5.3 Friction and equilibrium

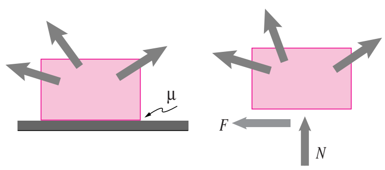
Preparatory Problems

5.3.1 For the block shown in the figure, what do you know about F if

a) the block is sliding to the right

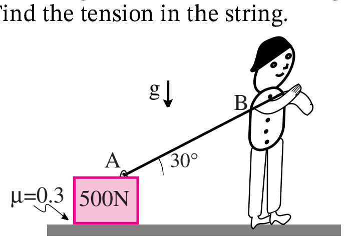
b) the block is sliding to the left

c) the block is not sliding.



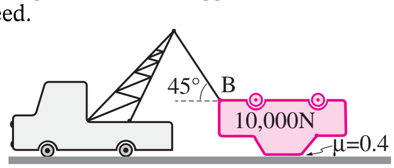
Problem 5.3.1

5.3.2 A block weighing 500 N is dragged slowly on the ground as shown in the figure. Find the tension in the string.



Problem 5.3.2

5.3.3 Find the tension in the cable assuming the car is dragged at constant speed.



Problem 5.3.3

5.3.4 Consider the tow truck dragging the car in Problem 5.3.3 again. In order to ensure safety, you would like to minimize the tension in the rope attached to the car. Assume that the angle shown at point B is θ.

a) What value of θ minimizes the tension in the rope?

b) What is the corresponding value of T?

c) What is the force of the ground on the car?

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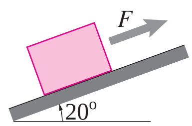
More-Involved Problems

5.3.5 A 30,000 N stone cube one meter on a side was dragged up a 20° ramp by 100 of a Pharaoh's slaves by a rope parallel to the slope. The coefficient of friction was $\mu = 0.2$. Assume all the ground contact is at the front and back edges of the cube.

a) Find the dragging force.

b) Find the force on the front and back edges of the cube.

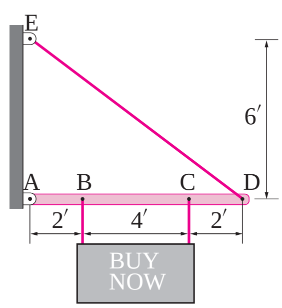


Problem 5.3.5

5.3.6 The 20 lbf uniform rectangular sign is suspended from the strut ABCD by two wires. The strut is supported by cable DE and a pin at A.

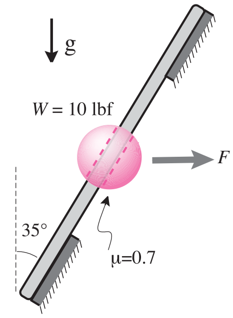
a) Find tension DE.

b) Suppose the workers who hung the sign forgot to pin the strut to the wall at point A. What is the least value of μ between the strut and wall for the system to maintain equilibrium.



Problem 5.3.6

5.3.7 A horizontal force F is applied to slide the bead on the rod shown in the figure. Find the value of F that is required to initiate sliding up the rod. Why is F so big or small?

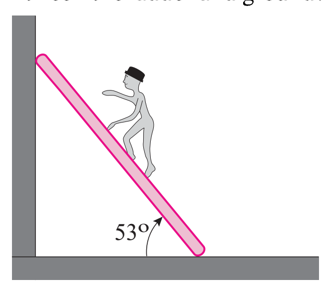


Problem 5.3.7

5.3.8 A 130 pound person climbs a 120 pound ladder that is 30 ft long. The ladder leans against a frictionless wall and makes an angle of 53° with the ground.

a) Find the force of the ground on the ladder when the person is one third of the way up the ladder.

b) When the person gets two thirds of the way up, the bottom of the ladder starts to slip. What is μ between the ladder and ground?



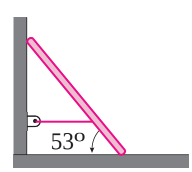
Problem 5.3.8

5.3.9 A uniform 200 N, 10 m ladder leans between a frictionless ground and a wall. It is kept from sliding away from the wall by a horizontal cable 2 m above the ground. Find

a) The tension in the cable.

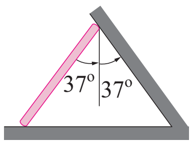
b) The force of the ground on the ladder.

c) The force of the wall on the ladder.



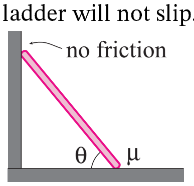
Problem 5.3.9

5.3.10 A uniform ladder of length ℓ and weight W rests against a frictionless slanted wall. What is the minimum μ between ladder and ground that is needed to hold the ladder in position?



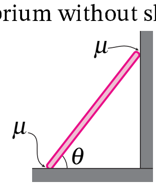
Problem 5.3.10

5.3.11 A uniform ladder with weight W and length ℓ leans against a frictionless vertical wall and makes an angle θ with the ground. In terms of the given quantities, find the values of μ at the ground for which the ladder will not slip.



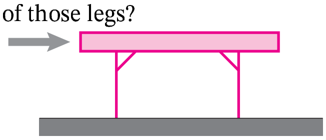
Problem 5.3.11

5.3.12 A uniform ladder with weight W and length ℓ leans against a frictional vertical wall and is supported by the frictional ground. The same coefficient of friction μ applies to the wall and to the ground. In terms of the given quantities, find the values of θ between the ladder and ground for which the ladder can be in equilibrium without slipping. *



Problem 5.3.12

5.3.13 A 2 m square 500 N 4-leg table is pushed across a floor by a horizontal force at its top surface and normal to one edge. Assume the table is 0.8 m high, that its center of mass is 0.6 m high and that all four legs slide on the floor with friction coefficient $\mu = 0.3$. Which legs carry the most load and what is the magnitude of the force from the ground on one of those legs?

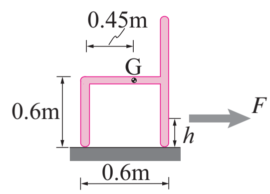


Problem 5.3.13

5.3.14 An 80 N chair is pulled steadily to the right by a rope. The coefficient of friction between the ground and floor is $\mu = 0.25$.

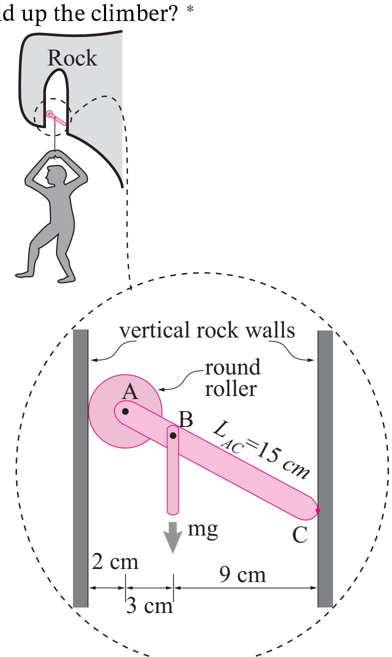
a) What is the force needed to pull the chair?

b) What is the highest point on the chair that the rope can be tied without the chair tipping over?



Problem 5.3.14

5.3.15 A candidate rock-climbing device consists of a roller (radius 2 cm) frictionlessly pinned at A to diagonal-member AC. The length of AC from point A to the wall-contact point at C is $L_{AC} = 15cm$. The climber ($m = 60$ kg) hangs from a rope connected to AC by a pin at B. B is on the line AC and located as shown in the figure. If needed, assume $g = 10$N/kg. What is the minimum coefficient of friction μ at C that is needed to hold up the climber? *



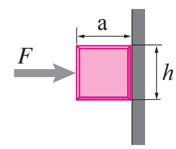
Problem 5.3.15

5.3.16 A uniform $W = 50$ N block with width a and height h is held against a wall with a horizontal force of F acting on the left side half way up the block. The block is prevented from sliding down the wall by friction. There is no glue (no tension between wall and block).

a) Assuming friction is high enough to prevent slip, what is the minimum F to keep the block from tipping away from the wall?

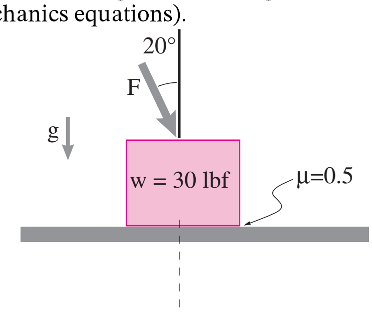
b) For twice that F what is the minimum friction to keep the block from sliding down the wall?

c) For $a = h$ and $F = 3W$ the resultant of all the wall normal and contact forces is a single force that acts on the right side of the block at what position y above the bottom of the block?



Problem 5.3.16

5.3.17 In the figure shown, what is force F required to push the block along the floor? This problem has no solution. Explain why (using free-body diagrams and mechanics equations).



Problem 5.3.17

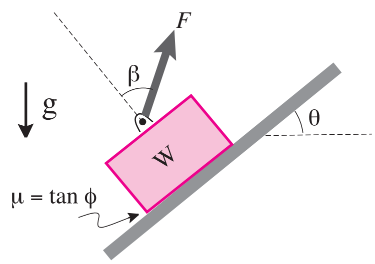
5.3.18 Consider the situation shown in the figure. Give your answers to the following questions in terms of some or all of W, θ, β, g, and ϕ or μ. Assume all values of β and $0 \leq \theta \leq \pi/2$, $0 \leq \mu$, $g > 0$, $W > 0$.

a) Assume the block slides steadily uphill. Find F. For what values of θ, β, and μ does no such F exist (allow $F < 0$)?

b) Assume the block slides downhill. What is F? For what values of θ, β, and μ does no such solution exist?

c) assume the block is not sliding. What are the possible values of F? For what values of θ, β, and μ does such a solution exist?

d) For what values of θ, β, and μ can you have the block slide up, slide down, or lock (that is, no incipient slip) depending on the value of F?

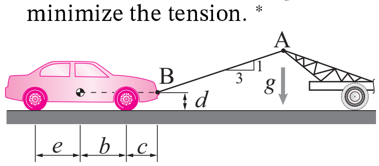


Problem 5.3.18

5.3.19 A car is being towed. Unfortunately all the wheels are locked and skidding with friction coefficient μ. The tow cable AB has a slope of 1/3.

a) In terms of some or all of e, b, c, d, m, g& μ, find the tension in the tow cable AB. *

b) Instead of an angle with slope 1/3, what should the cable angle be to minimize the tension. *



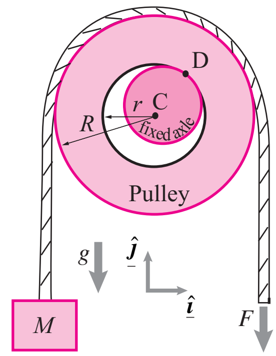
Problem 5.3.19

5.3.20 A weight M is steadily raised by pulling with a force F on a rope going over a negligible-mass pulley on an unlubricated journal bearing (no ball bearings). For an ideal frictionless pulley $F = Mg$. Here, however, we have a friction coefficient between the bearing and its axle which is $\mu = \tan\phi$. [Hint: Finding the location of the contact point D is part of the problem.]

a) Find F in terms of M, g, R, r and μ (or ϕ or $\sin\phi$ or $\cos\phi$ — whichever is most convenient. For example $\cos(\tan^{-1}(\mu))$ is more simply expressed as $\cos\phi$), *and*

b) Evaluate F in the special case that $M = 100\,\mathrm{kg}, g = 10\,\mathrm{m/s^2}, r = 1\,\mathrm{cm}, R = 2\,\mathrm{cm}$, and $\mu = \sqrt{3}/3$ (so $\phi = \pi/6, \sin\phi = 1/2, \cos\phi = \sqrt{3}/2$).

c) Referring back to the general case, for fixed r, R, M, and g what happens to F as $\mu \to \infty$ (does it go to ∞)?



Problem 5.3.20

5.3.21 A reel of mass M and outer radius R is connected by a horizontal string from point P across a pulley to a hanging object of mass m. The inner cylinder of the reel has radius $r = \frac{1}{2}R$. The slope has angle θ. There is no slip between the reel and the slope. There is gravity.

a) Find the ratio of the masses so that the system is at rest. *

b) Find the corresponding tension in the string, in terms of M, g, R, and θ. *

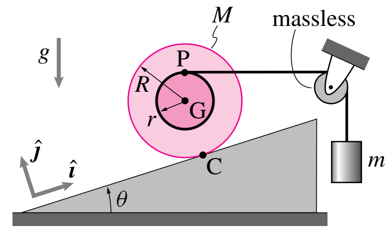
c) Find the corresponding force on the reel at its point of contact with the slope, point C, in terms of M, g, R, and θ. *

Harder Draw a careful sketch and find a point where the lines of action of the gravity force and string tension intersect. For the reel to be in static equilibrium, the line of action of the reaction force at C must pass through this point. Using this information, what must the tangent of the angle ϕ of the reaction force at C be, measured with respect to the normal to the slope? Does this answer agree with that you would obtain from your answer in part(c)? *

d) What is the relationship between the angle ϕ of the reaction at C, measured with respect to the normal to the ground, and the mass ratio required for static equilibrium of the reel? *

e) What is the minimum coefficient of friction μ at C needed to prevent slip.

Check that for $\theta = 0$, your solution gives $\frac{m}{M} = 0$ and $\vec{\boldsymbol{F}}_C = Mg\hat{\boldsymbol{\jmath}}$ and for $\theta = \frac{\pi}{2}$, it gives $\frac{m}{M} = 2$ and $\vec{\boldsymbol{F}}_C = Mg(\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}})$.



Problem 5.3.21

5.3.22 This problem is similar to problem 5.3.21. A reel of mass M and outer radius R is connected by an inextensible string from point P across a pulley to a hanging object of mass m. The inner cylinder of the reel has radius $r = \frac{1}{2}R$. The slope has angle θ. There is no slip between the reel and the slope. There is gravity. In terms of M, g, R, and θ, find:

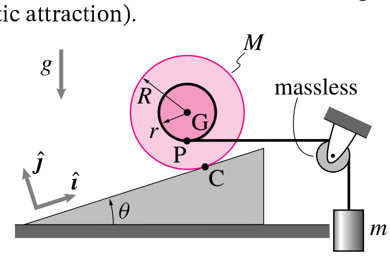
a) the ratio of the masses so that the system is at rest, *

b) the corresponding tension in the string, and *

c) the corresponding force on the reel at its point of contact with the slope, point C. *

d) What is the minimum coefficient of friction μ at C needed to prevent slip.

Check that for $\theta = 0$, your solution gives $\frac{m}{M} = 0$ and $\vec{\boldsymbol{F}}_C = Mg\hat{\boldsymbol{\jmath}}$ and for $\theta = \frac{\pi}{2}$, it gives $\frac{m}{M} = -2$ and $\vec{\boldsymbol{F}}_C = Mg(\hat{\boldsymbol{\imath}} - 2\hat{\boldsymbol{\jmath}})$.The negative mass ratio is impossible since mass cannot be negative and the negative normal force is impossible unless the wall or the reel or both can 'suck' or they can 'stick' to each other (that is, provide some sort of suction, adhesion, or magnetic attraction).

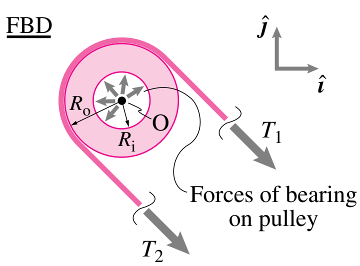


Problem 5.3.22

5.3.23 Assume a massless pulley is round and has outer radius R_2. It slides on a shaft that has radius R_i. Assume there is friction between the shaft and the pulley with coefficient of friction μ, and friction angle ϕ defined by $\mu = \tan(\phi)$. Assume the two ends of the line that are wrapped around the pulley are parallel.

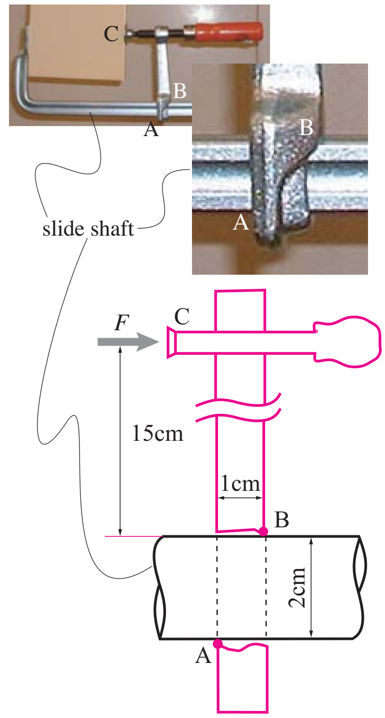
a) What is the relation between the two tensions when the pulley is turning? You may assume that the bearing shaft touches the hole in the pulley at only one point. *.

b) Plug in some reasonable numbers for R_i, R_o and μ (or ϕ) to see one reason why wheels (say pulleys) are such a good idea even when the bearings are not all that well lubricated. *



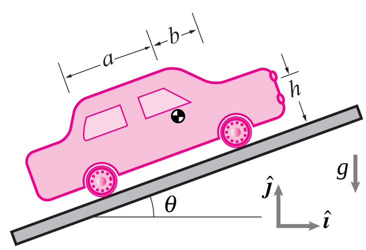
Problem 5.3.23

5.3.24 The so-called *pipe-clamp* has a bracket ABC which loosely fits around the slide-shaft (the‘pipe’). When not clamped there is no big force at C and the bracket freely slides on the shaft. However the bracket frictionally locks once the load F at C gets large. Neglecting gravity, find the minimum coefficient of friction μ at A and B for which this clamp holds well (which it does).



Problem 5.3.24

5.3.25 Find the minimum coefficient of friction μ needed for a front wheel drive car to go up hill. Answer in terms of some or all of a, b, h, m, g and θ.



Problem 5.3.25

5.3.26 Solve Problem 5.3.25 for a rear wheel drive car.

5.3.27 Solve Problem 5.3.25 for a four wheel drive car.