20.2 Dynamics of fixed axis rotation

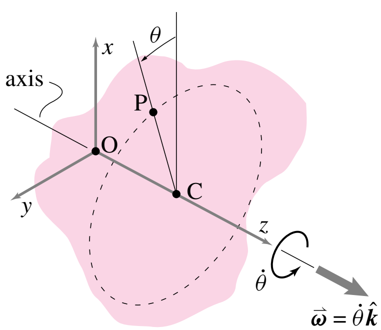
20.2.1 Let $P(x, y, z)$ be a point mass on an abstract massless body that rotates at a constant rate ω about the z-axis. Let $C(0, 0, z)$ be the center of the circular path that P describes during its motion.

a) Show that the rate of change of angular momentum of mass P about the origin O is given by

$$\dot{\vec{H}}_{/O} = -m\omega^2 Rz\hat{e}_\theta$$

where $R = \sqrt{x^2 + y^2}$ is the radius of the circular path of point P and m is its mass.

b) Is there any point on the axis of rotation (z-axis) about which $\dot{\vec{H}}$ is zero. Is this point unique?



Problem 20.2.1

20.2.2 System of Particles going in a circle. Assume for the problems below that you have two mass points going in circles about a common axis at a fixed common angular rate.

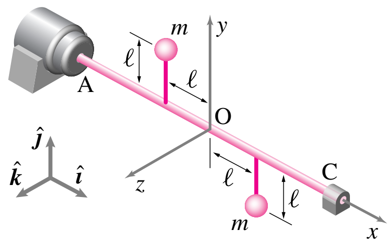
a) 2-D (particles in the plane, axis of rotation normal to the plane). Show, by calculating the sum in $\dot{\mathbf{H}}_C$ that the result is the same when using the two particles or when using a single particle at the center of mass (with mass equal to the sum of the two particles). You may do the general problem or you may pick particular appropriate locations for the masses, particular masses for the masses, a specific point C, a specific axis and a particular rotation rate.

b) 3-D (particles *not* in a common plane normal to the axis of rotation). Using particular mass locations that you choose, show that 2 particles give a *different* value for $\dot{\mathbf{H}}_C$ than a single particle at the center of mass. [Moral: In general, $\dot{\vec{H}}_{/C} \neq \vec{r}_{cm/C} \times m_{tot}\vec{a}_{cm}$.]

20.2.3 The rotating two-mass system shown in the figure has a constant angular velocity $\vec{\omega} = 12\,\text{rad/s}\hat{\imath}$. Assume that at the instant shown, the masses are in the xy-plane. Take $m = 1\,\text{kg}$ and $\ell = 1\,\text{m}$.

a) Find the linear momentum $\vec{L}$ and its rate of change $\dot{\vec{L}}$ for this system.

b) Find the angular momentum $\vec{H}_{/O}$ about point O and its rate of change $\dot{\vec{H}}_{/O}$ for this system.



Problem 20.2.3

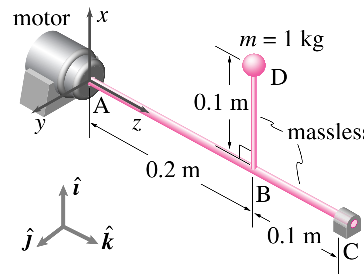
20.2.4 Particle attached to a shaft, no gravity. A shaft of negligible mass spins at constant rate. The motor at A imposes this rate if needed. The bearing at A prevents translation of its end of the shaft in any direction but causes no torques other than that of the motor (which causes a torque in the $\hat{k}$ direction). The frictionless bearing at C holds that end of the shaft from moving in the $\hat{\imath}$ and $\hat{\jmath}$ directions but allows slip in the $\hat{k}$ direction. The shaft is welded orthogonally to a bar, also of negligible mass, which is attached to a small ball of (non-negligible) mass of 1 kg. There is no gravity. The shaft is spinning counterclockwise (when viewed from the positive $\hat{k}$ direction) at 3 rad/s. In the configuration of interest the rod BD is parallel with the $\hat{\imath}$ direction.

a) What is the tension in the rod BD? *

b) What are all the reaction forces at A and C? *

c) What is the torque that the motor at A causes on the shaft? *

d) There are two ways to do problems (b) and (c): one is by treating the shaft, bar and mass as one system and writing the momentum balance equations. The other is to use the result of (a) with action-reaction to do statics on the shaft BC. Repeat problems (b) and (c) doing it the way you did not do it the first time.



Problem 20.2.4

20.2.5 Particle attached to a shaft, with gravity. Consider the same situation as in problem 20.2.4 with the following changes: There is gravity (pointing in the $-\mathbf{i}$ direction). At the time of interest the rod BD is parallel to the $+\mathbf{j}$ direction.

a) What is the force on D from the rod BD? (Note that the rod BD is *not* a 'two-force' member. Why not?)

b) What are all the reaction forces at A and C?

c) What is the torque that the motor at A causes on the shaft?

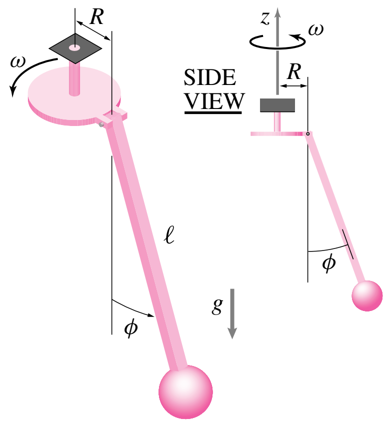
d) Do problems (b) and (c) again a different way.

20.2.6 A turntable rotates at the constant rate of $\omega = 2\,\text{rad/s}$ about the z axis. At the edge of the 2 m radius turntable swings a pendulum which makes an angle ϕ with the negative z direction. The pendulum length is $\ell = 10\,\text{m}$. The pendulum consists of a massless rod with a point mass $m = 2\,\text{kg}$ at the end. The gravitational constant is $g = 10\,\text{m/s}^2$ pointing in the $-z$ direction. A perspective view and a side view are shown.

a) What possible angle(s) could the pendulum make in a steady motion. In other words, find ϕ for steady circular motion of the system. It turns out that this quest leads to a transcendental equation. You are not asked to solve this equation but you should have the equation clearly defined. *

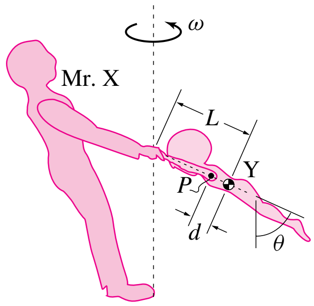
b) How many solutions are there? [hint, sketch the functions involved in your final equation.] *

c) Sketch very approximately any equilibrium solution(s) not already sketched on the side view in the picture provided.



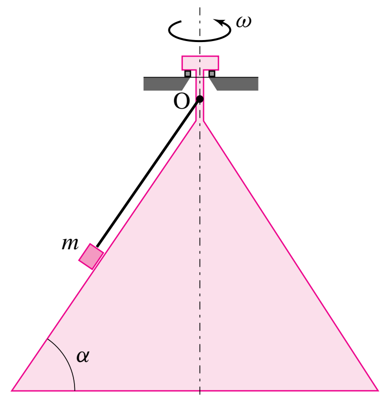
Problem 20.2.6

20.2.7 Mr. X swings his daughter Y about the vertical axis at a constant rate $\omega = 1\,\text{rad/s}$. Y is merely one year old and weighs 12 kg. Assuming Y to be a point mass located at the center of mass, estimate the force on each of her shoulder joints.



Problem 20.2.7

20.2.8 A block of mass $m = 2\,\text{kg}$ supported by a light inextensible string at point O sits on the surface of a rotating cone as shown in the figure. The cone rotates about its vertical axis of symmetry at constant rate ω. Find the value of ω in rpm at which the block loses contact with the cone.



Problem 20.2.8

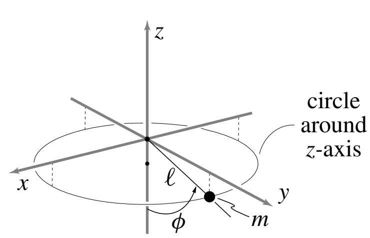
20.2.9 A small rock swings in circles at constant angular rate ω. It is hung by a string with length ℓ the other end of which is at the origin of a fixed coordinate system. The rock has mass m and the gravitational constant is g. The string makes an angle ϕ with the negative z-axis. The mass rotates so that if you follow it with the fingers of your right hand, your right thumb points up the z-axis. At the instant of interest the mass is passing through the yz-plane.

a) How long does the rock take to make one revolution? How does this compare with the period of oscillation of a simple pendulum. *

b) Find the following quantities in any order that pleases you. All solutions should be in terms of m, g, ℓ, ϕ, ω and the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

- The tension in the string (T).
- The velocity of the rock ($\vec{v}$).
- The acceleration of the rock ($\vec{a}$).
- The rate of change of the angular momentum about the $+x$ axis (dH_x/dt).

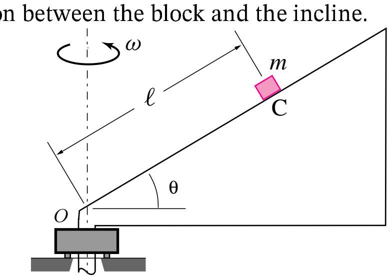
*



Problem 20.2.9

20.2.10 Consider the configuration in problem 20.2.9. What is the relation between the angle (measured from the vertical) at which the rock hangs and the speed at which it moves? Do an experiment with a string, weight, and stopwatch to check your calculation. *

20.2.11 A small block of mass $m = 5\,\text{kg}$ sits on an inclined plane which is supported by a bearing at point O as shown in the figure. At the instant of interest, the block is at a distance $\ell = 0.5\,\text{m}$ from where the surface of the incline intersects the axis of rotation. By experiments, the limiting rate of rotation of the turntable before the block starts sliding up the inclined plane is found to be $\omega = 125\,\text{rpm}$. Find the coefficient of friction between the block and the incline.

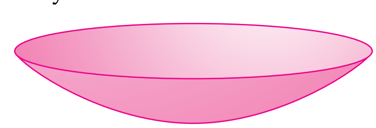


Problem 20.2.11

20.2.12 Particle sliding in circles in a parabolic bowl. As if in a James Bond adventure in a big slippery radar bowl, a particle-like human with mass m is sliding around in circles at speed v. The equation describing the bowl is $z = CR^2 = C(x^2 + y^2)$.

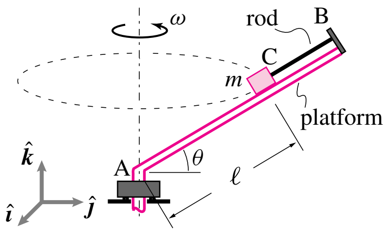
a) Find v in terms of any or all of R, g, and C.

b) Now say you are given ω, C and g. Find v and R if you can. Explain any oddities.



Problem 20.2.12

20.2.13 Mass on a rotating inclined platform. A block of mass $m = 5\,\text{kg}$ is held on an inclined platform by a rod BC as shown in the figure. The platform rotates at a constant angular rate ω. The coefficient of friction between the block and the platform is μ. $\theta = 30^o$, $\ell = 1\,\text{m}$, and $g = 9.81\,\text{m/s}^2$. (a) Assuming $\omega = 5\,\text{rad/s}$ and $\mu = 0$ find the tension in the rod. (b) If $\mu = .3$ what is the range of ω for which the block would stay on the ramp without slipping even if there were no rod? *

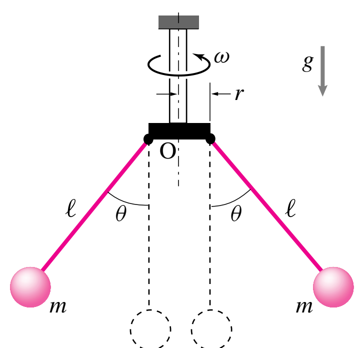


Problem 20.2.13

20.2.14 A circular plate of radius $r = 10\,\text{cm}$ rotates in the horizontal plane about the vertical axis passing through its center O. Two identical point masses hang from two points on the plate diametrically opposite to each other as shown in the figure. At a constant angular speed ω, the two masses maintain a steady state angle θ from their vertical static equilibrium.

a) Find the linear speed and the magnitude of the centripetal acceleration of either of the two masses.

b) Find the steady state ω as a function of θ. Do the limiting values of θ, $\theta \to 0$ and $\theta \to \pi/2$ give sensible values of ω?

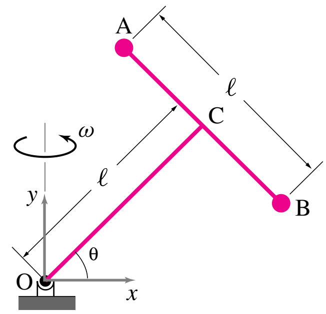


Problem 20.2.14

20.2.15 For the rod and mass system shown in the figure, assume that $\theta = 60°$, $\omega \equiv \dot{\theta} = 5\,\text{rad/s}$, and $\ell = 1\,\text{m}$. Find the acceleration of mass B at the instant shown by

a) calculating $\vec{a} = \vec{\omega}\times\vec{\omega}\times\vec{r}$ where $\vec{r}$ is the position vector of point B with respect to point O, i.e., $\vec{r} = \vec{r}_{B/O}$, and by

b) calculating $\vec{a} = -R\dot{\theta}^2\hat{e}_R$ where R is the radius of the circular path of mass B.



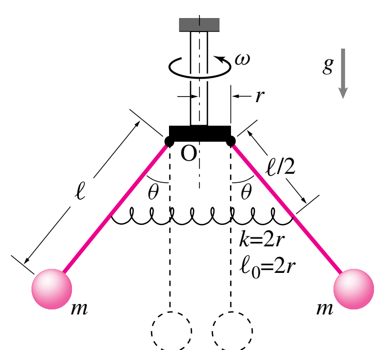
Problem 20.2.15

20.2.16 A spring of relaxed length ℓ_0 (same as the diameter of the plate) is attached to the mid point of the rods supporting the two hanging masses.

a) Find the steady angle θ as a function of the plate's rotation rate ω and the spring constant k. (Your answer may include other given constants.)

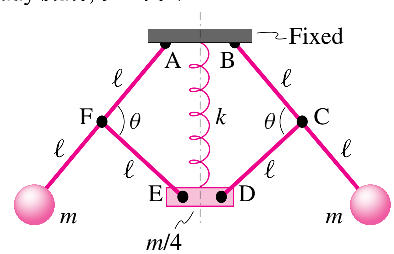
b) Verify that, for $k = 0$, your answer for θ reduces to that in problem 20.2.14.

c) Assume that the entire system sits inside a cylinder of radius 20 cm and the axis of rotation is aligned with the longitudinal axis of the cylinder. The maximum operating speed of the rotating shaft is 200 rpm (i.e., $\omega_{max} = 200\,\text{rpm}$). If the two masses are not to touch the walls of the cylinder, find the required spring stiffness k.



Problem 20.2.16

20.2.17 Flyball governors are used to control the flow of working fluid in steam engines, diesel engines, steam turbines, etc. A spring-loaded flyball governor is shown schematically in the figure. As the governor rotates about the vertical axis, the two masses tend to move outwards. The rigid and massless links that connect the outer arms to the collar, in turn, try to lift the central collar (mass $m/4$) against the spring. The central collar can move along the vertical axis but the top of the governor remains fixed. The mass of each ball is $m = 5$ kg, and the length of each link is $\ell = 0.25$ m. There are frictionless hinges at points A, B, C, D, E, F where the links are connected. Find the amount the string is elongated, Δx, when the governor rotates at 100 rpm, given that the spring constant $k = 500$ N/m, and that in this steady state, $\theta = 90°$. *



Problem 20.2.17

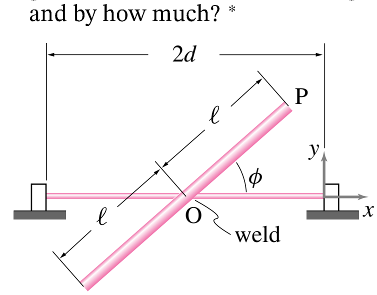
20.2.18 Crooked rod. A 2ℓ long uniform rod of mass m is welded at its center to a shaft at an angle ϕ as shown. The shaft is supported by bearings at its ends and spins at a constant rate ω.

a) What are $\dot{\vec{H}}_{/O}$ and $\dot{\vec{L}}$?

b) Calculate $\dot{\mathbf{H}}_{/O}$ using the integral definition of angular momentum. *

c) What are the six reaction components if there is no gravity? *

d) Repeat the calculations, including gravity g. Which reactions change and by how much? *



Problem 20.2.18

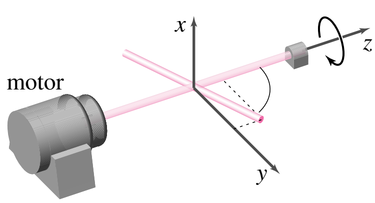
20.2.19 For the configuration in problem 20.2.18, find two positions along the rod where concentrating one-half the mass at each of these positions gives the same $\dot{\vec{H}}_{/O}$.

20.2.20 Rod on a shaft. A uniform rod is welded to a shaft which is connected to frictionless bearings. The rod has mass m and length ℓ and is attached at an angle γ at its midpoint to the midpoint of the shaft. The length of the shaft is $2d$. A motor applies a torque M_{motor} along the axis of the shaft. At the instant shown in the figure, assume m, ℓ, d, ω and M_{motor} are known and the rod lies in the yz-plane.

a) Find the angular acceleration of the shaft.

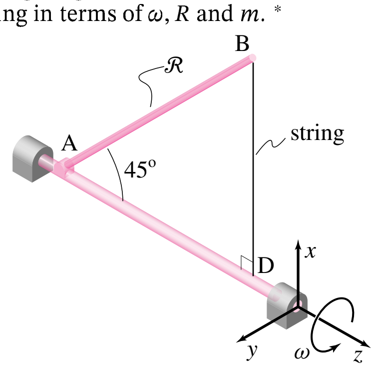
b) The moment that the shaft applies to the rod.

c) The reaction forces at the bearings.



Problem 20.2.20: Rod on a shaft.

20.2.21 Rod spins on a shaft. A uniform rod $\mathcal{R}$ with length R and mass m spins at constant rate ω about the z axis. It is held by a hinge at A and a string at B. Neglect gravity. Find the tension in the string in terms of ω, R and m. *

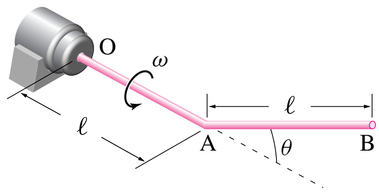


Problem 20.2.21

20.2.22 A uniform narrow shaft of length 2ℓ and mass $2M$ is bent at an angle θ half way down its length. It is spun at constant rate ω.

a) What is the magnitude of the force that must be applied at O to keep the shaft spinning?

b) (harder) What is the magnitude of the moment that must be applied at O to keep the shaft spinning?



Problem 20.2.22: **Spinning rod. No gravity.**

20.2.23 For the configuration in problem 20.2.22, find two positions along the bent portion of the shaft where concentrating one-half its mass at each of these positions gives the same $\dot{\vec{H}}_{/O}$ and $\dot{\vec{L}}$.

20.2.24 Rod attached to a shaft. A uniform 1 kg, $\sqrt{2}$ m rod is attached to a shaft which spins at a constant rate of 1 rad/s. One end of the rod is connected to the shaft by a ball-and-socket joint . The other end is held by two strings (or pin jointed rods) that are connected to a rigid cross bar. The cross bar is welded to the main shaft. Ignore gravity.

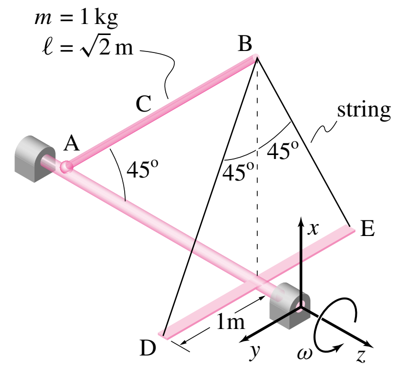
a) Draw a free-body diagram of the rod AB, including a bit of the strings.

b) Find the tension in the string DB. Two approaches:

(i) Write the six equations of linear and angular momentum balance (about any point of your choosing) and solve them (perhaps using a computer) for the tension T_{DB}.

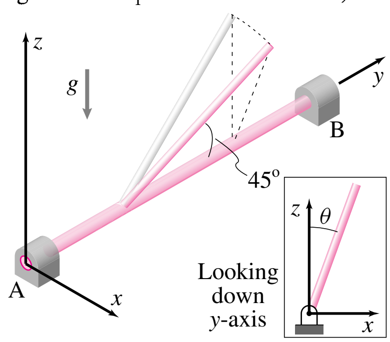
(ii) Write the equation of angular momentum balance about the axis AE. (That is, take the equation of angular momentum balance about point A and dot both sides with a vector in the direction of $\vec{r}_{E/A}$.) Use this calculation to find T_{DB} and compare to method ii.

c) Find the reaction <u>**force**</u> (a vector) at A.



Problem 20.2.24: Uniform rod attached to a shaft.

20.2.25 3D, crooked bar on shaft falls down. A massless, rigid shaft is connected to frictionless hinges at A and B. A bar with mass M_b and length ℓ is welded to the shaft at 45°. At time $t = 0$ the assembly is tipped a small angle θ_0 from the yz-plane but has no angular velocity. It then falls under the action of gravity. What is θ at $t = t_1$ (assuming t_1 is small enough so that θ_1 is much less than 1)?



Problem 20.2.25: A crooked rod welded to a shaft falls down.