6.2 The method of sections

Perhaps the most central concept in mechanics, and thus for truss analysis, is the free-body diagram. For truss analysis we have already found it fruitful to draw free-body diagrams of the whole structure, of the bars (to see that they are two-force bodies), and of the individual joints. But you can draw a free-body diagram of anything, for example of any part of a system you are studying. Assuming static equilibrium, force and moment balance apply to that subsystem.

In *the method of sections* you find bar tensions by drawing a free-body diagram of a part of the truss that includes more than one joint and less than the whole structure.

The place where the truss is cut is called the *section*.

What's wrong with the method of joints?

The method of joints can solve any solvable truss. So why learn a different method? There are two basic reasons.

1. Sometimes one only wants to know a little and the method of joints is cumbersome.

   Example: Difficulty in finding just one bar tension.
   Say you are interested only in T_{KM} in the truss of fig. 6.7 on page 320. With the method of joints we could find T_{KM} by working through the joints one at a time. To get to joint K we would have to draw free-body diagrams of at least 8 other joints first. And for each we would have to solve two simultaneous equations.

2. Sometimes the method of joints doesn't best reveal basic structural ideas.

   Example: Difficulty in understanding trends.
   Again look at the truss of fig. 6.7. With the method of joints we would find, after all the algebra, that all the bars on the bottom (AC, CE, EH, HJ, JL, LN, NP, PR) have compression (negative tension) and that each bar has more compression than the one to its right. Similarly the top is all tension with the tension increasing with the bars more to the left. Are these trends just a consequence of lots of algebra?

The method of sections provides a shortcut, particularly for elementary textbook-like problems. And the method of sections can explain some structural trends.

The basic method of sections recipe

Say you are just trying to find one bar tension, for example T_{KM} in the truss of fig. 6.7. For simplicity we limit our attention to 2D structures.

- Find a way to cut the structure into two parts, using a *section cut* that
  - cuts the bar of interest and
  - cuts at most 3 bars in total and
  - where one of the two parts of the truss has all loads known because
    - all loads are given applied loads, or
    - the loads are reactions that have been found using a free-body diagram of the whole structure.
- Write and solve the equations of moment balance for one side of the structure. This should be 3 equations in 3 unknowns.
  - Either use any three independent equations (say force balance (2) and moment balance (1)), or
  - Look for a shortcut. Try to find one equation that contains the unknown of interest and no other unknowns using
    - moment balance about the point of intersection of the lines of action of the two unknown forces that are not of interest, or
    - if the two uninteresting unknown forces are parallel, use force balance in a direction orthogonal to them.

For a given truss and given bar tension of interest there is no guarantee that the recipe applies. You can always find a section cut through the bar of interest, but there may be too-many unknowns in the free-body diagrams of both of the resulting sub-structures.

Because 2D statics of finite objects gives three scalar equations we can generally find all three unknown bar tensions from a section cut that goes through 3 bars.

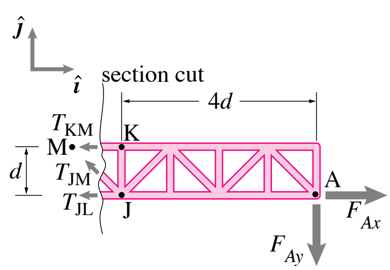


Figure 6.30: Free-body diagram of a 'section' of the structure.

Example: Three bar-forces from one FBD.
Look at the free-body diagram from a section cut in fig. 6.30. Moment balance about point J (about an axis through J in the z direction) gives:

$$\{\textstyle\sum \vec{\boldsymbol{M}}_{\mathrm{J}} = \vec{\boldsymbol{0}}\} \cdot \hat{\boldsymbol{k}} \quad \Rightarrow \quad T_{\mathrm{KM}} = 4F_{Ay}.$$

Using the FBD with this same section cut we can also find:

$$\{\textstyle\sum \vec{\boldsymbol{M}}_{\mathrm{M}} = \vec{\boldsymbol{0}}\} \cdot \hat{\boldsymbol{k}} \quad \Rightarrow \quad T_{\mathrm{JL}} = -5F_{Ay} + F_{Ax}, \text{ and}$$
$$\{\textstyle\sum \vec{\boldsymbol{F}}_{\mathrm{i}} = \vec{\boldsymbol{0}}\} \cdot \hat{\boldsymbol{\jmath}} \quad \Rightarrow \quad T_{\mathrm{JM}} = \sqrt{2}F_{Ay}.$$

Note that in the free-body diagram of fig. 6.30, moment balance about point J eliminates T_{JM} and T_{JL} and gives one equation for T_{KM}. And force balance in the $\hat{\boldsymbol{\jmath}}$ direction eliminates T_{KM} and T_{JL}, giving one equation for T_{JM}.

Using sections to gain insight

In the method of joints, as you worked your way along the structure fig. 6.7 from right to left you would have found the tensions getting bigger and bigger on the top bars and the compressions (negative tensions) getting bigger and bigger on the bottom bars. With the method of sections you can see that this comes from the lever arm of the load F being bigger and bigger for longer and longer sections of truss. The moment caused by the vertical load F_{Ay} is carried by the tension in the top bars and compression in the bottom bars.

A warning

Because of positive experiences with the method of sections for textbook-like problems and very simple structures, many people are left with the impression that the method of sections is more powerful than the method of joints. It isn't. The method of sections is of less general utility than the method of joints. And, unlike for the method of joints, there is no simple systematic way to find all of the bar tensions in all statically-determinate trusses (See fig. 6.31).

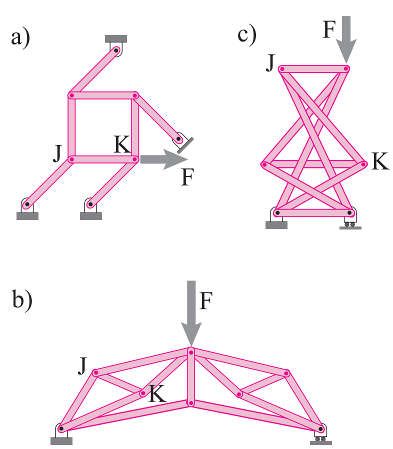


Figure 6.31: The method of sections is great when it works, but it doesn't always work. Like for the trusses here. For each of these trusses, there is no single section cut that leads to a free-body diagram from which you can find T_{JK}. Yet, these are all statically determinate trusses. For all of these trusses, to find T_{JK} one has to write the equilibrium equations for many joints and then solve these equations simultaneously. [In the truss (c), joints are marked by dots, there are no joints where the bars cross. This slightly unusual truss has no closed triangles).]

SAMPLE 6.7 The tower truss shown in the figure is fabricated with 19 rods. All the horizontal and vertical rods are one meter long. Joint J is halfway between joints K and H. The horizontal force applied at joint K is 1 kN. Find the tensions in

1. rod GJ, and
2. rod CE.

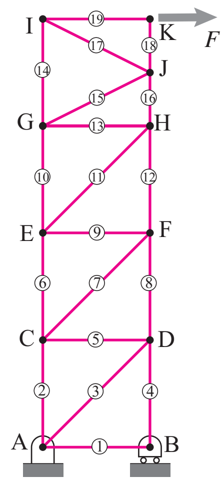


Figure 6.32

Solution To find the tension in rod GJ, numbered 15, let us make a cut through the truss as shown in fig. 6.33. The section taken here cuts rods 14, 15, and 16. The free-body diagram has only three unknown tensions acting on the part of the truss under consideration.

From the force balance in the x direction, we see at once,

$$
\begin{aligned}
F - T_{15}\cos\theta &= 0 \\
\Rightarrow \quad T_{15} &= \frac{F}{\cos\theta} \\
&= \frac{1\,\text{kN}}{\cos 26.56^\circ} \\
&= 1.12\,\text{kN}.
\end{aligned}
$$

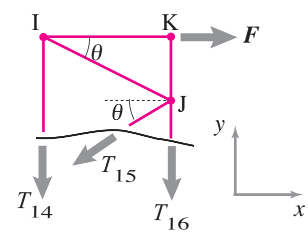


Figure 6.33

$$\boxed{T_{\text{GJ}} \equiv T_{15} = 1.12\,\text{kN}}$$

To determine the tension in rod CE, we consider a section that cuts rods CE, CF, and DF. The free-body diagram of the truss above this section is shown in fig. 6.34. Once again, we have only three unknown forces on the body under consideration (note that we will have six unknown forces that include three support reactions if we considered the lower part of the truss, below the selected section).

To find T_6, we write the scalar moment-balance equation in the z-direction about point F:

$$
\begin{aligned}
aT_6 - 2aF &= 0 \\
\Rightarrow \quad T_6 &= 2F \\
&= 2\,\text{kN}.
\end{aligned}
$$

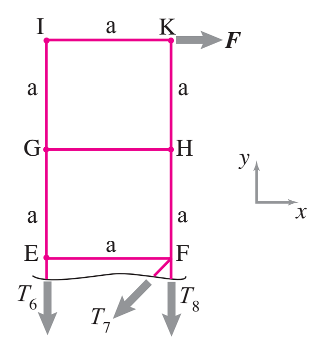


Figure 6.34

$$\boxed{T_{\text{CE}} \equiv T_6 = 2\,\text{kN}}$$

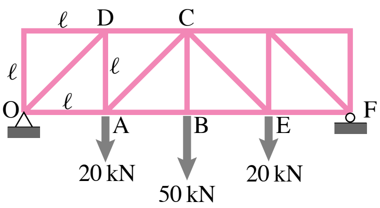


Figure 6.35

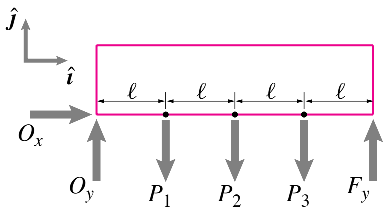


Figure 6.36

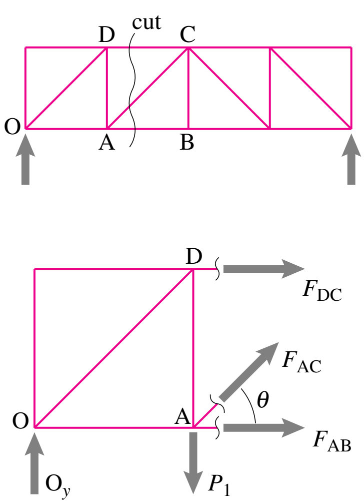


Figure 6.37

SAMPLE 6.8 **A 2-D truss:** The box truss shown in the figure is loaded by three vertical forces acting at joints A, B, and E. All horizontal and vertical bars in the truss are of length 2 m. Find the forces in members AB, AC, and DC.

Solution First, we need to find the support reactions at points O and F. We do this by drawing the free-body diagram of the whole truss and writing the equilibrium equations for it. Referring to Fig. 6.36, the force equilibrium, $\sum \vec{F} = \vec{0}$ implies,

$$O_x \hat{\imath} + (O_y + F_y - P_1 - P_2 - P_3)\hat{\jmath} = \vec{0}. \tag{6.11}$$

Dotting eqn. (6.11) with $\hat{\imath}$ and $\hat{\jmath}$, respectively, we get

$$\begin{aligned} O_x &= 0 \\ O_y + F_y &= P_1 + P_2 + P_3. \end{aligned} \tag{6.12}$$

The moment equilibrium about point O, $\vec{M}_O = \vec{0}$, gives

$$(-P_1\ell - P_2 2\ell - P_3 3\ell + F_y 4\ell)\hat{k} = \vec{0} \tag{6.13}$$

$$\text{or} \qquad F_y = \frac{1}{4}(P_1 + 2P_2 + 3P_3). \tag{6.14}$$

Solving eqns. (6.12) and (6.14), we get

$$F_y = 45\,\text{kN}, \qquad \text{and} \qquad O_y = 45kN.$$

In fact, from the symmetry of the structure and the loads, we could have guessed that the two vertical reactions must be equal, *i.e.*, $O_y = F_y$. Then, from eqn. (6.12) it follows that $O_y = F_y = (P_1 + P_2 + P_3)/2 = 45\,\text{kN}$.

Now, we proceed to find the forces in the members AB, AC, and DC. For this purpose, we make a cut in the truss such that it cuts members AD, AC, and DC, just to the right of joints A and D. Next, we draw the free-body diagram of the left (or right) portion of the truss and use the equilibrium equations to find the required forces. Referring to Fig. 6.37, the force equilibrium requires that

$$(F_{AB} + F_{DC} + F_{AC}\cos\theta)\hat{\imath} + (O_y - P_1 + F_{AC}\sin\theta)\hat{\jmath} = \vec{0}. \tag{6.15}$$

Dotting eqn. (6.15) with $\hat{\imath}$ and $\hat{\jmath}$, respectively, we get

$$F_{AB} + F_{DC} + F_{AC}\cos\theta = 0 \tag{6.16}$$

$$O_y - P_1 + F_{AC}\sin\theta = 0. \tag{6.17}$$

So far, we have two equations in three unknowns (F_{AB}, F_{DC}, F_{AC}). We need one more independent equation to be able to solve for the unknown forces. We now write moment equilibrium equation about point A, *i.e.*, $\sum \vec{M}_A = \vec{0}$,

$$\begin{aligned} (-O_y\ell - F_{DC}\ell)\hat{k} &= \vec{0} \\ \Rightarrow \qquad O_y + F_{DC} &= 0. \end{aligned} \tag{6.18}$$

We can now solve eqns. (6.16–6.18) any way we like, *e.g.*, using elimination or a computer. The solution we get (see next page for details) is:

$$F_{AC} = -25\sqrt{2}\,\text{kN}, \quad F_{DC} = -45\,\text{kN}, \qquad \text{and} \qquad F_{AB} = 70\,\text{kN}.$$

$$F_{AC} = -25\sqrt{2}\,\text{kN}, \quad F_{DC} = -45\,\text{kN}, \quad F_{AB} = 70\,\text{kN}.$$

Comments:

- Note that the values of F_{AC} and F_{DC} are negative which means that bars AC and DC are in compression, not tension, as we initially assumed. Thus the solution takes care of our incorrect assumptions about the directionality of the forces.
- **Short-cuts:** In the solution above, we have not used any tricks or any special points for moment equilibrium. However, with just a little bit of mechanics intuition we can solve for the required forces in five short steps as shown below.
  (i) No external force in $\hat{\imath}$ direction implies $O_x = 0$.
  (ii) Symmetry about the middle point B implies $O_y = F_y$. But,
  $$O_y + F_y = \sum P_i = 90\,\text{kN} \quad \Rightarrow \quad O_y = F_y = 45\,\text{kN}.$$
  (iii) $(\sum \vec{\boldsymbol{M}}_{\text{A}} = \vec{\mathbf{0}}) \cdot \hat{\boldsymbol{k}}$ gives
  $$O_y \ell + F_{DC}\ell = 0 \quad \Rightarrow \quad F_{DC} = -O_y = -45\,\text{kN}.$$
  (iv) $(\sum \vec{\boldsymbol{M}}_{\text{C}} = \vec{\mathbf{0}}) \cdot \hat{\boldsymbol{k}}$ gives
  $$-O_y\, 2\ell + P_1\ell + F_{AB}\ell = 0 \quad \Rightarrow \quad F_{AB} = 2O_y - P_1 = 70\,\text{kN}.$$
  (v) $(\sum \vec{\boldsymbol{F}} = \vec{\mathbf{0}}) \cdot \hat{\boldsymbol{\jmath}}$ gives
  $$O_y - P_1 + F_{AC}\sin\theta = 0 \quad \Rightarrow \quad F_{AC} = (P_1 - O_y)/\sin\theta = -25\sqrt{2}\,\text{kN}.$$
- **Solving equations:** On the previous page, we found F_{AB}, F_{DC}, and F_{AC} by solving eqns. (6.15–6.17) simultaneously. Here, we show you two ways to solve those equations.
  1. *By elimination:* From eqn. (6.17), we have
  $$F_{AC} = \frac{O_y - P_1}{\sin\theta} = \frac{20\,\text{kN} - 45\,\text{kN}}{1/\sqrt{2}} = -25\sqrt{2}\,\text{kN}.$$
  From eqn. (6.18), we get
  $$F_{DC} = -O_y = -45\,\text{kN},$$
  and finally, substituting the values found in eqn. (6.15), we get
  $$F_{AB} = -F_{DC} - F_{AC}\cos\theta = 45\,\text{kN} + 25\sqrt{2}\cdot\frac{1}{\sqrt{2}} = 70\,\text{kN}.$$
  2. *On a computer:* We can write the three equations in the matrix form:
  $$\underbrace{\begin{bmatrix} 1 & 1 & \cos\theta \\ 0 & 0 & \sin\theta \\ 0 & 1 & 0 \end{bmatrix}}_{\texttt{A}} \underbrace{\begin{Bmatrix} F_{AB} \\ F_{DC} \\ F_{AC} \end{Bmatrix}}_{\texttt{x}} = \begin{Bmatrix} 0 \\ P_1 - O_y \\ -O_y \end{Bmatrix} = \underbrace{\begin{Bmatrix} 0 \\ -25 \\ -45 \end{Bmatrix}}_{\texttt{b}} \text{kN}.$$
  We can now solve this matrix equation on a computer by keying in matrix `A` (with θ specified as $\pi/4$) and vector `b` as input and solving for `x`.①

① Pseudocode:
```
A = [1 1 cos(pi/4)
     0 0 sin(pi/4)
     0 1 0]
b = [0 -25 -45]
solve A*x = b for x
```

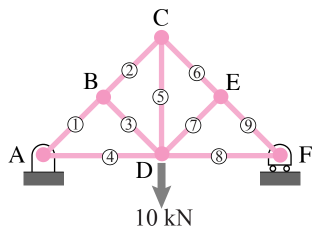


Figure 6.38

SAMPLE 6.9 Consider the truss shown in the figure. Rods AB, BC, EC, EF, BD, and DE are each 2 m long, and $\angle$ ABD $= \angle$DEF $= \pi/2$. Find the tensions in rods DE and CD.

Solution We do not need any analysis to find the tension in rod DE. Since DE is normal to CF, DE has to be a zero-force member for equilibrium of joint E.(2) However, let us find out the same result using the method of sections. Let us take a section just to the left of joint E that cuts through rods CE, DE and DF. The free-body diagram of the truss to the right of the section is shown in fig. 6.39.

The scalar moment-balance equation, $\sum M_z = 0$, about point F gives at once,

$$aT_7 = 0 \quad \Rightarrow \quad T_7 = 0.$$

Thus rod CE is tension free. Now, we make another cut, taking the section shown in fig. 6.40 to determine the tension in rod CD. Since $T_7 = 0$, we can write the scalar moment-balance equation in the z-direction about point A to give

$$\begin{aligned} \ell T_5 - \ell F &= 0 \\ \Rightarrow \quad T_5 &= F = 10\,\text{kN}. \end{aligned}$$

$$\boxed{T_7 = 0, \quad T_5 = 10\,\text{kN}}$$

(2) In fact, both rods BD and DE are zero-force members. Since they carry no tension, rods AD and DF must also be zero-force members for equilibrium of joint D

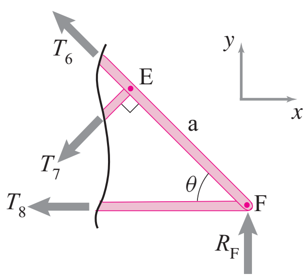


Figure 6.39: Free-body diagram with a sectional cut that cuts through rods CE, DE and DF.

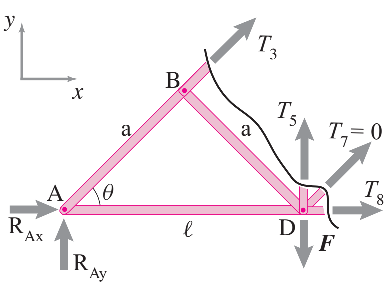


Figure 6.40: Free-body diagram with a sectional cut that cuts through rods BC, CD, DE, and DF.

6.2 Method of sections

Preparatory Problems

6.2.1 What is the method of sections?

6.2.2 When is the method of sections most useful?

6.2.3 With the free-body diagram associated with one section cut how many bar tensions can you hope to find?

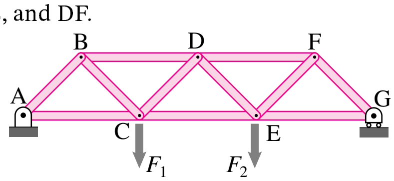
6.2.4 Given a truss and a particular bar in that truss

a) Can you always find one section cut with which you can find the desired bar tension?

b) If so, how do you find that cut? If not, why not?

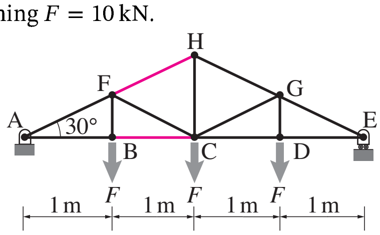
c) Whichever your answer above, give an example of a bar in a truss that illustrates your point.

6.2.5 This problem is exactly the same as Sample 6.3 where it was solved using method of joints. The truss is made up of five horizontal and six inclined rods. All inclined rods are 1 m long and at right angles to each other. The truss carries two vertical loads, $F_1 = 4\,\text{kN}$ and $F_2 = 1\,\text{kN}$ as shown. Find the tensions in rods CE, DE, and DF.



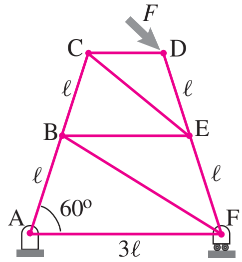
Problem 6.2.5

6.2.6 For the truss shown in the figure, find the tensions in rods BC and FH, assuming $F = 10\,\text{kN}$.



Problem 6.2.6

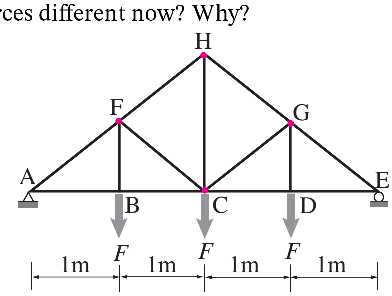
6.2.7 A force $F = 3\,\text{kN}$ acts at 45° with the horizontal at joint D of the truss shown in the figure. Find the tension in rod BE.



Problem 6.2.7

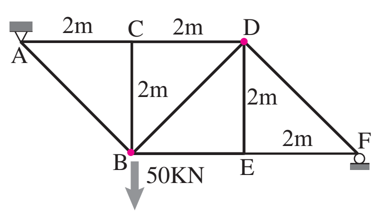
More-Involved Problems

6.2.8 For the 2m high truss shown, find the forces in bars FH, FB, and BC. Use $F = 10\,\text{kN}$. Now pretend that bars FC and CG are removed and two new bars BH and HD are put in. Find the forces in bars FH, FB, and BC again. Are the forces different now? Why?



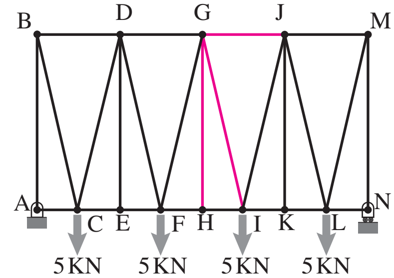
Problem 6.2.8

6.2.9 Find the forces in bars BC and BD in the truss shown in the figure. How does the force change in each of these bars if the load is moved to joint B from joint E?



Problem 6.2.9

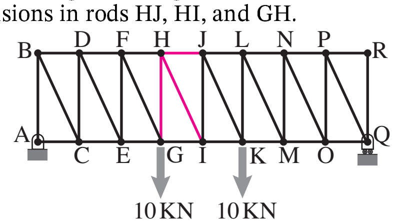
6.2.10 For the truss shown in the figure, assume that AC=CE=1 m, and AB=BD=2 m. The rest of the bays are identical to bay ABDE. For the given loads, find the tensions in rods GH, GI, and GJ. [Hint: you can use information about zero-force members.]



Problem 6.2.10

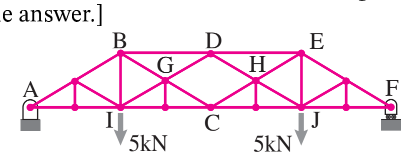
6.2.11 Consider the truss shown in Problem 6.2.6. Find the tension in rod CH. [Hint: you may have to use multiple sections or solve Problem 6.2.6 first.]

6.2.12 The truss shown in the figure consists of 8 'N' bays. In each bay, the vertical rod is 2 m long and the horizontal rod is 1 m long. For the given loads, find the tensions in rods HJ, HI, and GH.



Problem 6.2.12

6.2.13 A complex symmetric truss spanning a length of 16 m is shown in the figure. The outermost inclined rods make an angle of 30° with the horizontal. Find the tension in rod BD. [Note: you may have to use more than one section to get the answer.]



Problem 6.2.13

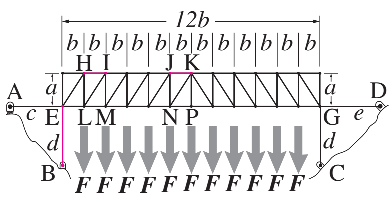
6.2.14 The 2D truss shown consists of 12 diagonally braced rectangles (each a high and b wide). Thus the slope of the diagonal elements is a/b. The whole structure is supported by 4 bars (with lengths c, d, and e as marked). The loading is idealized as 11 identical loads F shown. Give your answers in terms of some or all of a, b, c, d, e and F.

a) On a sketch of the figure below clearly mark all the zero-force members (put a '0' on the middle of each bar that has a 'bar force' of zero).

b) Find the 'bar-force' in bar EB. *

c) Find the 'bar-force' in bar HI. *

d) Find the 'bar-force' in bar JK. [Hint: Use the method of sections and, to reduce calculations, replace a group of the F forces with a single equivalent force.] *



Problem 6.2.14