SAMPLE 17.10 **Falling ladder:** The ends of a ladder of length $L = 3\,\text{m}$ slip along the frictionless wall and floor shown in Figure 17.38. At the instant shown, when $\theta = 60^\circ$, the angular speed $\dot\theta = 1.15\,\text{rad/s}$ and the angular acceleration $\ddot\theta = 2.5\,\text{rad/s}^2$. Find the absolute velocity and acceleration of end B of the ladder.

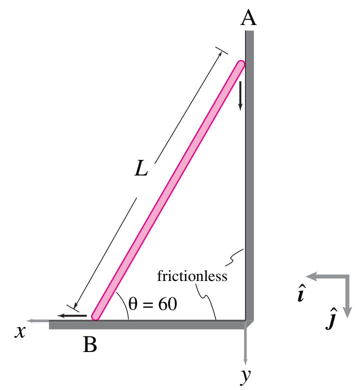


Figure 17.38

Solution Since the ladder is falling, it is rotating clockwise. From the given information:

$$\begin{aligned}\vec\omega &= \dot\theta\hat{\boldsymbol k} = -1.15\,\text{rad/s}\hat{\boldsymbol k}\\ \dot{\vec\omega} &= \ddot\theta\hat{\boldsymbol k} = -2.5\,\text{rad/s}^2\hat{\boldsymbol k}.\end{aligned}$$

We need to find $\vec v_B$, the absolute velocity of end B, and $\vec a_B$, the absolute acceleration of end B.

Since the end A slides along the wall and end B slides along the floor, we know the directions of $\vec v_A$, $\vec v_B$, $\vec a_A$ and $\vec a_B$.

Let $\vec v_A = v_A\hat{\jmath}$, $\vec a_A = a_A\hat{\jmath}$, $\vec v_B = v_B\hat{\imath}$ and $\vec a_B = a_B\hat{\imath}$ where the scalar quantities v_A, a_A, v_B and a_B are unknown.

$$\begin{aligned}\text{Now,}\quad \vec v_A &= \vec v_B + \vec v_{A/B} = \vec v_B + \vec\omega\times\vec r_{A/B}\\ \text{or}\quad v_A\hat{\jmath} &= v_B\hat{\imath} + \dot\theta\hat{\boldsymbol k}\times\underbrace{L(-\cos\theta\hat{\imath} - \sin\theta\hat{\jmath})}_{\vec r_{A/B}}\\ &= (v_B + \dot\theta L\sin\theta)\hat{\imath} - \dot\theta L\cos\theta\hat{\jmath}.\end{aligned}$$

Dotting both sides of the equation with $\hat{\imath}$, we get:

$$\begin{aligned}v_A\underbrace{\hat{\jmath}\cdot\hat{\imath}}_{0} &= (v_B + \dot\theta L\sin\theta)\underbrace{\hat{\imath}\cdot\hat{\imath}}_{1} + \dot\theta L\cos\theta\underbrace{\hat{\jmath}\cdot\hat{\imath}}_{0}\\ \Rightarrow\quad 0 &= v_B + \dot\theta L\sin\theta\\ \Rightarrow\quad v_B &= -\dot\theta L\sin\theta = -(-1.15\,\text{rad/s})\cdot 3\,\text{m}\cdot\frac{\sqrt3}{2}\\ &= 2.99\,\text{m/s}.\end{aligned}$$

$$\boxed{\vec v_B = 2.9\,\text{m/s}\hat{\imath}}$$

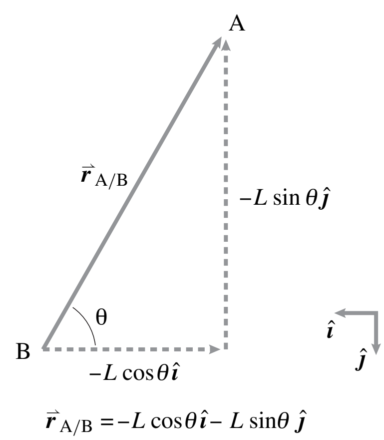


Figure 17.39

Similarly,

$$\begin{aligned}\vec a_A &= \vec a_B + \dot{\vec\omega}\times\vec r_{A/B} + \overbrace{\vec\omega\times(\vec\omega\times\vec r_{A/B})}^{-\omega^2\vec r_{A/B}}\\ a_A\hat{\jmath} &= a_B\hat{\imath} + \ddot\theta\hat{\boldsymbol k}\times L(-\cos\theta\hat{\imath} - \sin\theta\hat{\jmath}) - \dot\theta^2 L(-\cos\theta\hat{\imath} - \sin\theta\hat{\jmath})\\ &= (a_B + \ddot\theta L\sin\theta + \dot\theta^2 L\cos\theta)\hat{\imath} + (-\ddot\theta L\cos\theta + \dot\theta^2 L\sin\theta)\hat{\jmath}.\end{aligned}$$

Dotting both sides of this equation with $\hat{\imath}$ (as we did for velocity) we get:

$$\begin{aligned}0 &= a_B + \ddot\theta L\sin\theta + \dot\theta^2 L\cos\theta\\ \Rightarrow\quad a_B &= -\ddot\theta L\sin\theta - \dot\theta^2 L\cos\theta\\ &= -(-2.5\,\text{rad/s}^2\cdot 3\,\text{m}\cdot\frac{\sqrt3}{2}) - (-1.15\,\text{rad/s})^2\cdot 3\,\text{m}\cdot\frac12\\ &= 4.51\,\text{m/s}^2.\end{aligned}$$

$$\boxed{\vec a_B = 4.51\,\text{m/s}^2}$$

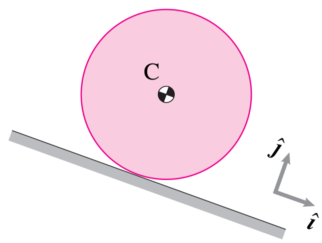


Figure 17.40

SAMPLE 17.11 A cylinder of diameter 500 mm rolls down an inclined plane with uniform acceleration (of the center-of-mass) $a = 0.1\,\mathrm{m/s^2}$. At an instant t_0, the mass-center has speed $v_0 = 0.5\,\mathrm{m/s}$.

1. Find the angular speed ω and the angular acceleration $\dot{\omega}$ at t_0.
2. How many revolutions does the cylinder make in the next 2 seconds?
3. What is the distance travelled by the center-of-mass in those 2 seconds?

Solution This problem is about simple kinematic calculations. We are given the velocity, $\dot{x}$, and the acceleration, $\ddot{x}$, of the center-of-mass. We are supposed to find angular velocity ω, angular acceleration $\dot{\omega}$, angular displacement θ in 2 seconds, and the corresponding linear distance x along the incline. The radius of the cylinder $R = \text{diameter}/2 = 0.25\,\mathrm{m}$.

1. From the kinematics of pure rolling,

$$\begin{aligned}\omega &= \frac{\dot{x}}{R} = \frac{0.5\,\mathrm{m/s}}{0.25\,\mathrm{m}} = 2\,\mathrm{rad/s},\\ \dot{\omega} &= \frac{\ddot{x}}{R} = \frac{0.1\,\mathrm{m/s^2}}{0.25\,\mathrm{m}} = 0.4\,\mathrm{rad/s^2}.\end{aligned}$$

$$\boxed{\omega = 2\,\mathrm{rad/s}, \quad \dot{\omega} = 0.4\,\mathrm{rad/s^2}}$$

2. We can find the number of revolutions the cylinder makes in 2 seconds by solving for the angular displacement θ in this time period. Since,

$$\ddot{\theta} \equiv \dot{\omega} = \text{constant},$$

we integrate this equation twice and substitute the initial conditions, $\dot{\theta}(t=0) = \omega = 2\,\mathrm{rad/s}$ and $\theta(t=0) = 0$, to get

$$\begin{aligned}\theta(t) &= \omega t + \frac{1}{2}\dot{\omega}t^2\\ \Rightarrow \quad \theta(t = 2\,\mathrm{s}) &= (2\,\mathrm{rad/s})\cdot(2\,\mathrm{s}) + \frac{1}{2}(0.4\,\mathrm{rad/s})\cdot(4\,\mathrm{s^2})\\ &= 4.8\,\mathrm{rad} = \frac{4.8}{2\pi}\,\mathrm{rev} = 0.76\,\mathrm{rev}.\end{aligned}$$

$$\boxed{\theta = 0.76\,\mathrm{rev}}$$

3. Now that we know the angular displacement θ, the distance travelled by the mass-center is the arc-length corresponding to θ, *i.e.*,

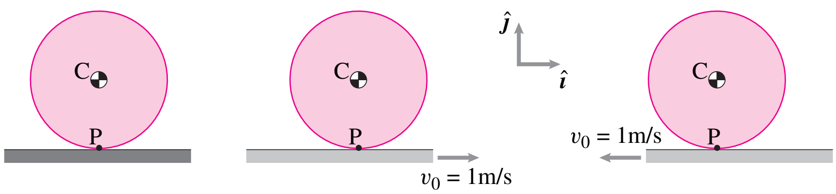
$$x = R\theta = (0.25\,\mathrm{m})\cdot(4.8) = 1.2\,\mathrm{m}.$$

$$\boxed{x = 1.2\,\mathrm{m}}$$

Note that we could have found the distance travelled by the mass-center by integrating the equation $\ddot{x} = 0.1\,\mathrm{m/s^2}$ twice.

SAMPLE 17.12 Condition of pure rolling. A cylinder of radius $R = 20\,\text{cm}$ rolls on a flat surface with absolute angular speed $\omega = 12\,\text{rad/s}$ under the conditions shown in the figure (In cases (ii) and (iii), you may think of the 'flat surface' as a conveyor belt). In each case,

1. Write the condition for pure rolling.
2. Find the velocity of the center C of the cylinder.



(i) Fixed base (ii) Base moves to the right (iii) Base moves to the left

Figure 17.41:

Solution At any instant during rolling, the cylinder makes a point-contact with the flat surface. Let the point of instantaneous contact on the cylinder be P, and let the corresponding point on the flat surface be Q. The condition of pure rolling, in each case, is $\vec{v}_P = \vec{v}_Q$, that is, there is no relative motion between the two contacting points (a relative motion will imply slip). Now, we analyze each case.

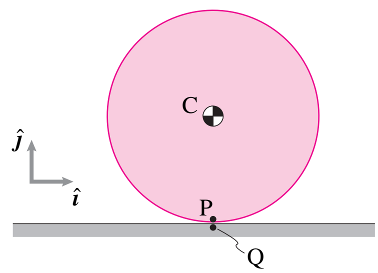


Figure 17.42: The cylinder rolls on the flat surface. Instantaneously, point P on the cylinder is in contact with point Q on the flat surface. For pure rolling, points P and Q must have the same velocity.

Case(i) In this case, the bottom surface is fixed. Therefore,

1. The condition of pure rolling is: $\vec{v}_P = \vec{v}_Q = \vec{\mathbf{0}}$.
2. Velocity of the center:

$$\begin{aligned} \vec{v}_C &= \vec{v}_P + \vec{\omega} \times \vec{r}_{C/P} = \vec{\mathbf{0}} + (-\omega\hat{k}) \times R\,\hat{\jmath} \\ &= \omega R\hat{\imath} = (12\,\text{rad/s}) \cdot (0.2\,\text{m})\hat{\imath} = 2.4\,\text{m/s}\hat{\imath}. \end{aligned}$$

Case(ii) In this case, the bottom surface moves with velocity $\vec{v} = 1\,\text{m/s}\hat{\imath}$. Therefore, $\vec{v}_Q = 1\,\text{m/s}\hat{\imath}$. Thus,

1. The condition of pure rolling is: $\vec{v}_P = \vec{v}_Q = 1\,\text{m/s}\hat{\imath}$.
2. Velocity of the center:

$$\begin{aligned} \vec{v}_C &= \vec{v}_P + \vec{\omega} \times \vec{r}_{C/P} = v_0\hat{\imath} + \omega R\hat{\imath} \\ &= 1\,\text{m/s}\hat{\imath} + 2.4\,\text{m/s}\hat{\imath} = 3.4\,\text{m/s}\hat{\imath}. \end{aligned}$$

Case(iii) In this case, the bottom surface moves with velocity $\vec{v} = -1\,\text{m/s}\hat{\imath}$. Therefore, $\vec{v}_Q = -1\,\text{m/s}\hat{\imath}$. Thus,

1. The condition of pure rolling is: $\vec{v}_P = \vec{v}_Q = -1\,\text{m/s}\hat{\imath}$.
2. Velocity of the center:

$$\begin{aligned} \vec{v}_C &= \vec{v}_P + \vec{\omega} \times \vec{r}_{C/P} = -v_0\hat{\imath} + \omega R\hat{\imath} \\ &= -1\,\text{m/s}\hat{\imath} + 2.4\,\text{m/s}\hat{\imath} = 1.4\,\text{m/s}\hat{\imath}. \end{aligned}$$

$$\begin{array}{llll} \text{(a)}: & \text{(i)}\vec{v}_P = \vec{\mathbf{0}}, & \text{(ii)}\vec{v}_P = 1\,\text{m/s}\hat{\imath}, & \text{(iii)}\vec{v}_P = -1\,\text{m/s}\hat{\imath}, \\ \text{(b)}: & \text{(i)}\vec{v}_C = 2.4\,\text{m/s}\hat{\imath}, & \text{(ii)}\vec{v}_C = 3.4\,\text{m/s}\hat{\imath}, & \text{(iii)}\vec{v}_C = 1.4\,\text{m/s}\hat{\imath} \end{array}$$

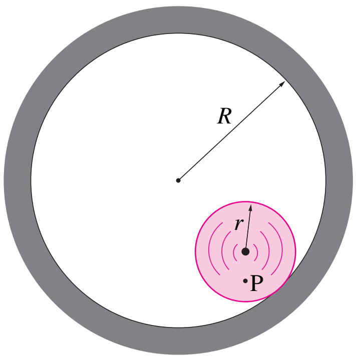


Figure 17.43: A uniform disk of radius r rolls without slipping inside a fixed cylinder.

SAMPLE 17.13 Motion of a point on a disk rolling inside a cylinder. A uniform disk of radius r rolls without slipping with constant angular speed ω inside a fixed cylinder of radius R. A point P is marked on the disk at a distance ℓ ($\ell < r$) from the center of the disk. At a general time t during rolling, find

1. the position of point P,
2. the velocity of point P, and
3. the acceleration of point P

Solution Let the disk be vertically below the center of the cylinder at $t = 0$ s such that point P is vertically above the center of the disk (Fig. 17.44). At this instant, Q is the point of contact between the disk and the cylinder. Let the disk roll for time t such that at instant t the line joining the two centers (line OC) makes an angle ϕ with its vertical position at $t = 0$ s. Since the disk has rolled for time t at a constant angular speed ω, point P has rotated counter-clockwise by an angle $\theta = \omega t$ from its original vertical position P'.

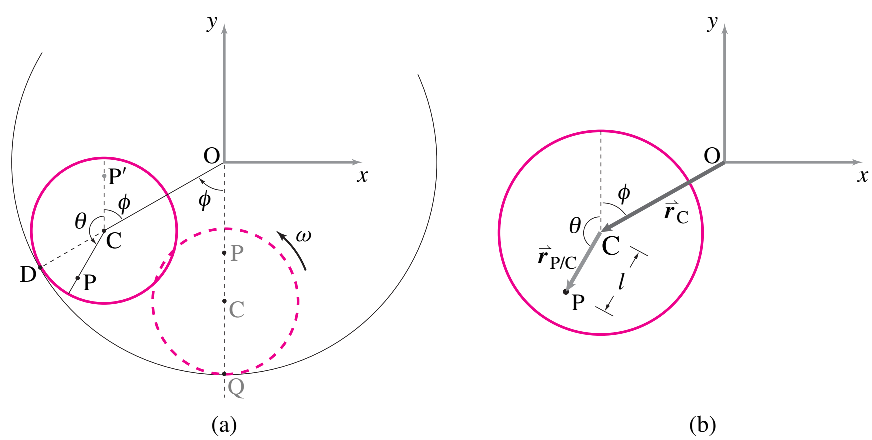


Figure 17.44: Geometry of motion: keeping track of point P while the disk rolls for time t, rotating by angle $\theta = \omega t$ inside the cylinder.

1. **Position of point P:** From Fig. 17.44(b) we can write

$$\vec{\boldsymbol{r}}_P = \vec{\boldsymbol{r}}_C + \vec{\boldsymbol{r}}_{P/C} = (R - r)\hat{\boldsymbol{\lambda}}_{OC} + \ell\hat{\boldsymbol{\lambda}}_{CP}$$

where

$$\begin{aligned} \hat{\boldsymbol{\lambda}}_{OC} &= \text{a unit vector along OC } = -\sin\phi\hat{\boldsymbol{\imath}} - \cos\phi\hat{\boldsymbol{\jmath}}, \\ \hat{\boldsymbol{\lambda}}_{CP} &= \text{a unit vector along CP } = -\sin\theta\hat{\boldsymbol{\imath}} + \cos\theta\hat{\boldsymbol{\jmath}}. \end{aligned}$$

Thus,

$$\vec{\boldsymbol{r}}_P = [-(R - r)\sin\phi - \ell\sin\theta]\hat{\boldsymbol{\imath}} + [-(R - r)\cos\phi + \ell\cos\theta]\hat{\boldsymbol{\jmath}}.$$

We have thus obtained an expression for the position vector of point P as a function of ϕ and θ. Since we also want to find velocity and acceleration of point P, it will be nice to express $\vec{\boldsymbol{r}}_P$ as a function of t. As noted above, $\theta = \omega t$; but how do we find ϕ as a function of t? Note that the center of the disk C is going around point O in circles with angular velocity $-\dot{\phi}\hat{\boldsymbol{k}}$. The disk, however, is rotating with angular velocity $\vec{\boldsymbol{\omega}} = \omega\hat{\boldsymbol{k}}$ about the instantaneous center of rotation, point D. Therefore, we can calculate the velocity of point C in two ways:

$$
\begin{array}{rrcl}
 & \vec{\boldsymbol{v}}_C & = & \vec{\boldsymbol{v}}_C \\
\text{or} & \vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}}_{C/D} & = & -\dot{\phi}\hat{\boldsymbol{k}} \times \vec{\boldsymbol{r}}_{C/O} \\
\text{or} & \omega\hat{\boldsymbol{k}} \times r(-\hat{\boldsymbol{\lambda}}_{OC}) & = & -\dot{\phi}\hat{\boldsymbol{k}} \times (R-r)\hat{\boldsymbol{\lambda}}_{OC} \\
\text{or} & -\omega r(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{\lambda}}_{OC}) & = & -\dot{\phi}(R-r)(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{\lambda}}_{OC}) \\
\Rightarrow & \dfrac{r}{R-r}\omega & = & \dot{\phi}.
\end{array}
$$

Integrating the last expression with respect to time, we obtain

$$\phi = \frac{r}{R-r}\omega t.$$

Let

$$q = \frac{r}{R-r},$$

then, the position vector of point P may now be written as

$$\vec{\boldsymbol{r}}_P = [-(R-r)\sin(q\omega t) - \ell\sin(\omega t)]\hat{\boldsymbol{\imath}} + [-(R-r)\cos(q\omega t) + \ell\cos(\omega t)]\hat{\boldsymbol{\jmath}}. \quad (17.37)$$

2. **Velocity of point P:** Differentiating Eqn. (17.37) once with respect to time we get

$$\vec{\boldsymbol{v}}_P = -\omega[(R-r)q\cos(q\omega t) + \ell\cos(\omega t)]\hat{\boldsymbol{\imath}} + \omega[(R-r)q\sin(q\omega t) - \ell\sin(\omega t)]\hat{\boldsymbol{\jmath}}.$$

Substituting $(R-r)q = r$ in $\vec{\boldsymbol{v}}_P$ we get

$$\vec{\boldsymbol{v}}_P = -\omega r[\{\cos(q\omega t) + \frac{\ell}{r}\cos(\omega t)\}\hat{\boldsymbol{\imath}} - \{\sin(q\omega t) - \frac{\ell}{r}\sin(\omega t)\}\hat{\boldsymbol{\jmath}}]. \quad (17.38)$$

3. **Acceleration of point P:** Differentiating Eqn. (17.38) once with respect to time we get

$$\vec{\boldsymbol{a}}_P = -\omega^2 r[-\{q\sin(q\omega t) + \frac{\ell}{r}\sin(\omega t)\}\hat{\boldsymbol{\imath}} - \{q\cos(q\omega t) - \frac{\ell}{r}\cos(\omega t)\}\hat{\boldsymbol{\jmath}}]. \quad (17.39)$$

$$
\begin{array}{rcl}
\vec{\boldsymbol{r}}_P & = & [-(R-r)\sin(q\omega t) - \ell\sin(\omega t)]\hat{\boldsymbol{\imath}} + [-(R-r)\cos(q\omega t) + \ell\cos(\omega t)]\hat{\boldsymbol{\jmath}} \\
\vec{\boldsymbol{v}}_P & = & -\omega r[\{\cos(q\omega t) + \frac{\ell}{r}\cos(\omega t)\}\hat{\boldsymbol{\imath}} - \{\sin(q\omega t) - \frac{\ell}{r}\sin(\omega t)\}\hat{\boldsymbol{\jmath}}] \\
\vec{\boldsymbol{a}}_P & = & -\omega^2 r[-\{q\sin(q\omega t) + \frac{\ell}{r}\sin(\omega t)\}\hat{\boldsymbol{\imath}} - \{q\cos(q\omega t) - \frac{\ell}{r}\cos(\omega t)\}\hat{\boldsymbol{\jmath}}]
\end{array}
$$

SAMPLE 17.14 The rolling disk: instantaneous kinematics. For the rolling disk in Sample 17.13, let $R = 4\,\text{ft}$, $r = 1\,\text{ft}$ and point P be on the rim of the disk. Assume that at $t = 0$, the center of the disk is vertically below the center of the cylinder and point P is on the vertical line joining the two centers. If the disk is rolling at a constant speed $\omega = \pi\,\text{rad/s}$, find

1. the position of point P and center C at $t = 1\,\text{s}$, $3\,\text{s}$, and $5.25\,\text{s}$,
2. the velocity of point P and center C at those instants, and
3. the acceleration of point P and center C at the same instants as above.

Draw the position of the disk at the three instants and show the velocities and accelerations found above.

Solution The general expressions for position, velocity, and acceleration of point P obtained in Sample 17.13 can be used to find the position, velocity, and acceleration of any point on the disk by substituting an appropriate value of ℓ in equations (17.37), (17.38), and (17.39). Since $R = 4r$,

$$q = \frac{r}{R-r} = \frac{1}{3}.$$

Now, point P is on the rim of the disk and point C is the center of the disk. Therefore,

$$\begin{aligned} \text{for point P:} \qquad \ell &= r, \\ \text{for point C:} \qquad \ell &= 0. \end{aligned}$$

Substituting these values for ℓ, and $q = 1/3$ in equations (17.37), (17.38), and (17.39) we get the following.

1. **Position:**

$$\begin{aligned} \vec{\boldsymbol{r}}_C &= -3r\left[\sin\left(\frac{\omega t}{3}\right)\hat{\boldsymbol{\imath}} + \cos\left(\frac{\omega t}{3}\right)\hat{\boldsymbol{\jmath}}\right], \\ \vec{\boldsymbol{r}}_P &= \vec{\boldsymbol{r}}_C + r\left[-sin\left(\omega t\right)\hat{\boldsymbol{\imath}} + \cos\left(\omega t\right)\hat{\boldsymbol{\jmath}}\right]. \end{aligned}$$

2. **Velocity:**

$$\begin{aligned} \vec{\boldsymbol{v}}_C &= -\omega r\left[\cos\left(\frac{\omega t}{3}\right)\hat{\boldsymbol{\imath}} - \sin\left(\frac{\omega t}{3}\right)\hat{\boldsymbol{\jmath}}\right], \\ \vec{\boldsymbol{v}}_P &= -\omega r\left[\left\{\cos\left(\frac{\omega t}{3}\right) + \cos\left(\omega t\right)\right\}\hat{\boldsymbol{\imath}} - \left\{\sin\left(\frac{\omega t}{3}\right) - \sin\left(\omega t\right)\right\}\hat{\boldsymbol{\jmath}}\right]. \end{aligned}$$

3. **Acceleration:**

$$\begin{aligned} \vec{\boldsymbol{a}}_C &= \frac{\omega^2 r}{3}\left[\sin\left(\frac{\omega t}{3}\right)\hat{\boldsymbol{\imath}} + \cos\left(\frac{\omega t}{3}\right)\hat{\boldsymbol{\jmath}}\right], \\ \vec{\boldsymbol{a}}_P &= \omega^2 r\left[\left\{\frac{1}{3}\sin\left(\frac{\omega t}{3}\right) + \sin\left(\omega t\right)\right\}\hat{\boldsymbol{\imath}} + \left\{\frac{1}{3}\cos\left(\frac{\omega t}{3}\right) - \cos\left(\omega t\right)\right\}\hat{\boldsymbol{\jmath}}\right]. \end{aligned}$$

We can now use these expressions to find the position, velocity, and acceleration of the two points at the instants of interest by substituting $r = 1\,\text{ft}$, $\omega = \pi\,\text{rad/s}$, and appropriate values of t. These values are shown in Table 17.1.

The velocity and acceleration of the two points are shown in Figures 17.45(a) and (b) respectively.

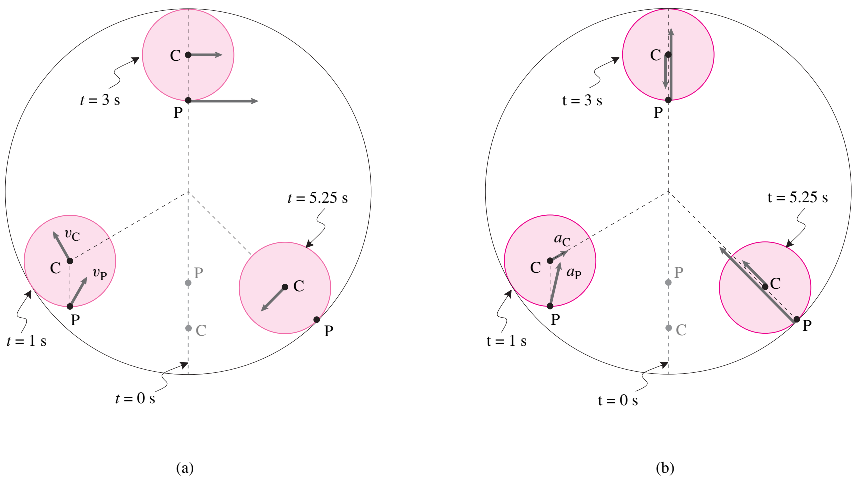


Figure 17.45: (a) Velocity and (b) Acceleration of points P and C at $t = 1\,\text{s},\ 3\,\text{s}$, and 5.25 s.

It is worthwhile to check the directions of velocities and the accelerations by thinking about the velocity and acceleration of point P as a vector sum of the velocity (same for acceleration) of the center of the disk and the velocity (same for acceleration) of point P with respect to the center of the disk. Since the motions involved are circular motions at constant rate, a visual inspection of the velocities and the accelerations is not very difficult. Try it.

t	1 s	3 s	5.25 s
$\vec{\boldsymbol{r}}_C$ (ft)	$3(-\frac{\sqrt{3}}{2}\hat{\boldsymbol{\imath}} - \frac{1}{2}\hat{\boldsymbol{\jmath}})$	$3\hat{\boldsymbol{\jmath}}$	$3(\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} - \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}})$
$\vec{\boldsymbol{r}}_P$ (ft)	$\vec{\boldsymbol{r}}_C - \hat{\boldsymbol{\jmath}}$	$\vec{\boldsymbol{r}}_C - \hat{\boldsymbol{\jmath}}$	$4(\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} - \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}})$
$\vec{\boldsymbol{v}}_C$ (ft/s)	$\pi(-\frac{1}{2}\hat{\boldsymbol{\imath}} + \frac{\sqrt{3}}{2}\hat{\boldsymbol{\jmath}})$	$\pi\hat{\boldsymbol{\imath}}$	$\pi(-\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} - \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}})$
$\vec{\boldsymbol{v}}_P$ (ft/s)	$\pi(\frac{1}{2}\hat{\boldsymbol{\imath}} + \frac{\sqrt{3}}{2}\hat{\boldsymbol{\jmath}})$	$2\pi\hat{\boldsymbol{\imath}}$	$\vec{\boldsymbol{0}}$
$\vec{\boldsymbol{a}}_C$ (ft/s^2)	$\frac{\pi^2}{3}(\frac{\sqrt{3}}{2}\hat{\boldsymbol{\imath}} + \frac{1}{2}\hat{\boldsymbol{\jmath}})$	$-\frac{\pi^2}{3}\hat{\boldsymbol{\jmath}}$	$\frac{\pi^2}{3}(-\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}})$
$\vec{\boldsymbol{v}}_P$ (ft/s^2)	$11.86(.24\hat{\boldsymbol{\imath}} + .97\hat{\boldsymbol{\jmath}})$	$\frac{2\pi^2}{3}\hat{\boldsymbol{\jmath}}$	$13.16(-\frac{1}{\sqrt{2}}\hat{\boldsymbol{\imath}} + \frac{1}{\sqrt{2}}\hat{\boldsymbol{\jmath}})$

Table 17.1: Position, velocity, and acceleration of point P and point C

SAMPLE 17.15 The rolling disk: path of a point on the disk. For the rolling disk in Sample 17.13, take $\omega = \pi$ rad/s. Draw the path of a point on the rim of the disk for one complete revolution of the center of the disk around the cylinder for the following conditions:

1. $R = 8r$,
2. $R = 4r$, and
3. $R = 2r$.

Solution In Sample 17.13, we obtained a general expression for the position of a point on the disk as a function of time. By computing the position of the point for various values of time t up to the time required to go around the cylinder for one complete cycle, we can draw the path of the point. For the various given conditions, the variable that changes in Eqn. (17.37) is q. We can write a computer program to generate the path of any point on the disk for a given set of R and r. Here is a pseudocode to generate the required path on a computer according to Eqn. (17.37).

A pseudocode to plot the path of a point on the disk:

```
(pseudo-code) program rollingdisk
%-------------------------------------------------------------
% This code plots the path of any point on a disk of radius
% 'r' rolling with speed 'w' inside a cylinder of radius 'R'.
% The point of interest is distance 'l' away from the center of
% the disk. The coordinates x and y of the specified point P are
% calculated according to the relation mentioned above.
%--------------------------------------------------------------
phi = pi/50*[1,2,3,...,100]   % make a vector phi from 0 to 2*pi
x = R*cos(phi)                % create points on the outer cylinder
y = R*sin(phi)
plot y vs x                   % plot the outer cylinder
hold this plot                % hold to overlay plots of paths
q = r/(R-r)                   % calculate q.
T = 2*pi/(q*w)                % calculate time T for going around-
                              % the cylinder once at speed 'w'.
t = T/100*[1,2,3, ..., 100]   % make a time vector t from 0 to T-
                              % taking 101 points.

rcx = -(R-r)*(sin(q*w*t))     % find the x coordinates of pt. C.
rcy = -(R-r)*(cos(q*w*t))     % find the y coordinates of pt. C.
rpx = rcx-l*sin(w*t)          % find the x coordinates of pt. P.
rpy = rcy + l*cos(q*t)        % find the y coordinates of pt. P.
plot rpy vs rpx               % plot the path of P and the path
plot rcy vs rcx               % of C. For path of C
```

Once coded, we can use this program to plot the paths of both the center and the point P on the rim of the disk for the three given situations. Note that for any point on the rim of the disk $l = r$ (see Fig 17.44).

1. Let $R = 4$ units. Then $r = 0.5$ for $R = 8r$. To plot the required path, we run our program `rollingdisk` with desired input,

```
R = 4
r = 0.5
w = pi
l = 0.5
execute rollingdisk
```

   The plot generated is shown in Fig.17.46 with a few graphic elements added for illustrative purposes.

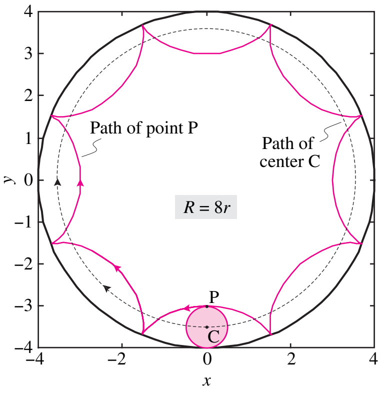


Figure 17.46: Path of point P and the center C of the disk for $R = 8r$.

2. Similarly, for $R = 4r$ we type:

```
R = 4
r = 1
w = pi
l = 1
execute rollingdisk
```

   to plot the desired paths. The plot generated in this case is shown in Fig.17.47

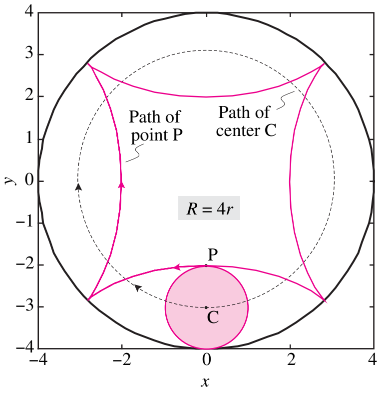


Figure 17.47: Path of point P and the center C of the disk for $R = 4r$.

3. The last one is the most interesting case. The plot obtained in this case by typing:

```
R = 4
r = 2
w = pi
l = 2
execute rollingdisk
```

   is shown in Fig.17.48. Point P just travels on a straight line! In fact, every point on the rim of the disk goes back and forth on a straight line. Most people find this motion odd at first sight. You can roughly verify the result by cutting a hole twice the diameter of a coin (say a US quarter or dime) in a piece of cardboard and rolling the coin around inside while watching a marked point on the perimeter.

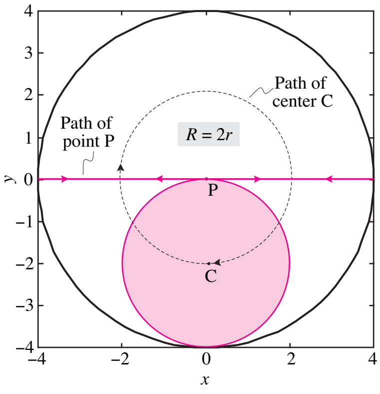


Figure 17.48: Path of point P and the center C of the disk for $R = 2r$.

A curiosity. We just discovered something simple about the path of a point on the edge of a circle rolling in another circle that is twice as big. The edge point moves in a straight line. In contrast one might think about the motion of the center G of a *straight* line segment that *slides* against two *straight* walls as in sample 17.23. A problem couldn't be more different. Naturally the path of point G is a circle (as you can check physically by looking at the middle of a ruler as you hold it sliding against a wall-floor corner).