re-inventing the "geodesic dome" a type of structure studied previously by Cauchy.

A property of redundant structures is that you can find more than one set of bar forces that satisfy the equilibrium equations. Even when the loads are all zero, these structures can have non-zero *locked-in* forces (sometimes called ('locked-in stress', or 'self stress'). In the structure of fig. 6.71c, for example, if one of the diagonals got cool and contracted, both it and the opposite diagonal would be put in tension while the outside was in compression. For structures whose parts are likely to expand or contract, or for which the foundation may shift, this locked-in stress can be a contributor to structural failure. So redundancy is not all good.

Finally, a structure can be both non-rigid and redundant as shown in fig. 6.71d. This structure can't carry all loads, but the loads it can carry it can carry with various locked in bar forces.

More examples of statically determinate, non-rigid, and redundant trusses are given on pages 9 and 10.

Note, one of the basic assumptions in elementary truss analysis which we have thus far used without comment is that

> motions and deformations of the structure are not taken into account when applying the equilibrium equations.

If a bar is vertical in the drawing, then it is taken as vertical for all joint equilibrium equations.

Example: Hanging rope
For elementary truss analysis, a hanging rope would be taken as hanging vertically even if side loads are applied to its end. This obviously ridiculous assumption manifests itself in truss analysis by the discovery that a hanging rope cannot carry any sideways loads (if it must stay vertical, this is true).

Determining determinacy: counting equations and unknowns

How can you tell if a truss is statically determinate? The only sure test is to write all the joint force-balance equations and see if they have a unique solution for all possible joint loads. Because this is an involved linear algebra calculation (which we skip in this book), it is nice to have shortcuts, even if not totally reliable. Here are three:

- See, using your intuition, if the structure can deform without any of the bars changing length. You can see that the structures of fig. 6.71b and d can distort. If a structure can distort, it is not rigid and thus is not statically determinate.

- See, using your intuition, if there are any redundant bars. A redundant bar is one that prevents a structural deformation that already is prevented. It is easy to see that the second diagonal in structures of fig. 6.71c and d is clearly redundant so these structures are not statically determinate.
- Count the total number of joint equations, two for each joint. See if this is equal to the number of unknown bar forces and reactions. If not, the structure is not statically determinate.

The counting formula in the third criterion above is:

$$2j = b + r \tag{6.28}$$

where j is the number of joints, including joints at reaction points, b is the number of bars, and r is the number of reaction components that shows on a free-body diagram of the whole structure (2 from pin joints, 1 from a pin on a roller).

If $2j > b + r$, the structure is necessarily not rigid because then there are more equations than unknowns②. For such a structure, there exist some loads for which there is no set of bar forces and reactions that can satisfy the joint equilibrium equations. A structure that is non-redundant and non-rigid always has $2j > b + r$ (see fig. 6.71b).

If $2j < b + r$, the structure is redundant because there are not as many equations as unknowns; if the equations can be solved, there is more than one combination of forces that solve them. A structure that is rigid and redundant always has $2j < b + r$ (see fig. 6.71c).

But the possibility of structures that are both non-rigid and redundant makes the counting formulas an imperfect way to classify structures.③Non-rigid redundant structures can have $2j < b+r$, $2j = b+r$, or $2j > b+r$. The redundant non-rigid structure in fig. 6.71d has $2j = b+r$.

The discussion above can be roughly summarized by this table (refer to fig. 6.71 for a simple example of each entry and to pages 9 and 10 for several more examples).

Truss Type	Rigid	Non-rigid
Non-redundant	a) $2j = b + r$ (Statically determinate)	b) $2j > b + r$
Redundant	c) $2j < b + r$	d) $2j < b + r$, $2j = b + r$, or $2j > b + r$

A basic summary is this:

②A non-rigid truss is sometimes called ‘over-determinate’ because there are more equations than unknowns. However, the term ‘over-determinate’ may incorrectly conjure up the image of there being too many bars (which we call *redundant*) rather than too many joints. So we avoid use of this phrase.

③In the language of mathematics we would say that satisfaction of the counting equation $2j = b + r$ is a *necessary* condition for static determinacy but it is not *sufficient.*

If

- $2j = b + r$ *and*
- you cannot see any ways the structure can distort, *and*
- you cannot see any redundant bars

then the truss is likely statically determinate. But the only way you can know for sure is through either a detailed study of the joint equilibrium equations, or familiarity with similar structures.

On the other hand if

- $2j > b + r$, *or*
- $2j < b + r$, *or*
- you can see a way the structure can distort, *or*
- you can see one or more redundant bars,

then the truss is *not* statically determinate.

Example: The classic statically determinate structure
A *triangulated truss* can be drawn as follows:

1. draw one triangle,
2. then another by adding two bars to an edge,
3. then another by adding two bars to an existent edge
4. and so on, but never adding a triangle by adding just one bar, and
5. you hold this structure in place with a pin at one joint and one pin on roller at another joint

then the structure is statically determinate. Many elementary trusses are of exactly this type. (*Note*: if you violate the 'but' in the 4th rule you can make a truss that looks 'triangulated' but is redundant, and therefore not statically determinate.)

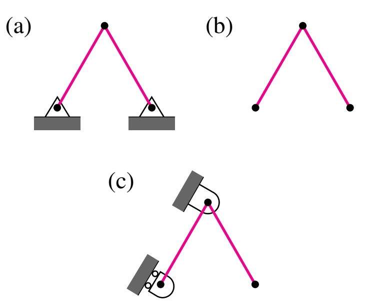


Figure 6.73: a) a determinate two bar truss connected to the ground, b) the same truss is not rigid when floating, which you can tell by seeing that c) it is not rigid when one bar is fixed to the ground.

Floating trusses

Sometimes one wants to know if a structure is rigid and non-redundant when it is floating unconnected to the ground (but still in 2D, say). For example, a triangle is rigid when floating and a square is not. The truss of fig. 6.73a is rigid as connected but not when floating (fig. 6.73b).A way to find out if a floating structure is rigid is to connect one bar of the truss to the ground by connecting one end of the bar with a pin and the other with a pin on a roller, as in fig. 6.73c. All determinations of rigidity for the floating truss are the same as for a truss grounded this way. The counting formula eqn. 6.28, is reduced to

$$2j = b + 3$$

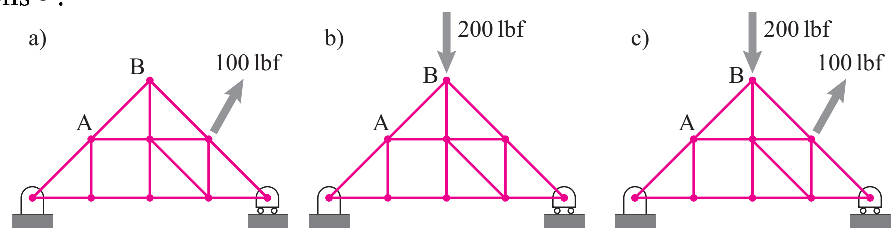
because this minimal way of holding the structure down uses $r = 3$ reaction force components.

The principle of superposition for trusses

Say you have solved a truss with a certain load and have also solved it with a different load. Then if both loads were applied the reactions would be the sums of the previously found reactions and the bar forces would be the sums of the previously found bar forces.

This useful fact follows from the linearity of the equilibrium equations④.

④A careful derivation would also show that the linearity depends on the nature of the foundation. Linearity holds for pins and pins on rollers, but not for frictional contact.



Example: Superposition and a truss
If for the loading (a) you found $T_{\text{AB}} = 50\,\text{lbf}$ and for loading (b) you found $T_{\text{AB}} = -140\,\text{lbf}$ then for loading (c) $T_{\text{AB}} = 50\,\text{lbf} - 140\,\text{lbf} = -90\,\text{lbf}$

The principle of superposition can only hold if the solution for zero load is zero tension in all the bars. Any truss that only has bars in tension when there is no load does not satisfy the principle of superposition.

Example: Spider web
A network of taut strings is a kind of a truss. So a spider web is a kind of a truss. But a spider web is only a coherent structure if it is kept taut. So there is tension in the strands even when there is no load (from, say the weight of a spider). Thus the principle of superposition does not apply. The tension in a given strand is not the tension due to the spider added to the tension due to an insect.

6.2 Structural rigidity and geometric congruence

This box is only for the curious. It will not help you solve truss homework problems.

In high school geometry one learns to prove that two shapes are congruent (the same shape and size) if they have enough in common. High school geometry proofs are based on triangles. For example one proof, called “side-side-side” (SSS), says that if two triangles have three sides with corresponding lengths then the corresponding angles are also equal.

Now, here, we claim that structures made of triangles tend to be rigid. Is there a relation between the central role of triangles in geometry proofs and their role in structural rigidity? The answer is yes, but more subtly than you may expect.

Consider one triangle. If the lengths are specified it is like three sticks connected with rubber bands (page 316). That two different triangles each with the same 3 side lengths are congruent means that one triangle whose side-lengths are given has no choice about its shape. So for one triangle the SSS proof corresponds exactly to structural rigidity.

More generally, imagine looking at a structure and thinking of certain aspects of it as fixed and others as not fixed. For example, think of a collection of bars with the lengths fixed (each bar cannot stretch or shrink) and the angles between them as not-fixed (the angles are flexible). This would be a model, say, of bars connected with pin joints. If one could find a geometry proof that these two structures had identical shapes it would mean that each one of them had no choice about its shape. So a geometry proof of congruency, based on the aspects of a structure that are approximately fixed, is a proof of structural rigidity. This shows that there *is* a connection between congruence proofs and structural rigidity.

Here’s the subtlety. Neither congruence proofs nor rigidity actually depend essentially on triangles. There are congruence proofs for shapes that do not have any closed triangles, and the related structures are rigid.

In fact, there is a whole arcane mathematics of rigidity. And the things mathematicians have learned about rigidity are incredible.

The case of K_{33}

Take 3 points on a plane and mark them with dots. Take 3 more points on the plane and mark them with little x’s. Connect each dot with each x. Thats 9 connection lines. In topology-speak they call this set of dots and lines “K three three” (*K*onnections between *three* dots and *three* x’s).

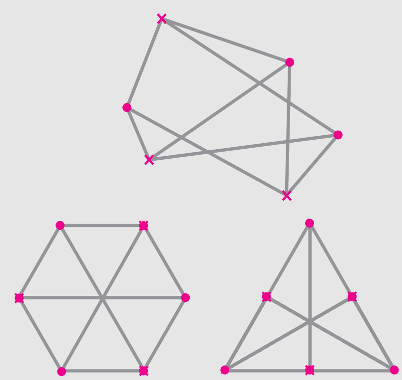
Now think of that criss-crossed K_{33} drawing as a structure made of sticks connected with hinges at the dots and x’s. Note that, neglecting where the sticks cross but are not connected, there are no closed triangles. Yet, incredibly, this structure is always rigid. Well, almost always. If all 6 points happen to lie on one circle, ellipse, parabola or hyperbola then the structure is not rigid.

Example: Regular hexagon

If you take a regular hexagon made of sticks (length ℓ and hinges and brace it with three cross bars (each with length 2ℓ) you will see that you have K33; every-other corner is a dot and the alternate ones are x’s. But the points on a hexagon are on a circle, so that structure is not rigid.

Example: Triangle with two bars per side

On the other hand, take an equilateral triangle and cut each side in half so you have six bars around the outside (each with length $\ell/2$). Now brace that hexagon (that is shaped like a triangle) with the three triangle altitudes (each with length $\sqrt{3}\ell$) and you again have K33. But this time it’s rigid.



The examples above are used in the text and homework to illustrate structures that don’t lend themselves to the simple joint-by-joint method-of-joints, nor the method of sections. For these trusses the method of joints leads to a set of equations that need to be solved simultaneously.

K_{nn}

The mathematical magic goes on. If you take any n dots and any n x’s and connect each dot to each x with a rigid rod (K_{nn}) you get a rigid structure. Unless all $2n$ dots happen to lie on a conic section.

The proofs of such rigidity theorems are way over our heads. But you can simply check such structures for rigidity with the computer program developed in section 6.3.

Rigidity and congruence?

So yes, geometric congruence and structural rigidity are the same subject. But that subject does not totally depend on triangles. Triangles just provide the simple examples and what we vaguely think of as the essence of both subjects.

6.3 Rigidity, redundancy, linear algebra and maps

This mathematical aside is only for people who have had a course in linear algebra. For definiteness this discussion is limited to 2D trusses, but the ideas also apply to 3D trusses.

For beginners trusses fall into two types, those that are uniquely solvable (statically determinate) and those that are not. Statically determinate trusses are rigid and non-redundant. However, a truss could be non-rigid and non-redundant, rigid and redundant, or non-rigid and redundant. These four possibilities are shown with a simple example each in fig. 6.71 on page 1, as a simple table on page 3, and as a big table of examples on pages 9 and 10.

Another approach is the table in fig. 6.74 which we now proceed to discuss in detail. It is a more abstract mathematical representation of this same set of possibilities.

• • •

To start with we use the matrix form of the truss joint equations from page 344. To make contact with linear algebra here we take the unknowns as $[v]$ with $[v] \equiv [T]$ being the n unknown tensions and reaction components. The set of lists of all conceivable tensions and reaction forces we call the "vector space" V (it is also R^n).

The m possible loads at the joints are written in the column vector $[w]$ (called $[L]$, for loads, in the numeric set up). The set of all possible loads we call the vector space W.

If we use the method of joints we can write two scalar equilibrium equations for each joint. These are linear algebraic equations. Thus we can write them in matrix form as (see eqn. (6.19) on page 344),

$$[A][v] = [w] \tag{6.29}$$

The classification of trusses is really a statement about the solutions of eqn. 6.29. This classification follows, in turn, from the properties of the matrix $[A]$.

Another point of view is to think of eqn. 6.29 as a function that maps one vector space onto another. For any $[v]$ eqn. 6.29 maps that $[v]$ to some $[w]$. That is, if one were given all the bar tensions and reactions one could uniquely determine the applied loads from eqn. 6.29. This map, from V to W we call T.

• • •

We can now discuss each of the truss categorizations in turn, with reference to the table at the end of this box.

The first column of the table corresponds to rigid trusses. These trusses have at least one set of bar forces that can equilibrate any particular load. This means that for every $[w]$ there is some $[v]$ that maps to (whose image is) $[w]$. In these cases the map T is onto. And the column space of $[A]$ is W. Thus $[A]$ needs to have at least as many columns as the dimension of W which is the number of rows of $[A]$.

On the other hand if the structure is not rigid there are some loads that cannot be equilibrated by any bar forces. This is the second column of the table. There is at least some $[w]$ with no pre-image $[v]$. Thus the map T is not onto and the column space of $[A]$ is less than all of W.

The first row of the table describes trusses which are not-redundant. Thus, any loads which can be equilibrated can be equilibrated with a unique set of bar tensions and reactions. Thus the columns of $[A]$ are linearly independent and the map T is one-to-one. The matrix $[A]$ must have at least as many rows as columns.

If a truss is redundant, as in the second row of the table, then there are various ways to equilibrate loads which can be carried. Points in W in the image of V have non-unique pre-images, T is not one to one, and the columns of A are linearly dependent.

• • •

We can now look at the four entries in the table. The top left case is the statically determinate case where the structure is rigid and non-redundant. The map T is one to one and onto, $V = W$, and the matrix $[A]$ is square and non-singular.

The bottom left case corresponds to a truss that is rigid and redundant. The map T is onto but not one to one. The columns of $[A]$ are linearly dependent and it has more columns than rows (it is wide).

The top right case is not rigid and not redundant. Some loads cannot be equilibrated and those that can be are equilibrated uniquely. T is one to one but not onto. The columns of $[A]$ are linearly independent but they do not span W. The matrix $[A]$ has more rows than columns and is thus tall.

The bottom right case is the most perverse. The structure is not rigid but is redundant. Not all loads can be equilibrated but those that can be equilibrated are equilibrated non-uniquely. The matrix $[A]$ could have any shape but its columns are linearly dependent and do not span W. The map T is neither one to one nor onto.

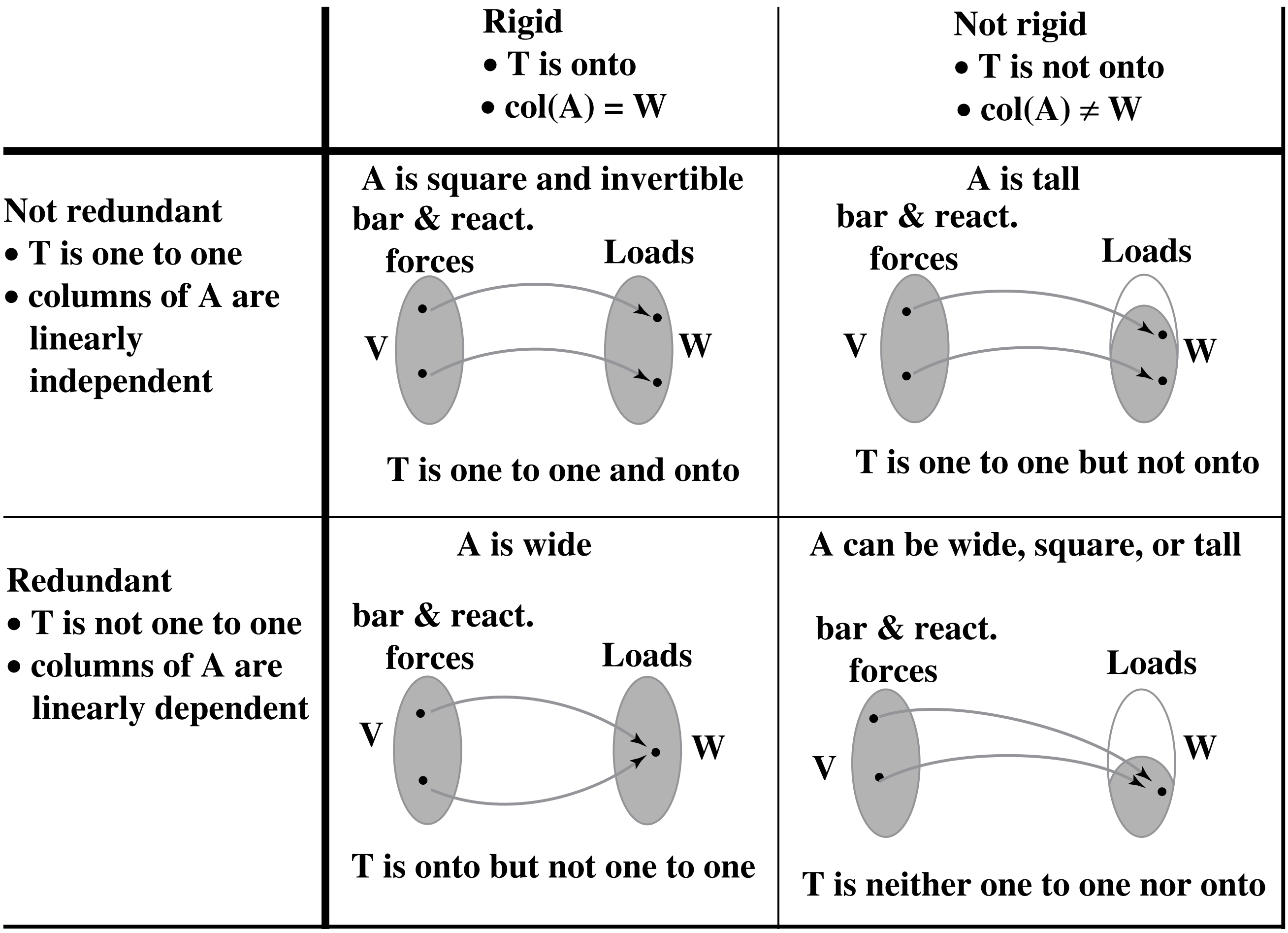


Figure 6.74: **Rigidity, redundancy and the structural matrices**. T is the linear transformation from the bar and reaction forces to the applied loads, it is represented with the matrix $[A]$. See box 6.3 on page 7 for a more detailed description.

2D TRUSS CLASSIFICATION
(page 1)

Rigid

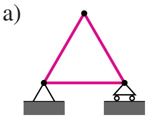
• loads can be equilibriated with bar forces

Not redundant

• Not indeterminate
• If there are bar forces that can equilibriate the loads they are unique
• No locked in stresses

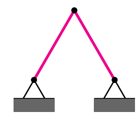
Statically determinate,
rigid and not redundant,
$b + r = 2j$,
One and only one set of bar forces can equilibriate any given load.

a)



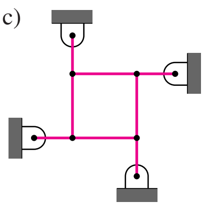
j=3, b=3, r=3

b)



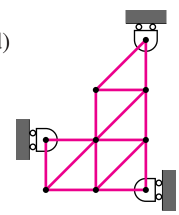
j=3, b=2, r=4

c)



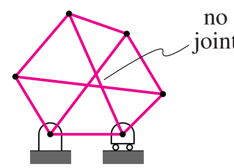
j=8, b=8, r=8

d)



j=9, b=15, r=3

e)



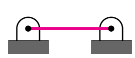
j=6, b=9, r=3

Redundant

• indeterminate
• locked in stress possible
• solutions not unique
  if they exist

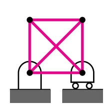
$b + r > 2j$, "too few equations", rigid and redundant,
Every possible load can be equilibrated
but the bar forces are not unique.

l)



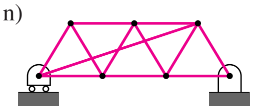
j=2, b=1, r=4

m)



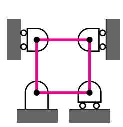
j=4, b=6, r=3

n)



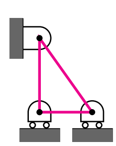
j=7, b=12, r=3

o)



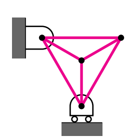
j=4, b=4, r=5

p)



j=3, b=3, r=4

q)



j=4, b=6, r=3

Figure 6.75: Examples of 2D trusses. These two pages concern the 2-fold system for identifying trusses. Trusses can be rigid or not rigid (the two columns) and they can be redundant or not redundant (the two rows). Elementary truss analysis is only concerned with rigid and not redundant trusses (*statically determinate* trusses). Note that the only difference between trusses (b) and (s) is a change of shape (likewise for the far more subtle examples (e) and (u)). Truss (e) is interesting as a rare example of a determinate truss with no triangles. Continued on page 10

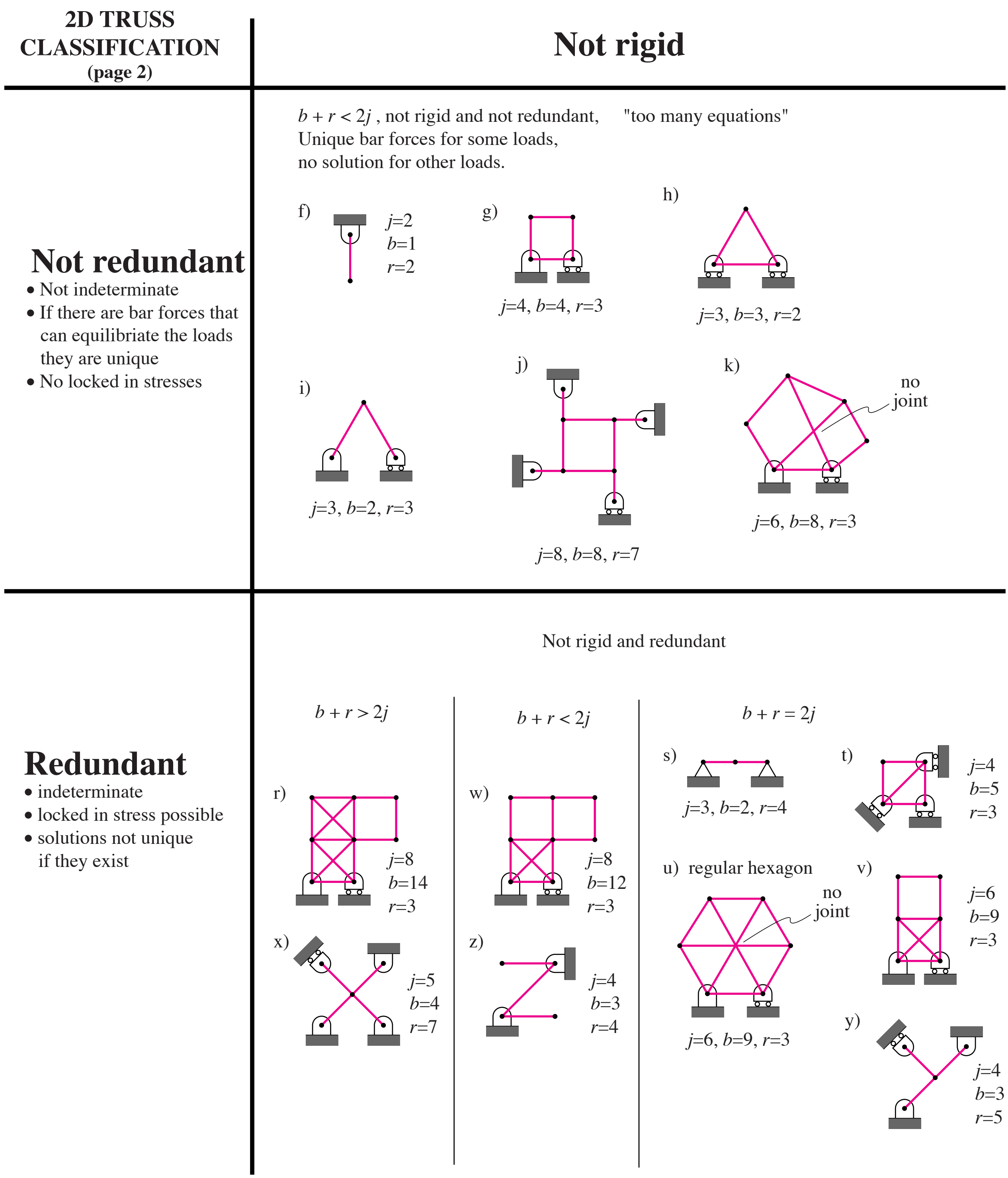


Figure 6.76: (Second page of a two page table.)

SAMPLE 6.16 **An indeterminate truss:** For the truss shown in the figure, find all support reactions.

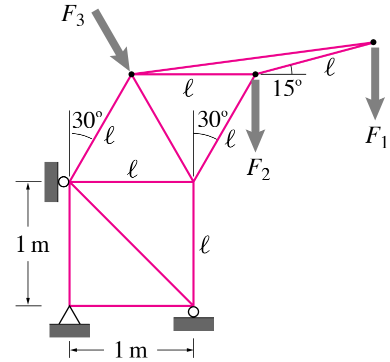


Figure 6.77

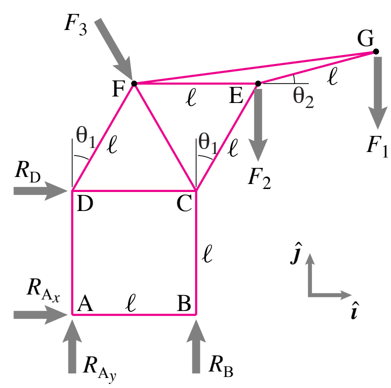


Figure 6.78: The free-body diagram of the structure.

Solution The free-body diagram of the truss is shown in Fig. 6.78. We need to find the support reactions R_{A_x}, R_{A_y}, R_B, and R_D.

The x and y components of the force equilibrium, $\sum \vec{F} = \vec{0}$, give

$$\sum F_x = 0 \quad \Rightarrow \quad R_{A_x} + R_D = -F_3 \cos\theta_1 \tag{6.30}$$

$$\sum F_y = 0 \quad \Rightarrow \quad R_{A_y} + R_B = F_1 + F_2 + F_3 \sin\theta_1. \tag{6.31}$$

Now we apply moment balance about point A, $\sum \vec{M}_{\mathrm{A}} = \vec{0}$. Let A be the origin of our xy-coordinate system (so that we can write $\vec{r}_{\mathrm{D/A}} = \vec{r}_{\mathrm{D}}$, etc.).

$$\vec{r}_{\mathrm{D}} \times \vec{R}_D + \vec{r}_{\mathrm{F}} \times \vec{F}_3 + \vec{r}_{\mathrm{G}} \times \vec{F}_1 + \vec{r}_{\mathrm{E}} \times \vec{F}_2 + \vec{r}_{\mathrm{B}} \times \vec{R}_B = 0$$

where,

$$\begin{aligned}
\vec{r}_{\mathrm{D}} \times \vec{R}_D &= \ell\,\hat{\jmath} \times R_D \hat{\imath} = -R_D \ell\,\hat{k} \\
\vec{r}_{\mathrm{F}} \times \vec{F}_3 &= (\vec{r}_{\mathrm{D}} + \vec{r}_{\mathrm{F/D}}) \times \vec{F}_3 \\
&= [\ell\,\hat{\jmath} + \ell(\sin\theta_1 \hat{\imath} + \cos\theta_1 \hat{\jmath})] \times F_3(\cos\theta_1 \hat{\imath} - \sin\theta_1 \hat{\jmath}) \\
&= F_3 \ell \cos\theta_1 \hat{k} - F_3 \ell \hat{k} = -F_3 \ell (1 + \cos\theta_1)\hat{k} \\
\vec{r}_{\mathrm{G}} \times \vec{F}_1 &= (r_{G_x} \hat{\imath} + r_{G_y} \hat{\jmath}) \times (-F_1 \hat{\jmath}) = -r_{G_x} F_1 \hat{k} \\
&= -F_1 \ell (1 + \sin\theta_1 + \cos\theta_2)\hat{k} \\
\vec{r}_{\mathrm{E}} \times \vec{F}_2 &= -F_2(\ell + \ell \sin\theta_1 \hat{k} = -F_2 \ell (1 + \sin\theta_1)\hat{k} \\
\vec{r}_{\mathrm{B}} \times \vec{R}_B &= \ell\,\hat{\imath} \times R_B \hat{\jmath} = R_B \ell\,\hat{k}.
\end{aligned}$$

Adding them together and dotting with $\hat{k}$ we get

$$-R_D \ell - F_3 \ell (1 + \cos\theta_1) - F_1 \ell (1 + \sin\theta_1 + \cos\theta_2) - F_2 \ell (1 + \sin\theta_1) + R_B \ell = 0$$

$$\begin{aligned}
\Rightarrow \quad R_B - R_D &= F_1(1 + \sin\theta_1 + \cos\theta_2) \\
&\quad + F_2(1 + \sin\theta_1) + F_3(1 + \cos\theta_1).
\end{aligned} \tag{6.32}$$

We have three equations (6.30–6.32) containing four unknowns R_{A_x}, R_{A_y}, R_B, and R_D. So, we cannot solve for the unknowns uniquely. This was expected as the truss is indeterminate. However, if we assume a value for one of the unknowns, we can solve for the rest in terms of the assumed one. For example, let $R_D = \alpha$. For simplicity let the right hand sides of eqns. (6.30, 6.31, and 6.32) be C_1, C_2, and C_3 (computed values), respectively. Then, we get $R_{A_x} = C_1 - \alpha$, $R_{A_y} = C_2 - C_3 - \alpha$, and $R_B = C_3 + \alpha$. The equilibrium is satisfied for any value of α. Thus there are infinite number of solutions! This is true for all indeterminate systems. However, when deformations of structures are taken into account (extra constraint equations), then solutions do turn out to be unique. You will learn about such things in courses dealing with strength of materials.

6.5 Advanced truss analysis: determinacy, rigidity, and redundancy

Preparatory Problems

6.5.1 Define these terms

a) statically determinate

b) rigid and non-rigid

c) redundant and non-redundant

6.5.2 For each set of conditions below, find 2 trusses both of which fit the description

a) rigid and non-redundant

b) rigid and redundant

c) not rigid and not redundant

d) not rigid and redundant

6.5.3 In 2D trusses we used the formula $b + r = 2j$. With what formula do we replace this for 3D trusses? Explain why.

6.5.4 For the 3D method of joints, for a whole truss how many independent scalar equilibrium equations can one write?

6.5.5 For one section cut in 3D how many bar tensions can you hope to find?

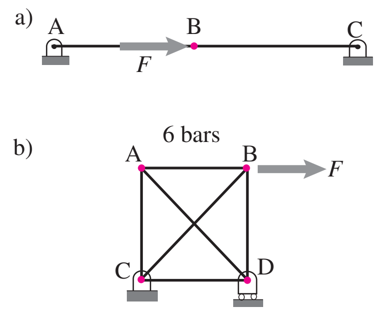
6.5.6 For a 3D truss that is rigid when not grounded, how many independent reaction components do you need to make it a statically determinate structure for any loading?

More-Involved Problems

6.5.7 For the following structures find at least 2 different sets of bar forces that can equilibrate the applied load shown.

a) Two bars in a line with a force in the same line.

b) A square with two diagonal braces.



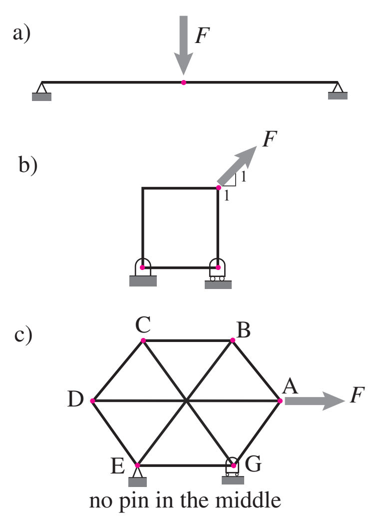
Problem 6.5.7

6.5.8 For the structures and loading shown show that there is no set of bar forces for which equilibrium is possible (at least with the geometry shown). All of these structures are not rigid, they require either infinite bar forces or some (or a lot of) deformation to withstand the load applied

a) Two bars in a straight line.

b) A square without a diagonal.

c) A regular hexagon with three diameters. [This problem is hard and might best be answered using linear algebra methods on the matrix form of the system of equilibrium equations.]



Problem 6.5.8

6.5.9 For each of the structures below and the shown loading answer these questions:
i) Does a set of equilibrium bar forces and ground reactions exist? ii) If so, find one such set. iii) Are the solutions, if they exist, unique? iv) If not find at least two solutions. v) Is the structure rigid? vi) If not, how can it deform?

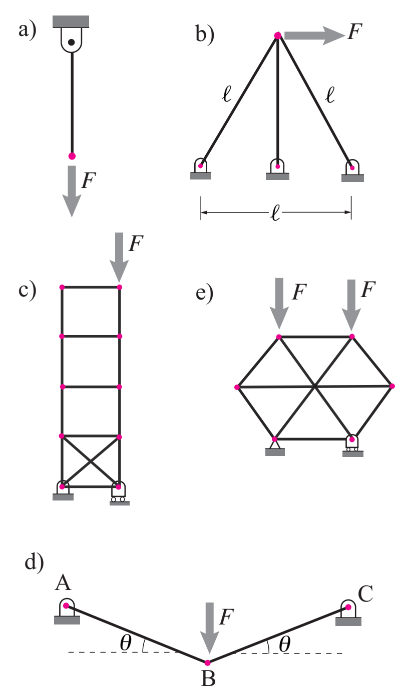
a) One hanging rod

b) A braced pole

c) A tower

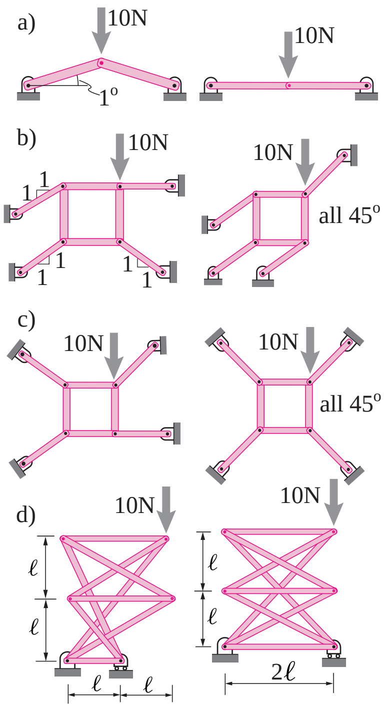
d) Two bars holding a vertical load. Comment in your answers how they change in the limit $\theta \to 0$.

e) A regular hexagon with three diagonals (this is a hard problem).



Problem 6.5.9

6.5.10 Use your program from problem 6.3.4 to analyze each pair of structures shown. In each case the output of your program should be radically different for the right structure than for the superficially similar structure on the left. **i)** Describe the difference in your computer program behavior, **ii)** As well as you can, explain what it is about the structures that causes this difference in computer behavior.



Problem 6.5.10