**SAMPLE 6.16** **An indeterminate truss:** For the truss shown in the figure, find all support reactions.

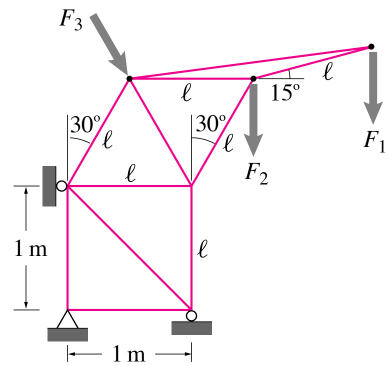


Figure 6.77

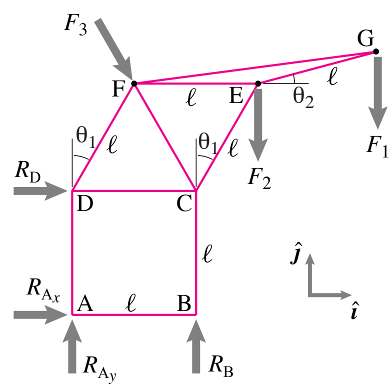


Figure 6.78: The free-body diagram of the structure.

**Solution** The free-body diagram of the truss is shown in Fig. 6.78. We need to find the support reactions $R_{A_x}, R_{A_y}, R_B$, and $R_D$.

The $x$ and $y$ components of the force equilibrium, $\sum \vec{F} = \vec{0}$, give

$$\sum F_x = 0 \quad \Rightarrow \quad R_{A_x} + R_D \quad = \quad -F_3 \cos\theta_1 \tag{6.30}$$

$$\sum F_y = 0 \quad \Rightarrow \quad R_{A_y} + R_B \quad = \quad F_1 + F_2 + F_3 \sin\theta_1. \tag{6.31}$$

Now we apply moment balance about point A, $\sum \vec{M}_{\text{A}} = \vec{0}$. Let A be the origin of our $xy$-coordinate system (so that we can write $\vec{r}_{\text{D/A}} = \vec{r}_{\text{D}}$, etc.).

$$\vec{r}_{\text{D}} \times \vec{R}_D + \vec{r}_{\text{F}} \times \vec{F}_3 + \vec{r}_{\text{G}} \times \vec{F}_1 + \vec{r}_{\text{E}} \times \vec{F}_2 + \vec{r}_{\text{B}} \times \vec{R}_B = 0$$

where,

$$\begin{aligned}
\vec{r}_{\text{D}} \times \vec{R}_D &= \ell\,\hat{\jmath} \times R_D \hat{\imath} = -R_D \ell\,\hat{k} \\
\vec{r}_{\text{F}} \times \vec{F}_3 &= (\vec{r}_{\text{D}} + \vec{r}_{\text{F/D}}) \times \vec{F}_3 \\
&= [\ell\,\hat{\jmath} + \ell(\sin\theta_1 \hat{\imath} + \cos\theta_1 \hat{\jmath})] \times F_3(\cos\theta_1 \hat{\imath} - \sin\theta_1 \hat{\jmath}) \\
&= F_3 \ell \cos\theta_1 \hat{k} - F_3 \ell \hat{k} = -F_3 \ell (1 + \cos\theta_1)\hat{k} \\
\vec{r}_{\text{G}} \times \vec{F}_1 &= (r_{G_x} \hat{\imath} + r_{G_y} \hat{\jmath}) \times (-F_1 \hat{\jmath}) = -r_{G_x} F_1 \hat{k} \\
&= -F_1 \ell (1 + \sin\theta_1 + \cos\theta_2)\hat{k} \\
\vec{r}_{\text{E}} \times \vec{F}_2 &= -F_2(\ell + \ell \sin\theta_1 \hat{k} = -F_2 \ell (1 + \sin\theta_1)\hat{k} \\
\vec{r}_{\text{B}} \times \vec{R}_B &= \ell\,\hat{\imath} \times R_B \hat{\jmath} = R_B \ell\,\hat{k}.
\end{aligned}$$

Adding them together and dotting with $\hat{k}$ we get

$$-R_D \ell - F_3 \ell (1 + \cos\theta_1) - F_1 \ell (1 + \sin\theta_1 + \cos\theta_2) - F_2 \ell (1 + \sin\theta_1) + R_B \ell = 0$$

$$\begin{aligned}
\Rightarrow \quad R_B - R_D \quad = \quad & F_1(1 + \sin\theta_1 + \cos\theta_2) \\
& + F_2(1 + \sin\theta_1) + F_3(1 + \cos\theta_1).
\end{aligned} \tag{6.32}$$

We have three equations (6.30–6.32) containing four unknowns $R_{A_x}, R_{A_y}, R_B$, and $R_D$. So, we cannot solve for the unknowns uniquely. This was expected as the truss is indeterminate. However, if we assume a value for one of the unknowns, we can solve for the rest in terms of the assumed one. For example, let $R_D = \alpha$. For simplicity let the right hand sides of eqns. (6.30, 6.31, and 6.32) be $C_1, C_2$, and $C_3$ (computed values), respectively. Then, we get $R_{A_x} = C_1 - \alpha$, $R_{A_y} = C_2 - C_3 - \alpha$, and $R_B = C_3 + \alpha$. The equilibrium is satisfied for any value of $\alpha$. Thus there are infinite number of solutions! This is true for all indeterminate systems. However, when deformations of structures are taken into account (extra constraint equations), then solutions do turn out to be unique. You will learn about such things in courses dealing with strength of materials.