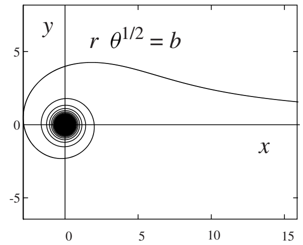
18.1.1 A particle moves along the two paths (1) and (2) as shown.

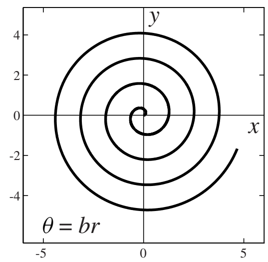
a) In each case, determine the velocity of the particle in terms of b, θ, and $\dot{\theta}$.

b) Find the x and y coordinates of the path as functions of b and r or b and θ.

(1)



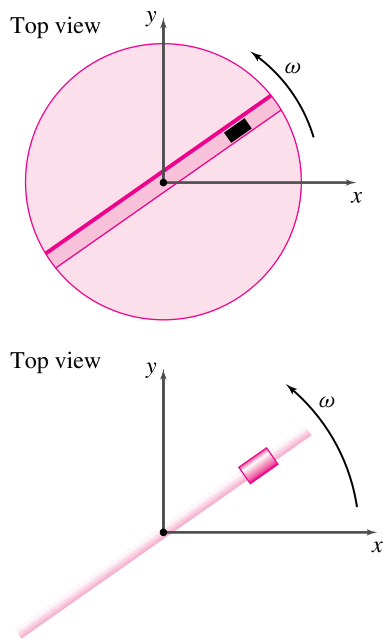
(2)



Problem 18.1

Answer:
plot(2): $\theta(t) = br(t)$ and $\vec{\boldsymbol{v}} = \frac{\dot{\theta}}{b}\hat{\boldsymbol{e}}_r + \frac{\theta\dot{\theta}}{b}\hat{\boldsymbol{e}}_t$.
plot(2): $x = r\cos(br)$ and $y = r\sin br$ or $x = \frac{\theta}{b}\cos(\theta)$ and $y = \frac{\theta}{b}\sin\theta$

18.1.5 Picking apart the polar coordinate formula for velocity. This problem concerns a small mass m that sits in a slot in a turntable. Alternatively you can think of a small bead that slides on a rod. The mass always stays in the slot (or on the rod). Assume the mass is a little bug that can walk as it pleases on the rod (or in the slot) and you control how the turntable/rod rotates. Name two situations in which one of the terms is zero but the other is not in the two term polar coordinate formula for velocity, $\dot{R}\hat{\boldsymbol{e}}_R + R\dot{\theta}\hat{\boldsymbol{e}}_\theta$. You should thus gain some insight into the meaning of each of the two terms in that formula.

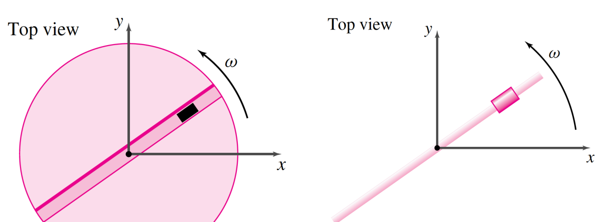


Problem 18.5

Answer:
One situation: $\vec{\boldsymbol{v}} = R\dot{\theta}\hat{\boldsymbol{e}}_\theta$;the case of a particle constrained to move in a circle.

17.1.5

Saturday, April 20, 2019 11:52 PM



Problem 17.1.5

Given: mass m in the slot / on the rod

Find: two situations, one of the terms is zero in polar coordinate formula for velocity:

$\dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$

Situation #1: $\dot{R}=0$, $\dot{\theta}\neq 0$, $R\neq 0$

Thus $\dot{R}\hat{e}_R=\vec{0}$, $R\dot{\theta}\hat{e}_\theta\neq\vec{0}$

In this case, the bug is lazy and does not move relatively in the slot / on the rod at all. The bug is thus relatively motionless to the slot. (but not at the origin)

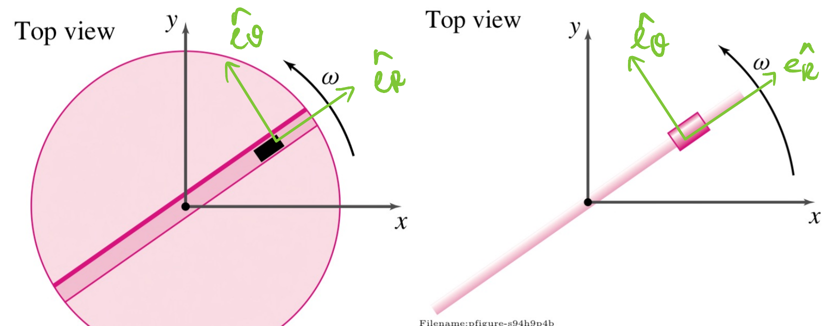
Situation #2: $\dot{\theta}=0$. $\dot{R}\neq 0$, R can be anything

Thus $R\dot{\theta}\hat{e}_\theta=\vec{0}$, $\dot{R}\hat{e}_R\neq\vec{0}$

In this case, I am lazy so that I don't want to rotate the turntable / rod, but the bug might be diligent and moves along the slot.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

18.1.5 Picking apart the polar coordinate formula for velocity. This problem concerns a small mass m that sits in a slot in a turntable. Alternatively you can think of a small bead that slides on a rod. The mass always stays in the slot (or on the rod). Assume the mass is a little bug that can walk as it pleases on the rod (or in the slot) and you control how the turntable/rod rotates. Name two situations in which one of the terms is zero but the other is not in the two term polar coordinate formula for velocity, $\dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$. You should thus gain some insight into the meaning of each of the two terms in that formula. *



Problem 18.1.5

Katherine Cao

Given: mass m in the slot / on the rod

Find:

Name 2 Situations in whic one of the terms is zero but the other is not in the two term polar coordinate formula for velocity

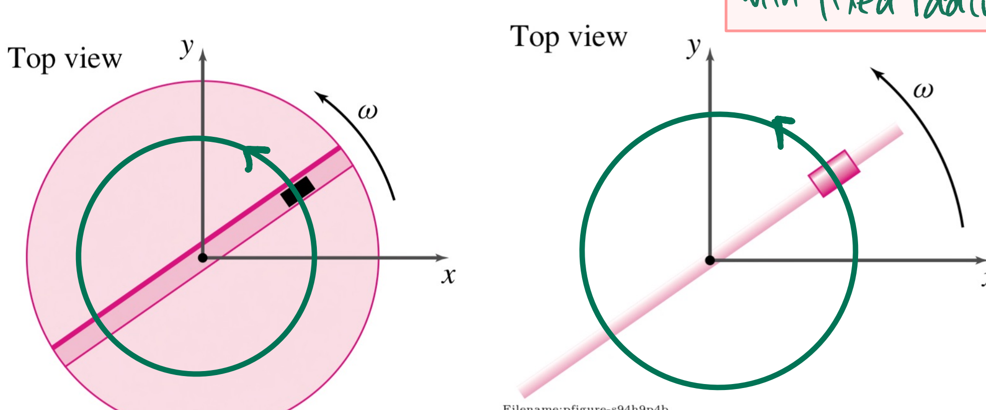
$\vec{V} = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$

$\boxed{\text{Situation 1}: \ \dot{R}\hat{e}_R = \vec{0} \ \Rightarrow \ \dot{R} = 0}$

$\vec{V} = \cancel{\dot{R}}\hat{e}_R + R\dot{\theta}\hat{e}_\theta = \boxed{R\dot{\theta}\hat{e}_\theta = R\omega\hat{e}_\theta}$

In this scenario, the little bug does not move relative to the turntable/rod. It only rotates together with the turntable/rod.

$\boxed{\text{Circular Trajectory with fixed radius}}$

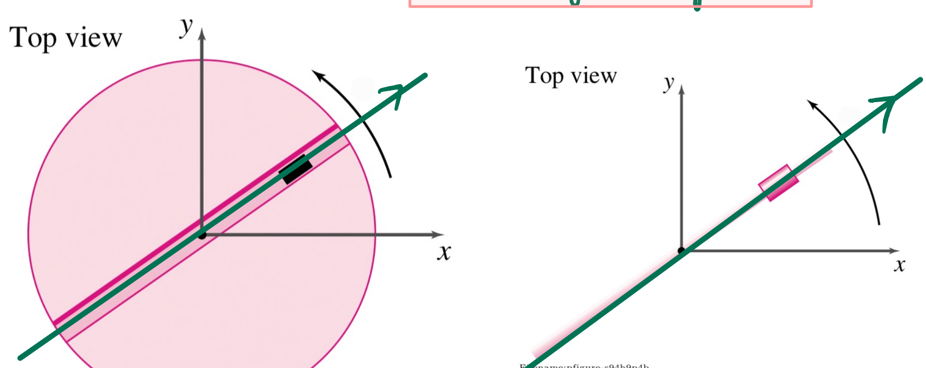


Problem 18.1.5

$\boxed{\text{Situation 2: } R\dot{\theta}\hat{e}_\theta = 0 \text{ with } R \neq 0,\ \dot{\theta} = 0}$

$$\vec{V} = \dot{R}\hat{e}_R + R\cancel{\dot{\theta}}^{\,0}\hat{e}_\theta = \boxed{\dot{R}\hat{e}_R}$$

In this scenario, the turntable / rod is stationary and does not rotate. The bug moves along the stationary slot on the stationary turntable / stationary rod. $\boxed{\text{Trajectory along } \hat{e}_R}$



Problem 18.1.5

Just showing trajectory, actual direction of $\vec{v}$ depends on $\dot{R}$ sign

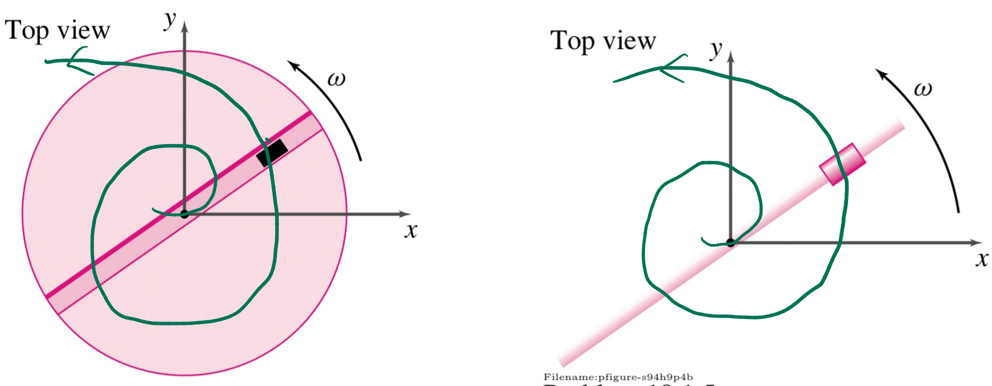
$\boxed{\text{Situation 3: } R\dot{\theta}\hat{e}_\theta = 0 \text{ with } R = 0,\ \dot{\theta} \neq 0,\ \dot{R} \neq 0}$

$$\vec{V} = \dot{R}\hat{e}_R + \cancel{R}^{\,0}\dot{\theta}\hat{e}_\theta = \boxed{\dot{R}\hat{e}_R}$$

In this scenario, the turntable / rod isn't stationary. At some moment the bug passes through origin, its velocity is along the slot / rod, $\vec{v} = \dot{R}\hat{e}_R$. Once moved away from origin:

$$\vec{V} = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta, \quad R\dot{\theta}\hat{e}_\theta \neq 0 \text{ since } R \neq 0,\ \dot{\theta} \neq 0$$

Trajectory depends on $\dot{R}$ and $\dot{\theta}$. If $\dot{R}, \dot{\theta}$ both constant, spiral shape.



Problem 18.1.5

(Sorry imagine those are smooth lines ...)

18.1.6 Picking apart the polar coordinate formula for acceleration. Reconsider the configurations in problem 18.1.5. This time, name four situations in which all of the terms, but one, in the four term polar coordinate formula for acceleration, $\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$, are zero. Each situation should pick out a different term. You should thus gain some insight into the meaning of each of the four terms in that formula.

Answer:
One situation: $\vec{a} = \ddot{R}\hat{e}_R$; the case of no rotation ($\dot{\theta} = \ddot{\theta} = 0$).

17.1.6

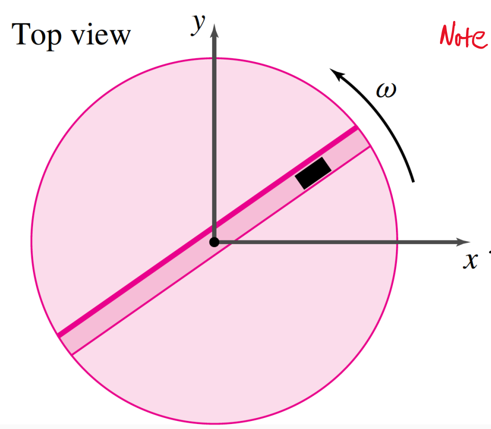
Friday, April 26, 2019 2:15 PM

Picking apart the polar coordinate formula for acceleration:

Find situation where only one term is non-zero

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta \qquad (4\text{-terms})$$

Note : value of θ is irrelevant



We can rotate the table the bug is in the slot can only walk along the slot

1° R = anything θ = anything

$\dot{R}$ = anything $\dot{\theta} = 0$ Non-rotating turntable

$\ddot{R} \neq 0$ $\ddot{\theta} = 0$

$$\vec{a} = (\ddot{R} - \cancel{R\dot{\theta}^2}^{\,0})\hat{e}_R + (2\dot{R}\cancel{\dot{\theta}}^{\,0} + \cancel{R\ddot{\theta}}^{\,0})\hat{e}_\theta$$

$$= \ddot{R}\hat{e}_R$$

We are too lazy and we do not rotate the turntable, while the bug is wandering along the slot with non-constant velocity, trying to find the food in the slot.

2° $R \neq 0$ θ = anything

$\dot{}$

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$\dot R = 0$ $\qquad\qquad$ $\dot\theta \neq 0$ $\qquad$ Const ω rotating turntable

$\ddot R = 0$ $\qquad\qquad$ $\ddot\theta = 0$

$$\vec a = (\cancelto{0}{\ddot R} - R\dot\theta^2)\hat e_R + (2\cancelto{0}{\dot R}\dot\theta + R\cancelto{0}{\ddot\theta})\hat e_\theta$$

$$= -R\dot\theta^2\hat e_R$$

we rotate the turntable at const angular velocity while the bug is too lazy to move at all. (not at origin)

3° $\quad R = 0$ $\qquad\qquad$ θ = anything

$\dot R \neq 0$ $\qquad\qquad$ $\dot\theta \neq 0$

$\ddot R = 0$ $\qquad\qquad$ $\ddot\theta$ = anything

$$\vec a = (\cancelto{0}{\ddot R} - \cancelto{0}{R\dot\theta^2})\hat e_R + (2\dot R\dot\theta + \cancelto{0}{R\ddot\theta})\hat e_\theta$$

$$= 2\dot R\dot\theta\,\hat e_\theta$$

as we are rotating the table, the bug happens to pass through the origin at constant velocity. Just at the moment when it is at the origin.

4° $\quad R \neq 0$ $\qquad\qquad$ θ = anything

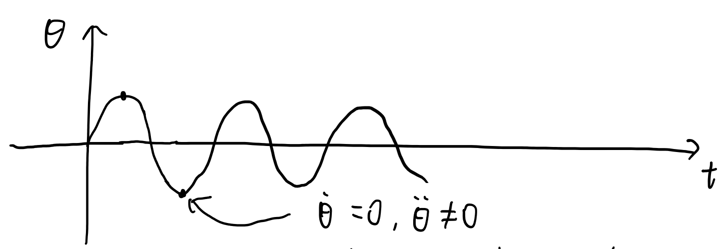
$\dot R$ = anything $\qquad\qquad$ $\dot\theta = 0$

$\ddot R = 0$ $\qquad\qquad$ $\ddot\theta \neq 0$

$$\vec a = (\cancelto{0}{\ddot R} - R\cancelto{0}{\dot\theta^2})\hat e_R + (2\dot R\cancelto{0}{\dot\theta} + R\ddot\theta)\hat e_\theta$$

$$= 2\dot R\ddot\theta\,\hat e_\theta$$

We rotate the table at a sinusoidal angular velocity



at any time when the angular velocity is zero

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

but the angular acceleration is non-zero.
The bug, however, still stays at a fixed point
but the origin.

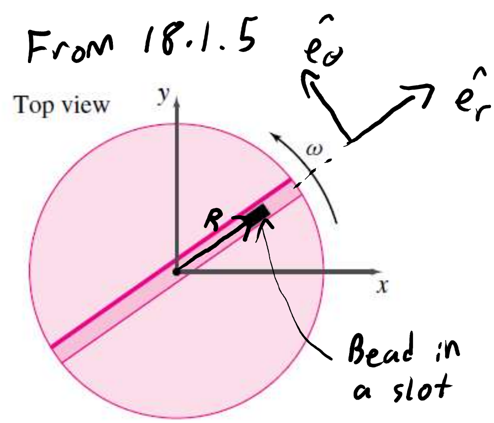
HW12-P63-18.1.6

Friday, April 30, 2021 8:47 AM

Solution by Michael Zakavorotny

18.1.6 Picking apart the polar coordinate formula for acceleration. Reconsider the configurations in problem 18.1.5. This time, name four situations in which all of the terms, but one, in the four term polar coordinate formula for acceleration, $\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$, are zero. Each situation should pick out a different term. You should thus gain some insight into the meaning of each of the four terms in that formula. *

From 18.1.5



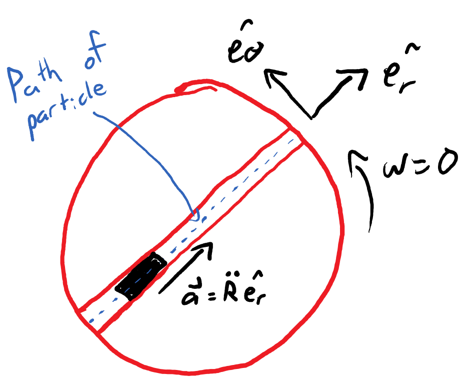
$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_r + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$$

Case 1: R = doesn't matter θ = doesn't matter

$\dot{R} \neq 0$ $\dot{\theta} = 0$

$\ddot{R} \neq 0$ $\ddot{\theta} = 0$

Therefore, acceleration term $\ddot{R} \neq 0$, and $-R\dot{\theta}^2 = 2\dot{R}\dot{\theta} = R\ddot{\theta} = 0$



The $\ddot{R}\hat{e}_r$ term represents the radial acceleration of the bead. This can exclusively be made non-zero if the disk is not rotating ($\omega = \dot{\theta} = 0$), and the bead is experiencing a linear acceleration in its slot.

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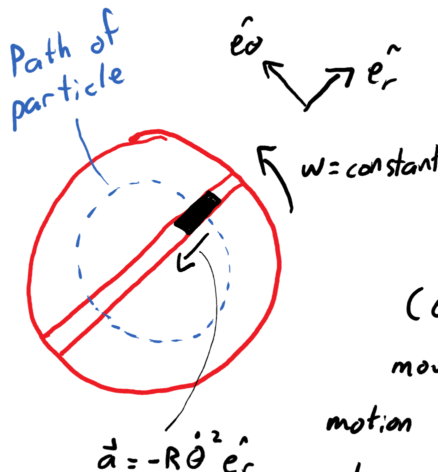
Case 2:

$R \neq 0$ θ = doesn't matter

$\dot{R} = 0$ $\dot{\theta}$ = constant

$\ddot{R} = 0$ $\ddot{\theta} = 0$

Therefore, $-R\dot{\theta}^2 \neq 0$, and $\ddot{R} = 2\dot{R}\dot{\theta} = R\ddot{\theta} = 0$



The $-R\dot{\theta}^2\hat{e}_r$ term is the centripetal acceleration of the bead. This can exclusively be made non-zero if the disk rotates at a constant angular speed ($\dot{\theta}$ = constant, $\ddot{\theta} = 0$) and the bead does not move along the length of the slot ($\dot{R} = 0$). This motion can be understood as constant speed circular motion.

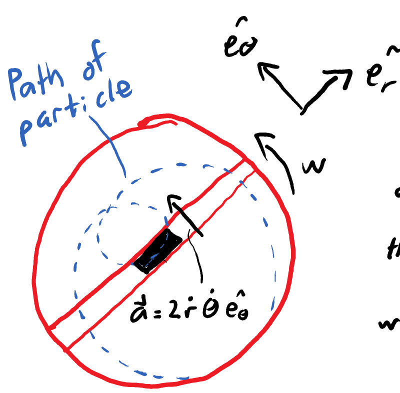
Case 3:

$R = 0$ θ = doesn't matter

$\dot{R}$ = constant $\dot{\theta} \neq 0$

$\ddot{R} = 0$ $\ddot{\theta}$ = doesn't matter

Therefore, $2\dot{R}\dot{\theta} \neq 0$, and $\ddot{R} = -R\dot{\theta}^2 = R\ddot{\theta} = 0$



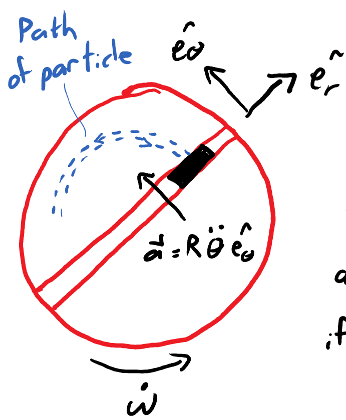
The term $2\dot{R}\dot{\theta}\hat{e}_\theta$ is the Coriolis acceleration. This can be isolated when the bead is at the center of rotation ($R = 0$) moving at a constant speed in the slot ($\dot{R}$ = constant), and the disk is rotating with any angular speed.

Case 4: $R \neq 0$ $\quad\quad \theta =$ doesn't matter

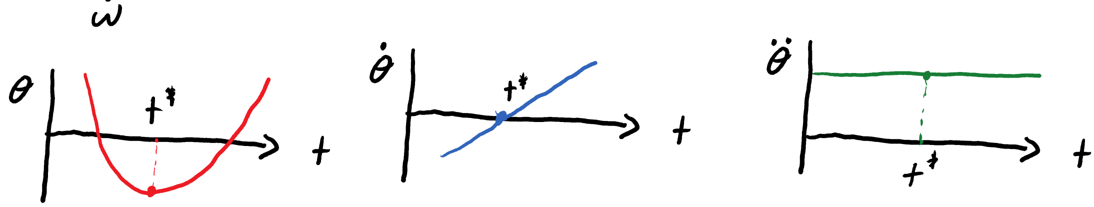
$\dot{R} = 0 \quad\quad \dot{\theta} = 0$

$\ddot{R} = 0 \quad\quad \ddot{\theta} \neq 0$

Therefore, $R\ddot{\theta} \neq 0$, and $\ddot{R} = -R\dot{\theta}^2 = 2\dot{R}\dot{\theta} = 0$



The term $R\ddot{\theta}\hat{e}_\theta$ is the tangential acceleration of the bead. This term exclusively can be made non-zero if the bead is not moving in the slot ($\dot{R} = 0$), and the angular speed $\dot{\theta}$ is instantaneously 0. However, the angular acceleration $\ddot{\theta}$ cannot be 0, which can occur if $\dot{\theta}$ is increasing or decreasing at that time



18.1.7 The two differential equations:

$$\begin{aligned} \ddot{R} - R\dot{\theta}^2 &= 0 \\ 2\dot{R}\dot{\theta} + R\ddot{\theta} &= 0 \end{aligned}$$

have the general solution

$$\begin{aligned} R &= \sqrt{d^2 + (v(t-t_0))^2} \\ \theta &= \theta_0 + \tan^{-1}(v(t-t_0)/d) \end{aligned}$$

where θ_0, d, t_0, and v are arbitrary constants. This solution could be checked by plugging back into the differential equations — you need not do this (tedious) substitution. The solution describes a curve in the plane. That is, if for a range of values of t, the values of R and θ were calculated and then plotted using polar coordinates, a curve would be drawn. *What can you say about the shape of this curve?*

[Hints:

a) Actually make a plot using some random values of the constants and see what the plot looks like.

b) Write the equation $\mathbf{F} = m\mathbf{a}$ for a particle in polar coordinates and think of a force that would be relevant to this problem.

c) The answer is something simple.]

6.54 Plot R vs θ in polar coords for

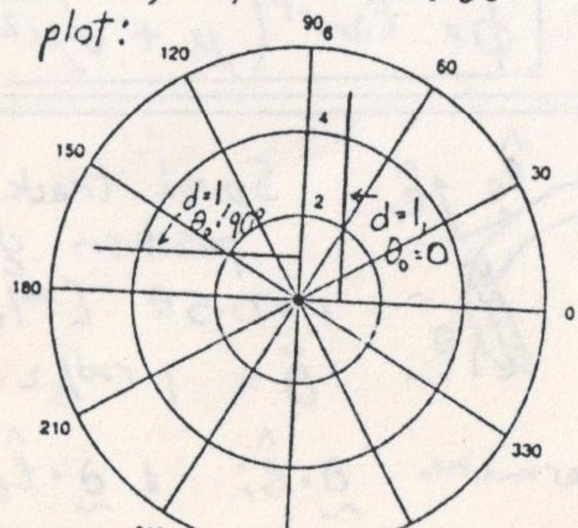
$$\ddot{R} - R\dot{\theta}^2 = 0$$

$$2\dot{R}\dot{\theta} + R\ddot{\theta} = 0$$

writing $\sum \underset{\sim}{F} = m\underset{\sim}{a}$ yields:

$$\sum \underset{\sim}{F} = m(\ddot{R} - R\dot{\theta}^2)\hat{e}_r + m(2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$$

dotting the above w/ $\hat{e}_r$ & $\hat{e}_\theta$, one can see that there must be no net force on the particle. Thus, the particle will move in a straight line. The orientation of this line depends on 1) the initial radial distance d, 2) the initial angle θ. See plot:



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18.1.9 Find expressions for $\hat{\boldsymbol{e}}_t$, a_t, a_n, $\hat{\boldsymbol{e}}_n$, and the radius of curvature ρ, at any position (or time) on the given particle paths for

a) problem **??**,
b) problem **??**,
c) problem **??**,
d) problem **??**,
e) problem **??**, and
f) problem **??**.

Answer:
(a)

$$\hat{\boldsymbol{e}}_t = \frac{\frac{2t}{\mathrm{s}}\hat{\boldsymbol{\imath}} + e^{\frac{t}{\mathrm{s}}}\hat{\boldsymbol{\jmath}}}{\sqrt{4\frac{t^2}{\mathrm{s}^2} + e^{\frac{2t}{\mathrm{s}}}}}$$

$$a_t = \frac{4t\frac{\mathrm{m}}{\mathrm{s}^3} + e^{\frac{2t}{\mathrm{s}}}\ \mathrm{m/s^2}}{\sqrt{4\frac{t^2}{\mathrm{s}^2} + e^{\frac{2t}{\mathrm{s}}}}}$$

$$\vec{\boldsymbol{a}}_n = \frac{(2 - 2\frac{t}{\mathrm{s}})e^{\frac{2t}{\mathrm{s}}}\ \mathrm{m/s^2}\hat{\boldsymbol{\imath}} + (4\frac{t^2}{\mathrm{s}^2} - 4\frac{t}{\mathrm{s}})e^{\frac{t}{\mathrm{s}}}\ \mathrm{m/s^2}\,\hat{\boldsymbol{\jmath}}}{4\frac{t^2}{\mathrm{s}^2} + e^{\frac{2t}{\mathrm{s}}}}$$

$$\hat{\boldsymbol{e}}_n = \frac{e^{\frac{t}{\mathrm{s}}}\hat{\boldsymbol{\imath}} - 2\frac{t}{\mathrm{s}}\hat{\boldsymbol{\jmath}}}{\sqrt{4\frac{t^2}{\mathrm{s}^2} + e^{\frac{2t}{\mathrm{s}}}}}$$

$$\rho = \frac{(4\frac{t^2}{\mathrm{s}^2} + e^{\frac{2t}{\mathrm{s}}})}{(2 - 2\frac{t}{\mathrm{s}})e^{\frac{t}{\mathrm{s}}}}\ \mathrm{m}$$

18.1.10 A particle travels at non-constant speed on an elliptical path given by $y^2 = b^2(1 - \frac{x^2}{a^2})$. Carefully sketch the ellipse for particular values of a and b. For various positions of the particle on the path, sketch the position vector $\vec{r}(t)$; the polar coordinate basis vectors $\hat{e}_r$ and $\hat{e}_\theta$; and the path coordinate basis vectors $\hat{e}_n$ and $\hat{e}_t$. At what points on the path are $\hat{e}_r$ and $\hat{e}_n$ parallel(or $\hat{e}_\theta$ and $\hat{e}_t$ parallel)?

17.1.10

Friday, April 26, 2019 2:56 PM

elliptical path : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

pick particular values for a,b

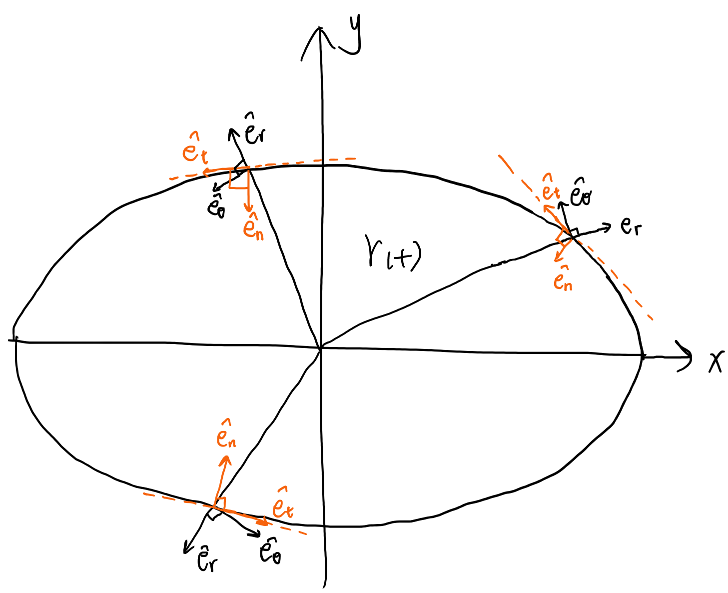
$\Rightarrow a = 2 , b = 1$

sketch $\vec{r}(t)$

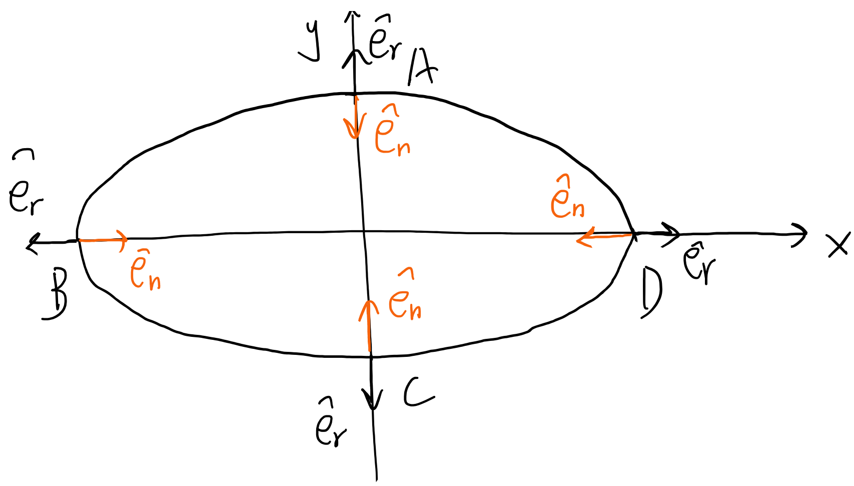
polar coordinate basis vectors $\hat{e}_r , \hat{e}_\theta$

path coordinate basis vectors $\hat{e}_n . \hat{e}_t$

for various positions



Find all points on the path $\hat{e}_r // \hat{e}_n$

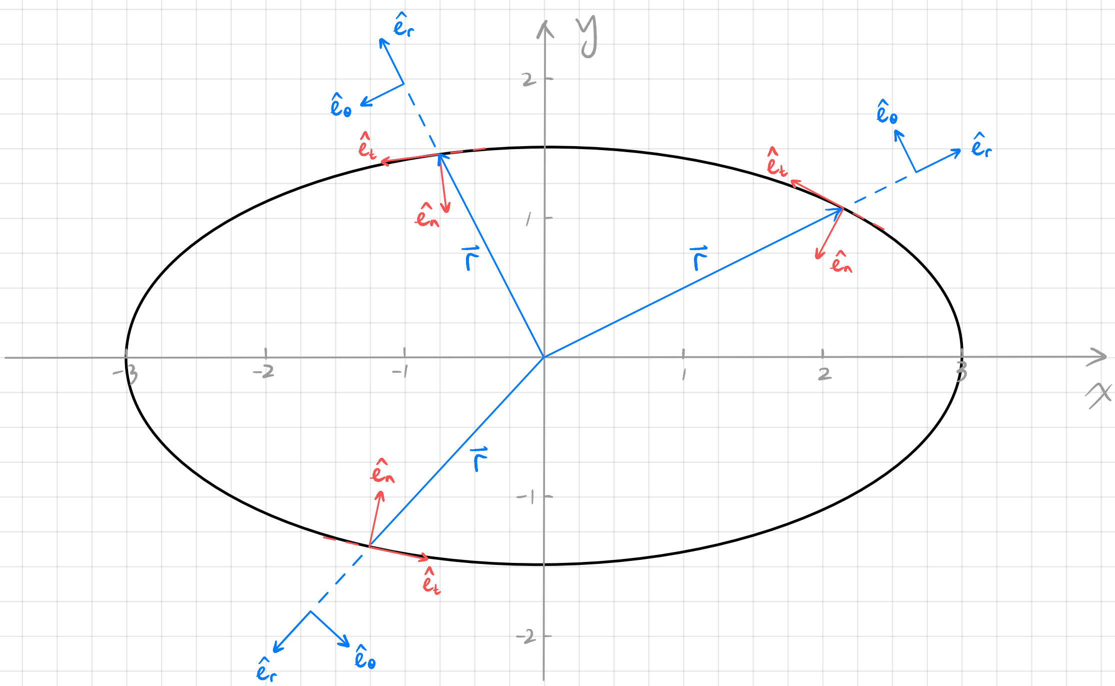

y
$\hat{e}_r$
A
$\hat{e}_n$
$\hat{e}_r$
$\hat{e}_n$
x
$\hat{e}_n$
$\hat{e}_r$
B
$\hat{e}_n$
D
C
$\hat{e}_r$

MAE 2030 HW P64 (18.1.10) Solution Duan Li

Given $y^2 = b^2(1 - \frac{x^2}{a^2})$

Define $a = 3$, $b = 1.5$

Sketch Ellipse, $\vec{r}(t)$, $\hat{e}_r$, $\hat{e}_\theta$, $\hat{e}_n$, $\hat{e}_t$



If drawing the ellipse in MATLAB, do

```
theta = linspace(0, 2*pi, 1000);
x = a*cos(theta);
y = b*sin(theta);
plot(x,y);
```

Find where $\hat{e}_r \parallel \hat{e}_n$

Answer. At the apoapses and the periapses