SAMPLE 6.1 The truss shown in the figure carries a load $F = 10\,\text{kN}$ at joint D. The truss is designed with nine rods, six of which (the inclined ones) have the same length $d = 2\,\text{m}$. Rods BC, EC, DE and BD form a square.

1. Find the support reactions at joints A and F.
2. Find the tensions in rods BD and BC.

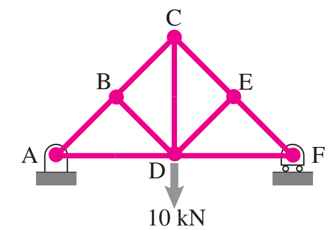

Figure 6.14

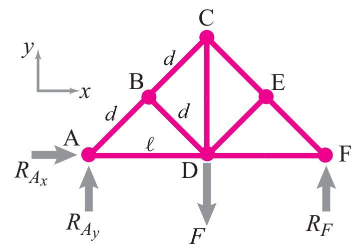

Figure 6.15

Solution

1. **Support reactions:** To find the support reactions at A and F, we draw the free-body diagram of the entire truss (see fig. 6.15). We are given that $d = 2\,\text{m}$ and that $\angle ABD = \angle DEF = \pi/2$. Therefore, $\ell = \sqrt{2}d = 2\sqrt{2}\,\text{m}$.

   The scalar force-balance equation in x-direction readily gives $R_{A_x} = 0$. The scalar moment-balance equation about point A gives

   $$2\ell R_F - \ell F = 0 \quad \Rightarrow \quad R_F = \frac{F}{2} = 5\,\text{kN}.$$

   Now, from the scalar force balance in the y-direction, we have

   $$R_{A_y} + R_F - F = 0 \quad \Rightarrow \quad R_{A_y} = F - R_F = 5\,\text{kN}.$$

   $$\boxed{R_{A_x} = 0, \quad R_{A_y} = 5\,\text{kN}, \quad R_F = 5\,\text{kN}}$$

2. **Tensions in BD and BC:** We can find the tensions in rods BC and BD by analyzing the equilibrium of joint B. As you can see, joint B has three unknown forces acting on it, namely the tensions of rods AB, BC and BD. Since the joint equilibrium equations (only two scalar equations) can only solve for two unknowns, we need to start at joint A, determine T_{AB} first and then move on to joint B.

   The free-body diagrams of the joints A and B are shown in fig. 6.16. Let us first consider the equilibrium of joint A. From the scalar force-balance equations, we have

   $$\begin{aligned}
   \sum F_y = 0 \quad &\Rightarrow \quad R_{A_y} + T_{AB}\sin\theta = 0 \\
   &\Rightarrow \quad T_{AB} = -R_{A_y}/\sin\theta = -5\,\text{kN}/(1/\sqrt{2}) = -7\,\text{kN}. \\
   \sum F_x = 0 \quad &\Rightarrow \quad T_{AB}\cos\theta + T_{AD} = 0 \\
   &\Rightarrow \quad T_{AD} = -T_{AB}\cos\theta = -7\,\text{kN}(1/\sqrt{2}) = 5\,\text{kN}.
   \end{aligned}$$

   Now, we analyze joint B. From the geometry of forces, it is clear that writing scalar force-balance equations in the x' and y' directions will be advantageous. For example, the force balance in the x' direction immediately gives $T_{BD} = 0$. The force balance in the y' direction gives

   $$-T_{AB} + T_{BC} = 0 \quad \Rightarrow \quad T_{BC} = T_{AB} = -7\,\text{kN}.$$

   $$\boxed{T_{BC} = -7\,\text{kN}, \quad T_{BD} = 0}$$

   Note that it is easy to spot bar BD as a zero-force member since it is perpendicular to rods AB and BC.

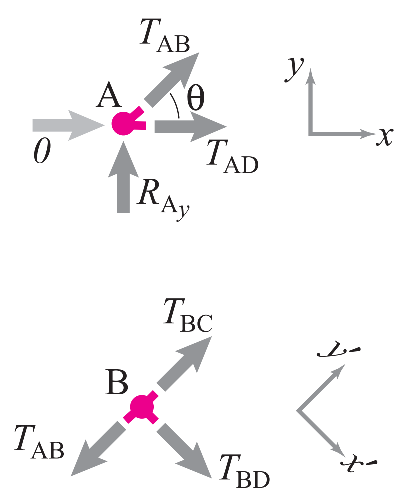

Figure 6.16: Free-body diagrams of joints A and B. From the given geometry, $\theta = pi/4$.

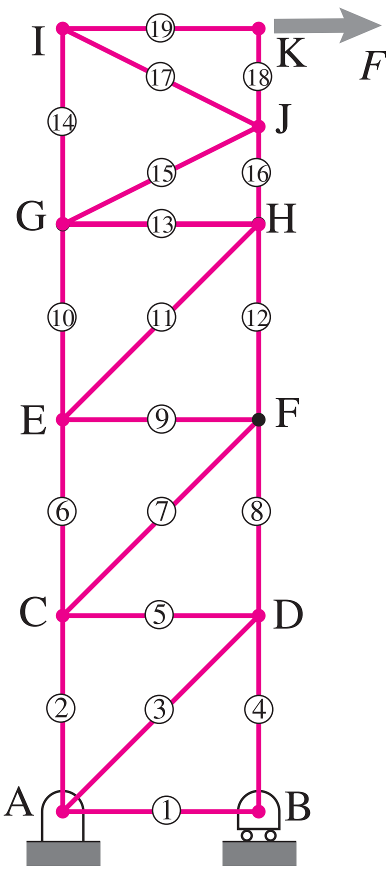

Figure 6.17

SAMPLE 6.2 For the truss tower shown in the figure, assume all horizontal and vertical rods to be 1 mlong and rods numbered 16 and 18 to be 0.5 mlong. Given that the horizontal load on the truss $F = 500\,\text{N}$, find the tension in rod 15.

Solution To find the tension in rod 15, we can use the equilibrium of either joint G or joint K. In either case, the free-body diagram will have four unknown bar tensions (for four bars connected to each of these joints) at the joint. Therefore, we will not be able to solve for them. So, let us start at joint K and work through joint I to joint J. This sequence gets us only two unknown forces at each joint.

The free-body diagrams of the three joints are shown in fig. 6.18. Let us first consider the equilibrium of joint K. A simple inspection (or force balance in the y-direction) shows that bar 18 is a zero-force member. The force balance in the horizontal direction then immediately gives $T_{19} = F = 500\,\text{N}$. Thus,

$$T_{19} = 500\,\text{N} \quad \text{and} \quad T_{18} = 0.$$

Next, we consider the equilibrium of joint I. Since T_{19} is already known, there are only two unknown forces, T_{14} and T_{17} at this joint. The force balance in the horizontal direction gives

$$\begin{aligned} T_{19} + T_{17}\cos\theta &= 0 \\ \Rightarrow \quad T_{17} &= -\frac{T_{19}}{\cos\theta} \\ &= -\frac{500\,\text{N}}{\cos(\tan^{-1}(0.5))} \\ &= -559\,\text{N}. \end{aligned}$$

Now we proceed to joint J. Note that we used only one scalar equation (force balance in the x-direction) at joint I, since we do not need T_{14}. Similarly, to find T_{15}, we only need the force balance in the horizontal direction at joint J:

$$\begin{aligned} -T_{17}\cos\theta - T_{15}\cos\theta &= 0 \\ \Rightarrow \quad T_{15} &= -T_{17} = 559\,\text{N}. \end{aligned}$$

$$\boxed{T_{15} = 559\,\text{N}}$$

Note: We did not have to find support reactions first in order to proceed to other joints as in the previous sample. As long as you can find a sequence of joints with just two unknown forces at each joint, up to the force that you need to determine, you can easily find the force with hand calculations.

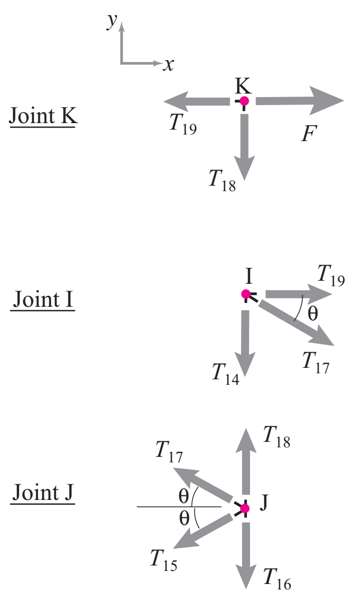

Figure 6.18: Free-body diagrams of joints K, I, and J. From the given geometry, $\theta = \tan^{-1}(\text{KJ}/\text{IK}) = 26.56^\circ$.

SAMPLE 6.3 The truss shown in the figure is made up of five horizontal and six inclined rods. All inclined rods are 1 m long and at right angles to each other. The truss carries two vertical loads, $F_1 = 4\,\text{kN}$ and $F_2 = 1\,\text{kN}$ as shown. Find the tensions in rods CE, DE, and DF.

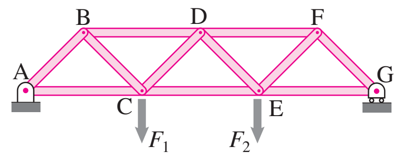

Figure 6.19

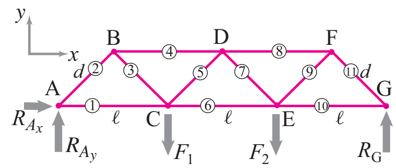

Figure 6.20

Solution To find tensions in rods CE, DE and DF, we can either use joints C and D, or joints E and F. However, for either set we need to start from other joints since there are more than two unknown forces at each joint. Let us start from joint G and work our way through joints F and E. To start at joint G, however, we first need to determine the support reaction G.

The free-body diagram of the entire truss is shown in fig. 6.20 where we have numbered the rods for convenience. The scalar moment-balance equation about point A in the z-direction gives

$$3\ell R_G - \ell F_1 - 2\ell F_2 = 0 \quad \Rightarrow \quad R_G = \frac{F_1 + 2F_2}{3} = 2\,\text{kN}.$$

The force-balance equations give

$$\sum F_x = 0 \quad \Rightarrow \quad R_{A_x} = 0$$
$$\sum F_y = 0 \quad \Rightarrow \quad R_{A_y} = F_1 + F_2 - R_G = 3\,\text{kN}.$$

Now, we are ready to proceed from joint G. The free-body diagrams of joints G, F, and E are shown in fig. 6.21.

At joint G:

$$\sum F_y = 0 \quad \Rightarrow \quad T_{11}\sin\theta + R_G = 0$$
$$\Rightarrow \quad T_{11} = -\frac{R_G}{\sin\theta} = -\sqrt{2}R_G = -2.83\,\text{kN}.$$
$$\sum F_x = 0 \quad \Rightarrow \quad -T_{11}\cos\theta - T_{10} = 0$$
$$\Rightarrow \quad T_{10} = -T_{11}\cos\theta = 2\,\text{kN}.$$

At joint F:

$$\sum F_y = 0 \quad \Rightarrow \quad -T_{11}\sin\theta - T_9\sin\theta = 0$$
$$\Rightarrow \quad T_9 = -T_{11} = 2.83\,\text{kN}$$
$$\sum F_x = 0 \quad \Rightarrow \quad (T_{11} - T_9)\cos\theta - T_8 = 0$$
$$\Rightarrow \quad T_8 = (T_{11} - T_9)\cos\theta = -4\,\text{kN}.$$

At joint E:

$$\sum F_y = 0 \quad \Rightarrow \quad (T_7 + T_9)\sin\theta - F_2 = 0$$
$$\Rightarrow \quad T_7 = \frac{F_2}{\sin\theta} - T_9 = -1.41\,\text{kN}.$$
$$\sum F_x = 0 \quad \Rightarrow \quad (T_9 - T_7)\cos\theta + T_{10} - T_6 = 0$$
$$\Rightarrow \quad T_6 = (T_9 - T_7\cos\theta + T_{10} = 5\,\text{kN}.$$

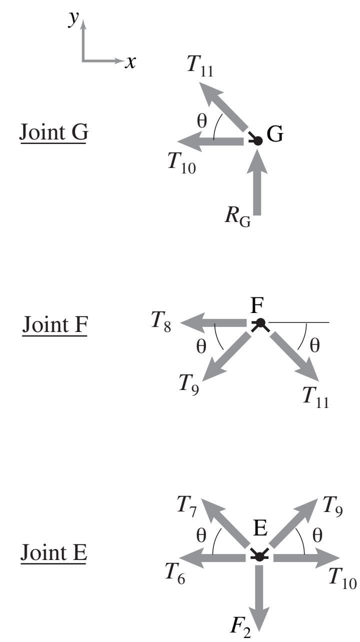

Figure 6.21: Free-body diagrams of joints G, F, and E. From the given geometry, $\theta = 45^\circ$.

$$T_{\text{CE}} = 5\,\text{kN},\ T_{\text{DE}} = -1.41\,\text{kN},\ T_{\text{DF}} = -4\,\text{kN},$$

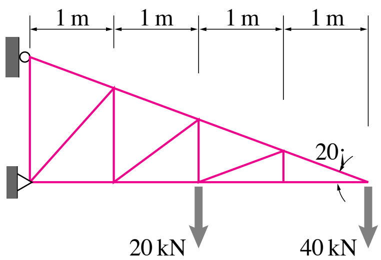

Figure 6.22

SAMPLE 6.4 The truss shown in the figure has four horizontal bays, each of length 1 m. The top bars make 20° angle with the horizontal. The truss carries two loads of 40 kN and 20 kN as shown. Find the forces in each bar. In particular, find the bars that carry the maximum tensile and compressive forces.

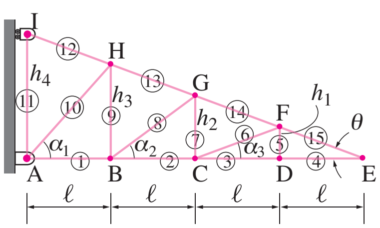

Figure 6.23: To keep track of joints and rods, we first label them. All joints are labeled with letters and rods with numbers.

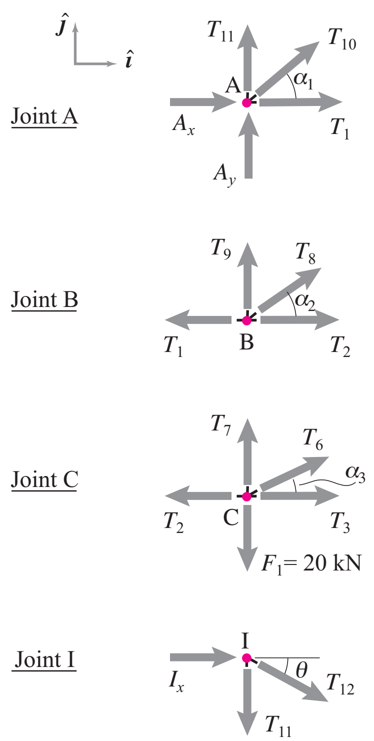

Figure 6.24: Free-body diagrams of some representative joints.

Solution Since we need to find the forces in all the 15 bars, we need to find enough equations to solve for these 15 forces in addition to 3 unknown reactions A_x, A_y, and I_x. Thus we have a total of 18 unknowns. Note that there are 9 joints and therefore, we can generate 18 scalar equations by writing force equilibrium equations (one vector equation per joint) for each joint.

Number of unknowns:	15 bar forces + 3 reactions	= 18
Number of joints:	(A, B, C, ..., and I)	= 9
Number of equations:	9 joint × 2 per joint	= 18.

So, we go joint by joint, draw the free-body diagram of each joint and write the equilibrium equations. After we get all the equations, we can solve them on a computer. All joint equations are just force equilibrium equations, *i.e.*, $\sum \vec{F} = \vec{0}$.

- Joint **A**:
$$(A_x + T_1 + T_{10}\cos\alpha_1)\hat{\imath} + (A_y + T_{11} + T_{10}\sin\alpha_1)\hat{\jmath} = \vec{0}. \tag{6.1}$$
- Joint **B**:
$$(-T_1 + T_2 + T_8\cos\alpha_2)\hat{\imath} + (T_9 + T_8\sin\alpha_2)\hat{\jmath} = \vec{0}. \tag{6.2}$$
- Joint **C**:
$$(-T_2 + T_3 + T_6\cos\alpha_3)\hat{\imath} + (T_7 + T_6\sin\alpha_3)\hat{\jmath} = F_1\hat{\jmath}. \tag{6.3}$$
- Joint **D**:
$$(T_4 - T_3)\hat{\imath} + T_5\hat{\jmath} = \vec{0}. \tag{6.4}$$
- Joint **E**:
$$(-T_4 - T_{15}\cos\theta)\hat{\imath} + T_{15}\sin\theta\,\hat{\jmath} = F_2\hat{\jmath}. \tag{6.5}$$
- Joint **F**:
$$(-T_6\cos\alpha_3 + (T_{15} - T_{14})\cos\theta)\hat{\imath} + (-T_6\sin\alpha_3 + (T_{14} - T_{15})\sin\theta - T_5)\hat{\jmath} = \vec{0}. \tag{6.6}$$
- Joint **G**:
$$(-T_8\cos\alpha_2 + (T_{14} - T_{13})\cos\theta)\hat{\imath} + ((T_{13} - T_{14})\sin\theta - T_8\sin\alpha_2 - T_7)\hat{\jmath} = \vec{0}. \tag{6.7}$$
- Joint **H**:
$$(-T_{10}\cos\alpha_1 + (T_{13} - T_{12})\cos\theta)\hat{\imath} + ((T_{12} - T_{13})\sin\theta - T_{10}\sin\alpha_1 - T_9)\hat{\jmath} = \vec{0}. \tag{6.8}$$
- Joint **I**:
$$(I_x + T_{12}\cos\theta)\hat{\imath} + (-T_{11} - T_{12}\sin\theta)\hat{\jmath} = \vec{0}. \tag{6.9}$$

Dotting each equation from (6.1) to (6.9) with $\hat{\imath}$ and $\hat{\jmath}$, we get the required 18 equations. We need to define all the angles that appear in these equations ($\alpha_1, \alpha_2, \alpha_3$, and θ) before we are ready to solve the equations on a computer.

Let ℓ be the length of each horizontal bar and let DF $= h_1$, CG $= h_2$, and BH $= h_3$. Then, $h_1/\ell = h_2/2\ell = h_3/3\ell = \tan\theta$. Therefore,

$$\begin{aligned}
\tan\alpha_1 = \frac{h_3}{\ell} = \frac{3\ell\tan\theta}{\ell} \qquad &\Rightarrow \qquad \alpha_1 = \tan^{-1}(3\tan\theta)\\
\tan\alpha_2 = \frac{h_2}{\ell} = 2\tan\theta \qquad &\Rightarrow \qquad \alpha_2 = \tan^{-1}(2\tan\theta)\\
\tan\alpha_3 = \frac{h_1}{\ell} = \tan\theta \qquad &\Rightarrow \qquad \alpha_3 = \tan^{-1}(\tan\theta) = \theta.
\end{aligned}$$

Now, we are ready for a computer solution. You can enter the 18 equations in matrix form or as your favorite software package requires and get the solution by solving for the unknowns. Here is a pseudocode to set up and solve the matrix equation. Let us order the unknown forces in the form

$$x = [T_1 \quad T_2 \quad \dots \quad T_{15} \quad A_x \quad A_y \quad I_x]^T$$

so that x_1 to $x_{15} = T_1$ to T_{15}; $x_{16} = A_x$, $x_{17} = A_y$, and $x_{18} = I_x$.

Entering and solving full matrix equation:

```
theta = pi/9                        % specify theta in radians
alpha1 = atan(3*tan(theta)          % calculate alpha1
alpha2 = atan(2*tan(theta))         % calculate alpha2 from arctan
alpha3 = theta                      % calculate alpha3 from arctan
C = cos(theta), S = sin(theta)      % compute all sines and cosines
C1 = cos(alpha1), S1 = sin(alpha1)
C2 = .. ..

F1 = 20;                             % input given external loads
F2 = 40;

A = [1 0 0 0 0 0 0 0 0 C1 0 0 0 0 0 1 0 0     % enter matrix A row-
wise
     0 0 0 0 0 0 0 0 0 S1 1 0 0 0 0 0 1 0
     :
     :
     0 0 0 0 0 0 0 0 0 0 -1 -S 0 0 0 0 0 0]
b = [0 0 0 0 0 F1 0 0 0 F2 0 0 0 0 0 0 0 0]'  % enter column vector b

solve A*x = b for x
```

The solution obtained from the computer is

$$\begin{array}{lcrlcrlcr}
T_1 &=& -128.22\,\text{kN}, & T_2 &=& -109.9\,\text{kN}, & T_3 &=& -109.9\,\text{kN},\\
T_4 &=& -109.9\,\text{kN}, & T_5 &=& 0, & T_6 &=& 0,\\
T_7 &=& 20\,\text{kN}, & T_8 &=& -22.66\,\text{kN}, & T_9 &=& 13.33\,\text{kN},\\
T_{10} &=& -13.56\,\text{kN}, & T_{11} &=& -50\,\text{kN}, & T_{12} &=& 146.19\,\text{kN},\\
T_{13} &=& 136.44\,\text{kN}, & T_{14} &=& 116.95\,\text{kN}, & T_{15} &=& 116.95\,\text{kN},\\
A_x &=& 137.37\,\text{kN}, & A_y &=& 60\,\text{kN}, & I_x &=& -137.37\,\text{kN}.
\end{array}$$

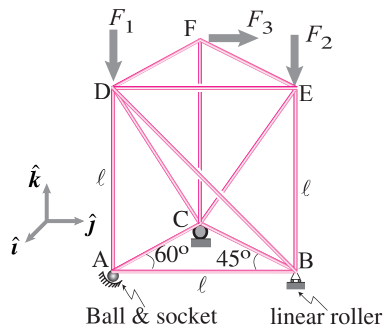

Figure 6.25: This 3-D truss has a triangular base ABC with support points A, B, and C on the ground, and a triangular top DEF. The structure is thus like a prism.

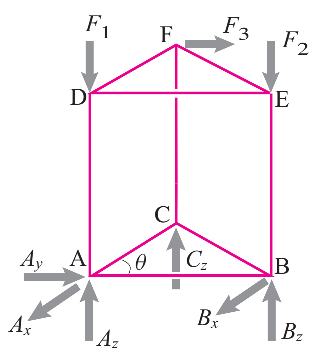

Figure 6.26: The free-body diagram of the structure as a whole. Note that we do not need to show the internal braces DB, DC, or EC here.

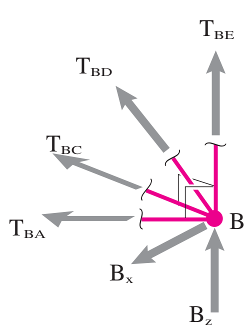

Figure 6.27: The free-body diagram of joint B.

SAMPLE 6.5 A simple 3-D truss: The 3-D truss shown in the figure has 12 bars and 6 joints. Nine of the 12 bars that are either horizontal or vertical have length $\ell = 1\,\text{m}$. The truss is supported at A on a ball and socket joint, at B on a linear roller, and at C on a planar roller (all three supports are on the ground). The loads on the truss are $\vec{F}_1 = -50\,\text{N}\hat{\boldsymbol{k}}, \vec{F}_2 = -60\,\text{N}\hat{\boldsymbol{k}}$, and $\vec{F}_3 = 30\,\text{N}\hat{\boldsymbol{j}}$. Find all support reactions and the tension in bar BC.

Solution The free-body diagram of the entire structure is shown in fig. 6.26. Let the support reactions at A, B, and C be $\vec{\boldsymbol{R}}_A = R_{A_x}\hat{\boldsymbol{\imath}} + R_{A_y}\hat{\boldsymbol{\jmath}} + R_{A_z}\hat{\boldsymbol{k}}$, $\vec{\boldsymbol{R}}_B = R_{B_x}\hat{\boldsymbol{\imath}} + R_{B_z}\hat{\boldsymbol{k}}$, and $\vec{\boldsymbol{R}}_C = R_C\hat{\boldsymbol{k}}$. Then the moment balance about point A, $\sum \vec{\boldsymbol{M}}_{\text{A}} = \vec{\boldsymbol{0}}$, gives

$$\vec{\boldsymbol{r}}_{\text{B/A}} \times \vec{\boldsymbol{R}}_B + \vec{\boldsymbol{r}}_{\text{C/A}} \times \vec{\boldsymbol{R}}_C + \vec{\boldsymbol{r}}_{\text{E/A}} \times \vec{\boldsymbol{F}}_2 + \vec{\boldsymbol{r}}_{\text{F/A}} \times \vec{\boldsymbol{F}}_3 = \vec{\boldsymbol{0}}. \tag{6.10}$$

Note that $\vec{\boldsymbol{F}}_1$ passes through A and, therefore, produces no moment about A. Now we compute each term in the equation above.

$$\begin{aligned}
\vec{\boldsymbol{r}}_{\text{B/A}} \times \vec{\boldsymbol{R}}_B &= \ell\hat{\boldsymbol{\jmath}} \times (R_{B_x}\hat{\boldsymbol{\imath}} + R_{B_z}\hat{\boldsymbol{k}}) &&= -R_{B_x}\ell\hat{\boldsymbol{k}} + R_{B_z}\ell\hat{\boldsymbol{\imath}}, \\
\vec{\boldsymbol{r}}_{\text{C/A}} \times \vec{\boldsymbol{R}}_C &= \ell(\cos\theta\,\hat{\boldsymbol{\jmath}} - \sin\theta\,\hat{\boldsymbol{\imath}}) \times R_C\hat{\boldsymbol{k}} &&= R_C\frac{\ell}{2}\hat{\boldsymbol{\imath}} + R_C\frac{\sqrt{3}\ell}{2}\hat{\boldsymbol{\jmath}} \\
\vec{\boldsymbol{r}}_{\text{E/A}} \times \vec{\boldsymbol{F}}_2 &= (\ell\hat{\boldsymbol{\jmath}} + \ell\hat{\boldsymbol{k}}) \times (-F_2\hat{\boldsymbol{k}}) &&= -F_2\ell\hat{\boldsymbol{\imath}}, \\
\vec{\boldsymbol{r}}_{\text{F/A}} \times \vec{\boldsymbol{F}}_3 &= [\ell(\cos\theta\,\hat{\boldsymbol{\jmath}} - \sin\theta\,\hat{\boldsymbol{\imath}}) + \ell\hat{\boldsymbol{k}}] \times F_3\hat{\boldsymbol{\jmath}} &&= -F_3\ell\hat{\boldsymbol{\imath}} - F_3\frac{\sqrt{3}\ell}{2}\hat{\boldsymbol{k}}.
\end{aligned}$$

Substituting these products in eqn. (6.10), and dotting the resulting equation with $\hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}}$, and $\hat{\boldsymbol{\imath}}$, respectively, we get

$$\begin{aligned}
R_C &= 0 \\
R_{B_x} &= -\frac{\sqrt{3}}{2}F_3 = -15\sqrt{3}\,\text{N} \\
R_{B_z} &= -\frac{1}{2}R_C + F_2 + F_3 = 90\,\text{N}.
\end{aligned}$$

Thus, $\vec{\boldsymbol{B}} = R_{B_x}\hat{\boldsymbol{\imath}} + R_{B_z}\hat{\boldsymbol{k}} = -15\sqrt{3}\,\text{N}\hat{\boldsymbol{\imath}} + 30\,\text{N}\hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{R}}_C = \vec{\boldsymbol{0}}$. Now from the force balance, $\sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}}$, we find $\vec{\boldsymbol{R}}_A$ as

$$\begin{aligned}
\vec{\boldsymbol{R}}_A &= -\vec{\boldsymbol{R}}_B - \vec{\boldsymbol{R}}_C - \vec{\boldsymbol{F}}_1 - \vec{\boldsymbol{F}}_2 - \vec{\boldsymbol{F}}_3 \\
&= -(-15\sqrt{3}\,\text{N}\hat{\boldsymbol{\imath}} + 90\,\text{N}\hat{\boldsymbol{k}}) - (-50\,\text{N}\hat{\boldsymbol{k}}) - (-60\,\text{N}\hat{\boldsymbol{k}}) - (30\,\text{N}\hat{\boldsymbol{\jmath}}) \\
&= 15\sqrt{3}\,\text{N}\hat{\boldsymbol{\imath}} - 30\,\text{N}\hat{\boldsymbol{\jmath}} + 20\,\text{N}\hat{\boldsymbol{k}}.
\end{aligned}$$

To find the force in bar BC, we draw a free-body diagram of joint B (which connects BC) as shown in fig. 6.27. Now, writing the force balance for the joint in the x-direction, *i.e.*, $[\sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{0}}] \cdot \hat{\boldsymbol{\imath}}$, gives

$$\begin{aligned}
R_{B_x} + \vec{\boldsymbol{T}}_{\text{BC}} \cdot \hat{\boldsymbol{\imath}} &= 0 \\
\text{or} \qquad R_{B_x} + T_{BC}\sin\theta &= 0 \\
\Rightarrow \qquad T_{BC} &= -\frac{R_{B_x}}{\sin\theta} \\
&= -\frac{-15\sqrt{3}\,\text{N}}{\sqrt{3}/2} = 30\,\text{N}.
\end{aligned}$$

Thus, the force in bar BC is $T_{BC} = 30\,\text{N}$ (tensile force).

$$\vec{\boldsymbol{R}}_A = 15\sqrt{3}\,\text{N}\hat{\boldsymbol{\imath}} - 30\,\text{N}\hat{\boldsymbol{\jmath}} + 20\,\text{N}\hat{\boldsymbol{k}},\ \vec{\boldsymbol{R}}_B = -15\sqrt{3}\,\text{N}\hat{\boldsymbol{\imath}} + 90\,\text{N}\hat{\boldsymbol{k}},\ \vec{\boldsymbol{R}}_C = \vec{\boldsymbol{0}},\ T_{BC} = 30\,\text{N}$$

SAMPLE 6.6 A 3-D truss solved on the computer: The 3-D truss shown in the figure is fabricated with 12 bars. Bars 1–5 are of length $\ell = 1\,\mathrm{m}$, bars 6–9 have length $\ell/\sqrt{2}(\approx 0.71\,\mathrm{m})$, and bars 10–12 are cut to size to fit between the joints they connect. The truss is supported at A on a ball and socket, at B on a linear roller, and at C on a planar roller. A load $F = 2\,\mathrm{kN}$ is applied at D as shown. Write all equations required to solve for all bar forces and support reactions and solve the equations using a computer.

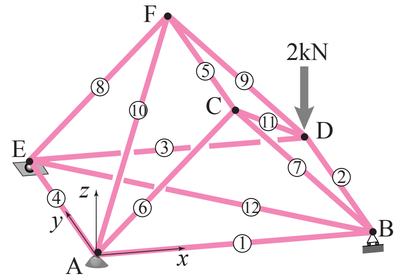

Figure 6.28

Solution There are 12 bars and 6 joints in the given truss. The unknowns are 12 bar forces and six support reactions (3 at A ($R_{A_x}, R_{A_y}, R_{A_z}$), 2 at B ($R_{B_y}, R_{B_z}$), and 1 at E ($R_{E_z}$)). Therefore, we need 18 independent equations to solve for all the unknowns. Since the force equilibrium at each joint gives one vector equation in 3-D, *i.e.,* three scalar equations, the 6 joints in the truss can generate the required number ($6 \times 3 = 18$) of equations. Therefore, we go joint by joint, draw the free-body diagram of the joint, write the force equilibrium equation, and extract the 3 scalar equations from each vector equation. We switch from the letters to denote the bars in the force vectors to numbers in its scalar representation (T_1, T_2, *etc.*) to facilitate computer solution.

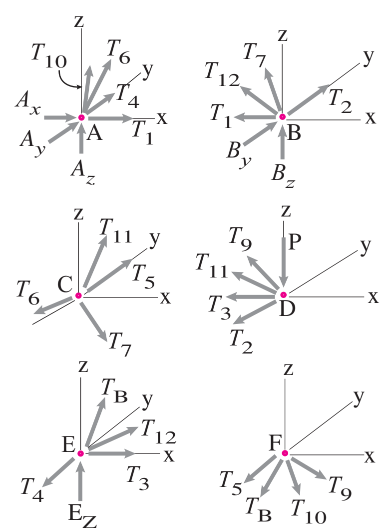

Figure 6.29

- **Joint A:**

$$T_1\hat{\imath} + \frac{T_6}{\sqrt{2}}(\hat{\imath} + \hat{k}) + \frac{T_{10}}{\sqrt{6}}(\hat{\imath} + 2\hat{\jmath} + \hat{k}) + T_4\hat{\jmath} + R_{A_x}\hat{\imath} + R_{A_y}\hat{\jmath} + R_{A_z}\hat{k} = \vec{0}.$$

- **Joint B:**

$$-T_1\hat{\imath} + \frac{T_7}{\sqrt{2}}(-\hat{\imath} + \hat{k}) + T_2\hat{\jmath} + \frac{T_{12}}{\sqrt{2}}(-\hat{\imath} + \hat{\jmath}) + R_{B_y}\hat{\jmath} + R_{B_z}\hat{k} = \vec{0}.$$

- **Joint C:**

$$-\frac{T_6}{\sqrt{2}}(\hat{\imath} + \hat{k}) - \frac{T_7}{\sqrt{2}}(-\hat{\imath} + \hat{k}) + T_5\hat{\jmath} + \frac{T_{11}}{\sqrt{6}}(\hat{\imath} + 2\hat{\jmath} - \hat{k}) = \vec{0}.$$

- **Joint D:**

$$-T_2\hat{\jmath} - \frac{T_{11}}{\sqrt{6}}(\hat{\imath} + 2\hat{\jmath} - \hat{k}) - T_3\hat{\imath} + \frac{T_9}{\sqrt{2}}(-\hat{\imath} + \hat{k}) - F\hat{k} = \vec{0}.$$

- **Joint E:**

$$-T_4\hat{\jmath} + \frac{T_{12}}{\sqrt{2}}(\hat{\imath} - \hat{\jmath}) + T_3\hat{\imath} + \frac{T_8}{\sqrt{2}}(\hat{\imath} + \hat{k}) + R_{E_z}\hat{k} = \vec{0}.$$

- **Joint F:**

$$-T_5\hat{\jmath} - \frac{T_8}{\sqrt{2}}(\hat{\imath} + \hat{k}) - \frac{T_{10}}{\sqrt{6}}(\hat{\imath} + 2\hat{\jmath} + \hat{k}) - \frac{T_9}{\sqrt{2}}(-\hat{\imath} + \hat{k}) = \vec{0}.$$

Now we can separate out 3 scalar equations from each of the joint vector equations by dotting them with $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

Joint	[Eqn.] $\cdot\,\hat{\imath}$	[Eqn.] $\cdot\,\hat{\jmath}$	[Eqn.] $\cdot\,\hat{k}$
A	$T_1 + \frac{1}{\sqrt{2}}T_6 + \frac{1}{\sqrt{6}}T_{10} + R_{A_x} = 0,$	$\frac{2}{\sqrt{6}}T_{10} + T_4 + R_{A_y} = 0,$	$\frac{1}{\sqrt{2}}T_6 + \frac{1}{\sqrt{6}}T_{10} + R_{A_z} = 0$
B	$-T_1 - \frac{1}{\sqrt{2}}T_7 - \frac{1}{\sqrt{2}}T_{12} = 0,$	$T_2 + \frac{1}{\sqrt{2}}T_{12} + R_{B_y} = 0,$	$\frac{1}{\sqrt{2}}T_7 + R_{B_z} = 0$
C	$-\frac{1}{\sqrt{2}}T_6 + \frac{1}{\sqrt{2}}T_7 + \frac{1}{\sqrt{6}}T_{11} = 0,$	$T_5 + \frac{2}{\sqrt{6}}T_{11} = 0,$	$\frac{1}{\sqrt{2}}T_6 + \frac{1}{\sqrt{2}}T_7 + \frac{1}{\sqrt{6}}T_{11} = 0$
D	$-\frac{1}{\sqrt{6}}T_{11} - T_3 - \frac{1}{\sqrt{2}}T_9 = 0,$	$-T_2 - \frac{2}{\sqrt{6}}T_{11} = 0,$	$\frac{1}{\sqrt{6}}T_{11} + \frac{1}{\sqrt{2}}T_9 = F$
E	$\frac{1}{\sqrt{2}}T_{12} + T_3 + \frac{1}{\sqrt{2}}T_8 = 0,$	$-T_4 - \frac{1}{\sqrt{2}}T_{12} = 0,$	$\frac{1}{\sqrt{2}}T_8 + R_{E_z} = 0$
F	$-\frac{1}{\sqrt{2}}T_8 - \frac{1}{\sqrt{6}}T_{10} + \frac{1}{\sqrt{2}}T_9 = 0,$	$-T_5 - \frac{2}{\sqrt{6}}T_{10} = 0,$	$\frac{1}{\sqrt{2}}T_8 + \frac{1}{\sqrt{6}}T_{10} + \frac{1}{\sqrt{2}}T_9 = 0.$

Thus, we have 18 required equations for the 18 unknowns. Before we go to the computer, we need to do just one more little thing. We need to order the unknowns in some way in a one-dimensional array. So, let

$$x = [R_{A_x} \quad R_{A_y} \quad R_{A_z} \quad R_{B_x} \quad R_{B_y} \quad R_{E_z} \quad T_1 \dots T_{12}].$$

Thus $x_1 = R_{A_x},\ x_2 = R_{A_y},\ \dots, x_7 = T_1,\ x_8 = T_2,\ \dots, x_{18} = T_{12}$. Now we are ready to go to the computer, feed these equations, and get the solution. We enter each equation as part of a matrix [A] and a vector {b} such that `[A]{x} = {b}`. Here is the pseudocode:

```
sq2i = 1/sqrt(2)            % define a constant
sq6i = 1/sqrt(6)            % define another constant
F = 2                       % specify given load
A(1,[1 7 12 16]) = [1 1 sq2i sq6i]
A(2,[2 10 16]) = [1 1 2*sq6i]
.
.
A(18,[14 15 16]) = [sq2i sq2i sq6i]
b(12,1) = F
form A and b setting all other entries to zero
solve A*x = b for x
```

The solution obtained from the computer is the one-dimensional array `x` which after decoding according to our numbering scheme gives the following answer.

$$R_{A_x} = R_{A_y} = 0,\ \ R_{A_z} = -2\,\mathrm{kN},\ \ R_{B_y} = 0,\ \ R_{B_z} = 2\,\mathrm{kN},\ \ R_{E_z} = 2\,\mathrm{kN},$$
$$T_1 = T_3 = -2\,\mathrm{kN},\ \ T_2 = T_4 = T_5 = -4\,\mathrm{kN},\ \ T_6 = 0,$$
$$T_7 = T_8 = -2.83\,\mathrm{kN},\ \ T_9 = 0,\ \ T_{10} = T_{11} = 4.9\,\mathrm{kN},\ \ T_{12} = 5.66\,\mathrm{kN},$$