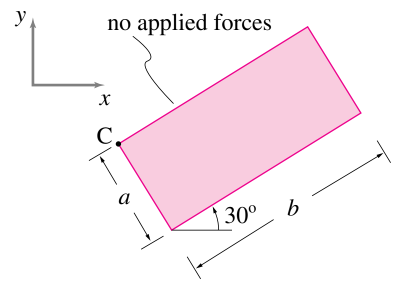
17.2.1 The uniform rectangle of width $a = 1\,\text{m}$, length $b = 2\,\text{m}$, and mass $m = 1\,\text{kg}$ in the figure is sliding on the xy-plane with no friction. At the moment in question, point C is at $x_C = 3\,\text{m}$ and $y_C = 2\,\text{m}$. The linear momentum is $\vec{L} = 4\hat{\imath} + 3\hat{\jmath}\ (\text{kg}\cdot\text{m/s})$ and the angular momentum about the center of mass is $\vec{H}_{cm} = 5\hat{k}(\text{kg}\cdot\text{m/s}^2)$. Find the acceleration of any point on the body that you choose. (Mark it.) [Hint: You have been given some redundant information.]



Problem 17.1

①

Chapter - 16

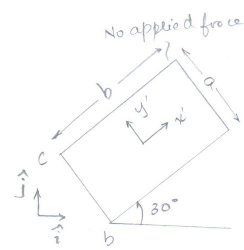
16.2.1

$a = 1\text{m};\ b = 2\text{m};\ m = 1\text{kg}$

$(x_c, y_c) \equiv (3\,\text{m}, 2\,\text{m})$

$\vec{L} = 4\hat{i} + 3\hat{j}\ [\text{kg}\cdot\text{m/sec}]$

$\vec{H}_{cm} = 5\hat{k}\ [\text{kg m/sec}^2]$



$\Rightarrow \sum \vec{F} = m\vec{a}_{cm} = 0 \Rightarrow \vec{a}_{cm} = 0$

$\vec{H}_{cm} = [I]^{cm}\vec{\omega} \Rightarrow 5\hat{k} = \frac{m}{2}(a^2+b^2)\vec{\omega}$

$\vec{\omega} = \frac{10}{4+1}\hat{k} = 2\hat{k}\ \text{rad/sec}$

$\dot{\vec{\omega}} = 0$ [No net torque on body]

Acceleration of an arbitrary point on plate

$\vec{a}_p = \vec{a}_{cm} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{P/cm})$

$\vec{a}_p = -\omega^2\vec{r}_{P/cm}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

2

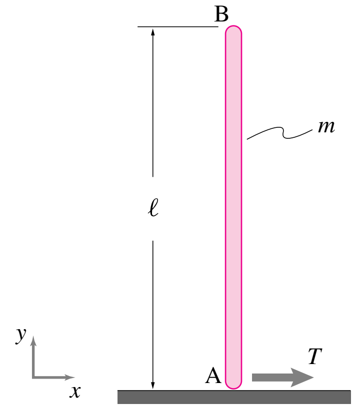
For point B

$$\vec{a}_b = -4 \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} -b/2 \\ -a/2 \end{bmatrix}$$

$$= -4 \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -1 \\ -1/2 \end{bmatrix}$$

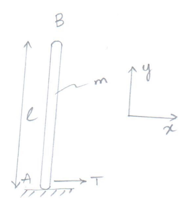
$$\vec{a}_b = -4 \left[\left(-\sqrt{3}/2 + 1/4 \right) \hat{i} + \left(-1/2 - \sqrt{3}/4 \right) \hat{j} \right] \ m/sec^2$$

17.2.2 The vertical pole AB of mass m and length ℓ is initially, *at rest* on a frictionless surface. A tension T is suddenly applied at A. What is $\ddot{x}_{cm}$? What is $\ddot{\theta}_{AB}$? What is $\ddot{x}_B$? Gravity may be ignored.



Problem 17.2

a) $\vec{F} = m\vec{a}_{cm}$



$$\vec{a}_{cm} = \left(\frac{T}{m}\right)\hat{i} = \ddot{x}_{cm}\hat{i} + \ddot{y}_{cm}\hat{j}$$

$$\ddot{x}_{cm} = \frac{T}{m}$$

b) $\vec{M}_{/A} = 0 = \vec{r}_{cm/A} \times m\vec{a}_{cm} + I_{cm}\dot{\vec{\omega}} + (\vec{\omega} \times I\vec{\omega})$

$$0 = \frac{\ell}{2}\hat{j} \times m\left(\frac{T}{m}\right)\hat{i} + \frac{m\ell^2}{12}\dot{\vec{\omega}}$$

$$\dot{\vec{\omega}} = \frac{6T}{m\ell}\hat{k}$$

$$\ddot{\theta}_{AB} = \frac{6T}{m\ell} \quad \text{rad/sec}^2$$

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3

$$\vec{a}_B = \vec{a}_{cm} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/cm}) + \dot{\vec{\omega}} \times \vec{r}_{B/cm}$$

$$= \left(\frac{T}{m}\right)\hat{i} + \left(\frac{6T}{m\ell}\right)\hat{k} \times \frac{\ell}{2}\hat{j}$$

$$= \frac{T}{m}\hat{i} + \left(\frac{-3T}{m}\right)\hat{i}$$

$$\vec{a}_B = -\frac{2T}{m}\hat{i}$$

$$\ddot{x}_B = -\frac{2T}{m} \quad m/sec^2$$

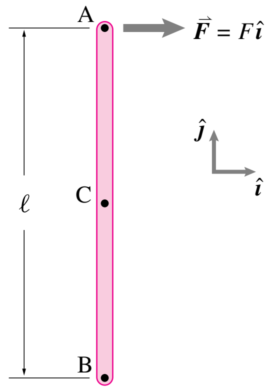
17.2.3 Force on a stick in space. 2-D . No gravity. A uniform thin stick with length ℓ and mass m is, at the instant of interest, parallel to the y axis and has no velocity and no angular velocity. The force $\vec{F} = F\hat{\imath}$ with $F > 0$ is suddenly applied at point A. The questions below concern the instant after the force $\vec{F}$ is applied.

a) What is the acceleration of point C, the center of mass?

b) What is the angular acceleration of the stick?

c) What is the acceleration of the point A?

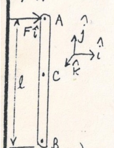
d) (relatively harder) What additional force would have to be applied to point B to make point B's acceleration zero?



Problem 17.3

Answer:
(a) $\vec{a}_{cm} = \frac{F}{m}\hat{\imath}$.
(b) $\vec{\alpha} = -\frac{6F}{m\ell}\hat{k}$.
(c) $\vec{a}_A = \frac{4F}{m}\hat{\imath}$.
(d) $\vec{F}_B = \frac{F}{2}\hat{\imath}$

7·11



stick; length ℓ; mass m
The instant after $\underset{\sim}{F} = F\hat{\imath}$ is applied,

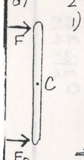
a) Find $\underset{\sim}{a}_{cm}$.
LMB:
$$\Sigma \underset{\sim}{F} = F\hat{\imath} = m_{tot}\underset{\sim}{a}_{cm}$$
$$\Rightarrow \boxed{\underset{\sim}{a}_{cm} = \frac{F}{m}}$$

b) Find $\underset{\sim}{\alpha}$.
AMB:
$$\Sigma \underset{\sim}{M}_{/cm} = \dot{\underset{\sim}{H}}_{/cm} \Rightarrow \frac{\ell}{2}\hat{\jmath} \times F\hat{\imath} = \cancel{\underset{\sim}{\omega} \times I^{cm}_{zz}\underset{\sim}{\omega}}^{\,0 \text{ in 2d}} + I^{cm}_{zz}\dot{\underset{\sim}{\omega}}$$
note that $I^{cm}_{zz} = \frac{m\ell^2}{12}$
$$\Rightarrow -\frac{F\ell}{2}\hat{k} = \frac{m\ell^2}{12}\dot{\underset{\sim}{\omega}} \Rightarrow \boxed{\dot{\underset{\sim}{\omega}} = \frac{-6F}{m\ell}\hat{k}}$$

c) Find $\underset{\sim}{a}_A$:
$$\underset{\sim}{a}_A = \underset{\sim}{a}_{cm} + \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r}_{A/cm} - \cancel{\dot{\theta}^2 \underset{\sim}{r}_{A/cm}}^{\,0}$$
since $\dot{\theta} = 0$ at this instant

7·11 continued P.8
$$\underset{\sim}{a}_A = \frac{F}{m}\hat{\imath} + \left(\frac{-6F}{m\ell}\hat{k} \times \frac{\ell}{2}\hat{\jmath}\right) = \left(\frac{F}{m} + \frac{3F}{m}\right)\hat{\imath} =$$
$$\boxed{\underset{\sim}{a}_A = \frac{4F}{m}\hat{\imath}}$$

d) 2 methods:
1) Brute force: (essentially same calcs. as in a)-c) above)

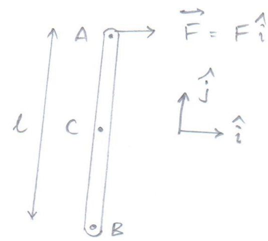


LMB:
$$\Sigma \underset{\sim}{F} = \dot{\underset{\sim}{L}} : (F + F_B)\hat{\imath} = m\underset{\sim}{a}_{cm}$$
AMB:
$$\Sigma \underset{\sim}{M} = \dot{\underset{\sim}{H}} : \frac{\ell}{2}\hat{\jmath} \times F\hat{\imath} + \left(-\frac{\ell}{2}\hat{\jmath} \times F_B\hat{\imath}\right) = I^{cm}_{zz}\dot{\underset{\sim}{\omega}}$$
$$\Rightarrow \dot{\underset{\sim}{\omega}} = \frac{6}{m\ell}(F_B - F)\hat{k}$$
Find accel of pt. B & set equal to zero:
$$\underset{\sim}{a}_B = \underset{\sim}{0} = \underset{\sim}{a}_{cm} + \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r}_{B/cm} \quad \left(\underset{\sim}{r}_{B/cm} = -\frac{\ell}{2}\hat{\jmath}\right)$$
$$\underset{\sim}{0} = \frac{F + F_B}{m}\hat{\imath} + \frac{3}{m}(F - F_B)\hat{\imath}$$
$$\Rightarrow F + 3F = 3F_B - F_B \Rightarrow \boxed{F_B = \frac{F}{2}}$$

method 2: more clever
Use superposition. Note that $\underset{\sim}{a}_A$ is $\frac{4F}{m}\hat{\imath}$ when the force F is applied at A. Before we apply F_B at pt. B, then,
$$\underset{\sim}{a}_B = \underset{\sim}{a}_{cm} + \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r}_{B/cm} = \frac{F}{m}\hat{\imath} + \left(\frac{-6F}{m\ell}\hat{k} \times -\frac{\ell}{2}\hat{\jmath}\right)$$
$$= \left(\frac{F}{m} - \frac{3F}{m}\right)\hat{\imath} = -2F/m\,\hat{\imath}.$$
Which is 1/2 as large as $\underset{\sim}{a}_A$ ($4F/m$ vs. $2F/m$).
Thus, a force 1/2 as large as that being applied at pt. A is necessary for $\underset{\sim}{a}_B$ to equal 0.
$$\Rightarrow \boxed{\underset{\sim}{F}_B = \frac{F}{2}\hat{\imath}}$$
same as above.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

(i) Force Balance



$$\vec{F} = m\vec{a}_c$$

$$\vec{a}_c = \frac{F}{m}\hat{i}$$

$$\vec{a}_c = \frac{F}{m}\hat{i}$$

(ii) Momentum balance about point 'A' gives

$$\vec{\tau}_{/A} = \vec{M}_{/A} = 0 = \dot{\vec{H}}_{/A}$$

$$0 = \vec{r}_{cm/A} \times m\vec{a}_c + I_{cm}\dot{\vec{\omega}}$$

$$-\frac{\ell}{2}\hat{j} \times m\left(\frac{F}{m}\hat{i}\right) + \frac{m\ell^2}{12}\vec{\alpha} = 0$$

$$\vec{\alpha} = -\frac{6F}{m\ell}\hat{k} \ \ rad/sec^2$$

(iii) $\;{}^{f}\vec{a}_A = {}^{f}\vec{a}_C + {}^{B}\vec{a}_{A/C} \qquad = \vec{a}_c + \dot{\vec{\omega}} \times \vec{r}_{A/C}$

$$= \frac{F}{m}\hat{i} - \frac{6F}{m\ell} \times \frac{\ell}{2}\,(-\hat{i})$$

$$\vec{a}_A = \frac{4F}{m}\hat{i} \quad m/sec^2$$

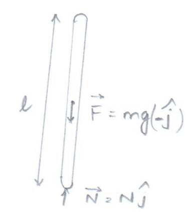
17.2.4 A uniform thin rod of length ℓ and mass m stands vertically, with one end resting on a frictionless surface and the other held by someone's hand. The rod is released from rest, displaced slightly from the vertical. No forces are applied during the release. There is gravity.

a) Find the path of the center of mass.

b) Find the force of the floor on the end of the rod just before the rod is horizontal.

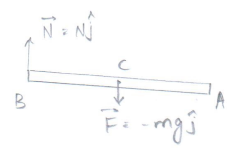
a) Path of centre of Mass

$$\vec{a}_{cm} = -g\hat{j} + \frac{N}{m}\hat{j}$$



So, centre of mass moves on straight line along $-\hat{j}$ ie falls straight down.

b) Normal force will act such that $(\vec{a}_B)_v = 0$, momentum balance just before rod becomes horizontal.



$$\vec{M}_{/c} = -\frac{\ell}{2}\hat{i} \times N\hat{j} = -\frac{N\ell}{2}\hat{k}$$

$$= \dot{\vec{H}}_{/c} = \frac{m\ell^2}{12}\dot{\vec{\omega}}$$

$$\dot{\vec{\omega}} = -\frac{6N}{m\ell}\hat{k}$$

$$\vec{a}_B = 0 = \vec{a}_{cm} + \dot{\vec{\omega}} \times \vec{r}_{B/cm} = -g\hat{j} + \frac{N}{m}\hat{j} + \left(-\frac{6N}{m\ell}\hat{k}\right) \times \left(-\frac{\ell}{2}\hat{i}\right)$$

$$-g\hat{j} + \frac{N}{m}\hat{j} + \frac{3N}{m}\hat{j} = 0$$

Normal force $\vec{N} = \frac{mg}{4}\hat{j}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.2.5 A uniform disk, with mass center labeled as point G, is sitting motionless on the frictionless xy plane. A massless peg is attached to a point on its perimeter. This disk has radius of 1 m and mass of 10 kg. A constant force of $F = 1000\,\text{N}\hat{\imath}$ is applied to the peg for .0001 s (one ten-thousandth of a second).

a) What is the velocity of the center of mass of the center of the disk after the force is applied?

b) Assuming that the idealizations named in the problem statement are exact is your answer to (a) exact or approximate?

c) What is the angular velocity of the disk after the force is applied?

d) Assuming that the idealizations named in the problem statement are exact is your answer to (c) exact or approximate?

5

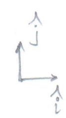
16-2-5

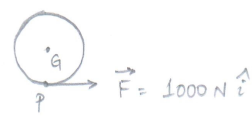
$r = 1\,m$

$m = 10\,kg$

$\vec{F} = 1000\,N\,\hat{i}$

$\Delta t = 0.0001$ sec





a) $\vec{a}_{cm} = \dfrac{F}{m} = 100\,\hat{i}\ m/sec^2$

$\dfrac{\Delta \vec{V}_{cm}}{\Delta t} = 100\,\hat{i}\ m/sec^2 \Rightarrow \vec{V}_{cm} = 100 \times 10^{-4}\,\hat{i}\ m/sec$

$\vec{V}_{cm} = 10^{-2}\,\hat{i}\ m/sec$

b) exact

c) moment balance about G:

$\vec{r}_{P/G} \times \vec{F} = I\,\dot{\vec{\omega}}$

Say $\vec{r}_{P/G} = -r\hat{j}$

$\dot{\vec{\omega}} = \dfrac{-r\hat{j} \times 1000\,\hat{i}}{mr^2/2} = 200\hat{k}\ rad/sec^2$

$\dfrac{\Delta\vec{\omega}}{\Delta t} = 200\hat{k}$; $\vec{\omega}_{final} = 2\times10^{-2}\,\hat{k}\ rad/sec$

d) Approximate:

Since $F = 1000\,\hat{i}$; it always acts on point 'p' during that time interval, the torque changes with the change in orientation and the Angular acceleration is not constant.

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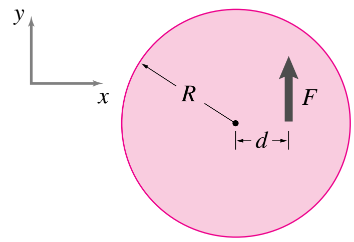
17.2.6 A uniform thin flat disc is floating in space. It has radius R and mass m. A force F is applied to it at a distance d from the center in the y direction. Treat this problem as two-dimensional.

a) What is the acceleration of the center of the disc?

b) What is the angular acceleration of the disk?

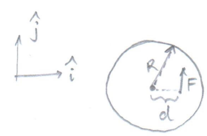


Problem 17.6

Answer:
(a) $\vec{a} = \frac{F}{m}\hat{\boldsymbol{j}}$.
(b) $\vec{\alpha} = \frac{Fd}{m\frac{R^2}{2}}\hat{\boldsymbol{k}}$.

6

16.2.6



a) $\vec{a}_c = ?$

$\sum \vec{F} = m\vec{a}_c$

$\vec{a}_c = \frac{F}{m}\hat{j}$

b) Moment balance about 'C':

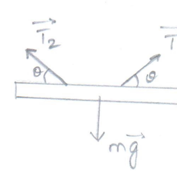
$\vec{M}/_c = \vec{r}_{P/c} \times \vec{F} = I\dot{\vec{\omega}}$

$d\hat{i} \times F\hat{j} = \frac{mr^2}{2}\dot{\vec{\omega}}$

$\dot{\vec{\omega}} = \frac{2Fd}{mr^2}\hat{k}$

16.2.8

a) FBD



$\vec{T}_1 = T\cos\theta\,\hat{i} + T\sin\theta\,\hat{j}$

$\vec{T}_2 = -T\cos\theta\,\hat{i} + T\sin\theta\,\hat{j}$

$\vec{F}_G = -mg\,\hat{j}$

$\Rightarrow \sum \vec{F} = (2T\sin\theta - mg)\,\hat{j}$

b) After the string is cut

$\sum \vec{F} = T\cos\theta\,\hat{i} + T\sin\theta\,\hat{j} - mg\,\hat{j}$

$\sum \vec{F} = T\cos\theta\,\hat{i} + (T\sin\theta\cancel{\hat{j}} - mg)\,\hat{j}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

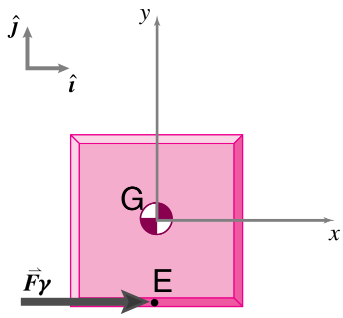
17.2.7 A uniform 1kg plate that is one meter on a side is initially at rest in the position shown. A constant force $\vec{F} = 1\,\text{N}\hat{\imath}$ is applied at $t = 0$ and maintained henceforth. If you need to calculate any quantity that you don't know, but can't do the calculation to find it, assume that the value is given.

a) Find the position of G as a function of time (the answer should have numbers and units).

b) Find a differential equation, and initial conditions, that when solved would give θ as a function of time. θ is the counterclockwise rotation of the plate from the configuration shown.

c) Write computer commands that would generate a drawing of the outline of the plate at $t = 1$ s. You can use hand calculations or the computer for as many of the intermediate commands as you like. Hand work and sketches should be provided as needed to justify or explain the computer work.

d) Run your code and show clear output with labeled plots. Mark output by hand to clarify any points.



Problem 17.7

10:10AM Wed section 205
TA : Pranav Bhounsule

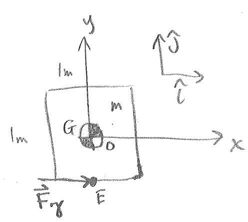
Alan Argondizza
TAM 2030
HW 17 Due 3/26/09

14.12, 14.19, 14.21, 14.31

10/10

$\boxed{14.12}$ See $\boxed{\text{SOLUTION}}$

$\boxed{14.19}$ ✓



$\vec{F} = 1\text{N}\hat{\imath}$
$m = 1\text{kg}$

a) LMB: $\sum\vec{F} = m\vec{a}_G$

$\vec{a}_G = \frac{\vec{F}}{m}$

$\left\{ \vec{a}_G = \frac{1\text{N}\hat{\imath}}{1\text{kg}} = a_x\hat{\imath} + a_y\hat{\jmath} \right\}$

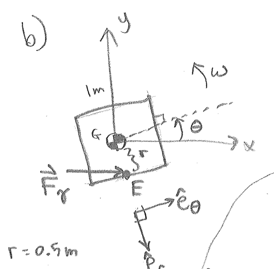
$\{..\}\cdot\hat{\imath} \Rightarrow \; 1\,\text{m/s}^2 = a_x = \ddot{x}$

Position:

$\vec{r}_{G/O} = x\hat{\imath} = (\cancel{x_0}^{0} + \cancel{\dot{x}_0}^{0} + \frac{1}{2}\ddot{x}t^2)\hat{\imath}$

$$\boxed{\vec{r}_{G/O} = \frac{t^2}{2}\hat{\imath}\ \text{m}}$$

b)



$r = 0.5\text{m}$

Pretend G is fixed:

AMB $\sum\vec{M}_{/G} = \dot{\vec{H}}_{/G}$

$I_{cm} = \frac{m}{12}(a^2+b^2) = \frac{m}{12}(1^2+1^2)$

$= \frac{m}{6} = \underline{\frac{1}{6} = I_{cm}}$

$r(\vec{F}_\gamma\cdot\hat{e}_\theta)\hat{k} = I^{cm}\dot{\vec{\omega}} + \cancel{\vec{\omega}\times(I^{cm}\vec{\omega})}^{0}$

$\{ rF_\gamma\cos\theta\,\hat{k} = I_{cm}\ddot{\theta}\hat{k} \}$

$\{..\}\cdot\hat{k} \Rightarrow \; rF_\gamma\cos\theta = I_{cm}\ddot{\theta} \Rightarrow (\frac{1}{2})(1)\cos\theta = \frac{1}{6}\cdot\ddot{\theta}$

OR

$\vec{r}_{E/G}\times\vec{F}_r$
$= r\hat{e}_r\times\{F_r\cos\theta\,\hat{e}_\theta + F_r\sin\theta\,\hat{e}_r\}$
$= rF_r\cos\theta\,\hat{k}$

$$\boxed{\ddot{\theta} = 3\cos\theta}$$

$$\boxed{\begin{array}{l}\theta|_{t=0} = 0\\ \dot{\theta}|_{t=0} = 0\end{array}}$$

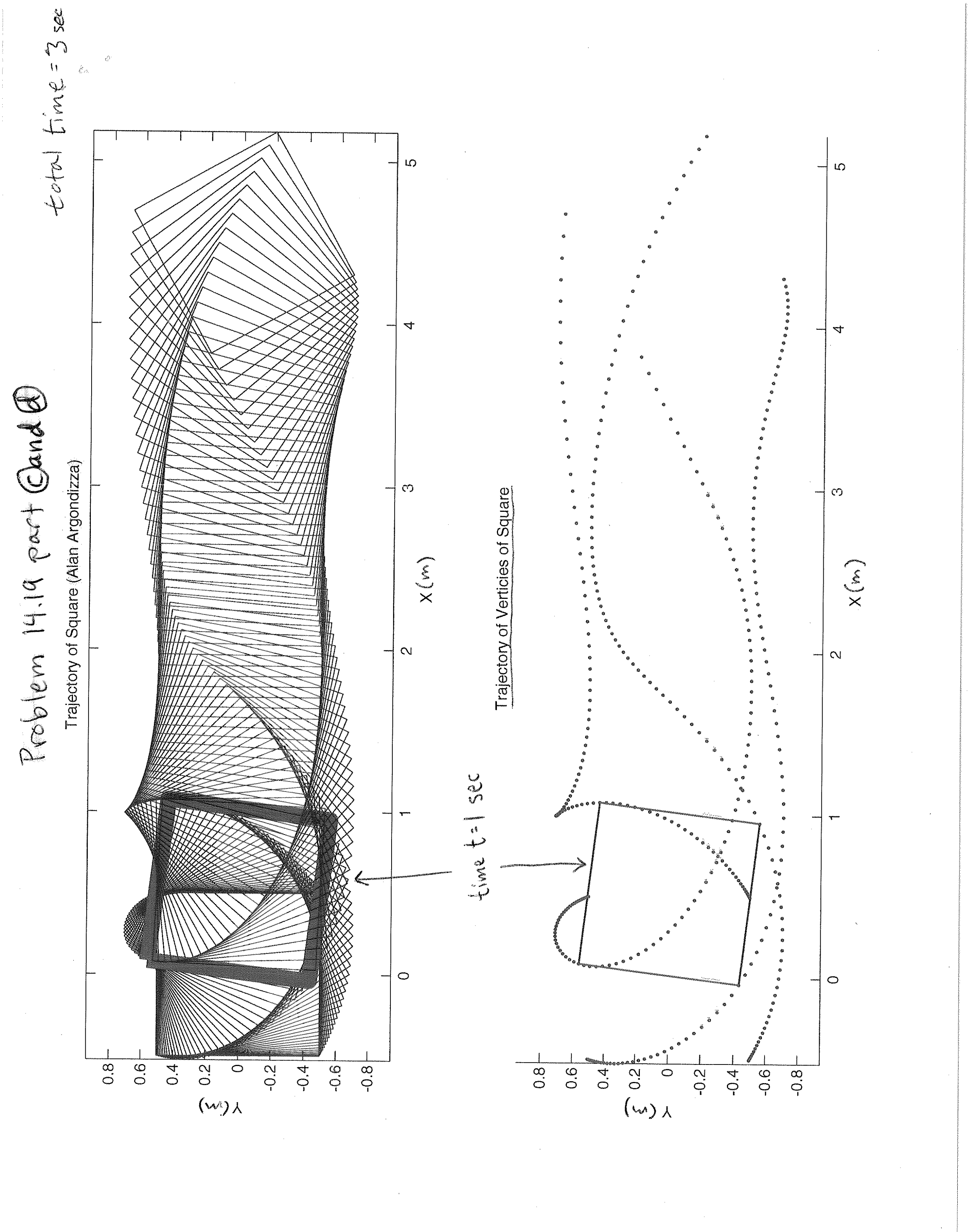
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

```
function square_plate
% Alan Argondizza
% Solution to 14.19 part c and d

% Initial conditions and time span
time= 3;
tspan= linspace(0,time,101);  %Integrate for time seconds
z0 = [0,0]';                  % initial [angle,omega] both zero

% solve the ODE:
[t,z] = ode45(@rhs, tspan, z0);

% Unpack the variables
theta = z(:,1);      %first column of z
thetadot = z(:,2);   %second column of z

clf
tag=0

%plot using a loop:
for i= 1:1:length(t)

    %entire square:
    subplot(2,1,1)
    %create initial square:
    square= [.5,-.5,-.5,.5,.5;.5,.5,-.5,-.5,.5];
    %create rotation matrix:
    R= [cos(theta(i)),-sin(theta(i));...
        sin(theta(i)),cos(theta(i))];
    %determine displacement of G:
    xdisp= .5*t(i)^2;
    rotatedsquare= R*square + [xdisp,xdisp,xdisp,xdisp,xdisp;0,0,0,0,0];

    plot(rotatedsquare(1,:),rotatedsquare(2,:));

    %this conditional marks the square at time t= 1 second:
    if floor(t(i)) == 1
        if tag ~= 55
            line(rotatedsquare(1,:),rotatedsquare(2,:),'LineWidth',10,'Color','red');
        end
    end
    title('Trajectory of Square (Alan Argondizza)');
    xlabel('X');
    ylabel('Y');
    axis('equal');
    hold on

    %verticicies of square:
    subplot(2,1,2)
    line(rotatedsquare(1,:),rotatedsquare↙
(2,:),'LineStyle','none','Color','red','Marker','.');
```

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

```
    %this conditional marks the square at time t= 1 second:
    if floor(t(i)) == 1
        if tag ~= 55
            line(rotatedsquare(1,:),rotatedsquare(2,:),'LineWidth',1,'Color','red');
        end
        tag=55;
    end
    title('Trajectory of Verticies of Square');
    xlabel('X');
    ylabel('Y');
    axis('equal');
    hold on
end
end

function zdot = rhs(t,z)

theta = z(1);       % unpack z into readable variables
thetadot = z(2);

%RHS:
omega = thetadot;
omegadot = 3*cos(theta);
% pack up the derivatives:
z1dot = omega;
z2dot = omegadot;
%function return:
zdot = [z1dot z2dot]';
end
```

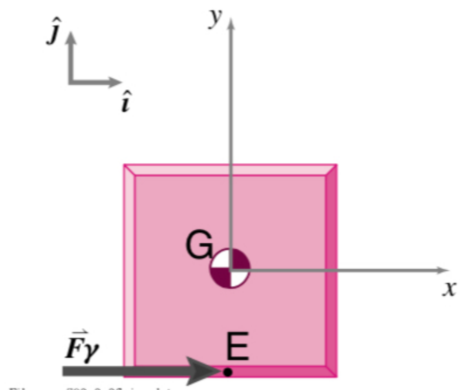
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*



Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1 Trajectory of Plate

Given:



$$L = 1m$$

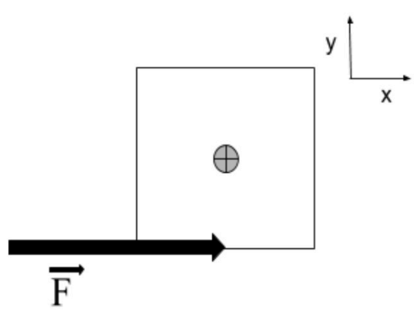
$$m = 1Kg$$

$$\vec{F} = 1N\hat{\imath}$$

@ t = 0s

a) Find $\vec{r}_G(t)$ (Position as a function of time)

FBD



LMB: $\sum \vec{F} = m\vec{a}$; $\vec{F}_\gamma = m(a_x\hat{\imath} + a_y\hat{\jmath})$

dot with $\hat{\imath}$:

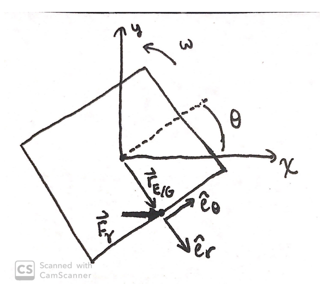
$$1N = ma_x$$

$$a_x = 1\frac{m}{s^2}$$
$$\ddot{x} = a_x = 1\frac{m}{s^2}$$
$$\dot{x} = \int_0^t \ddot{x}(t')dt' + \dot{x}(0) = t - 0 + 0 = t\frac{m}{s}$$
$$x = \int_0^t \dot{x}(t')dt' + x(0) = \frac{t^2}{2} - 0 + 0 = \frac{t^2}{2}m$$
$$\vec{x}(t) = (\frac{t^2}{2}\hat{i})m$$

dot with $\hat{j}$:

$$0N = ma_y$$
$$a_y = 0\frac{m}{s^2}$$
$$\dot{y}(0) = 0\frac{m}{s}; y(0) = 0m$$
$$\vec{r_G}(t) = \vec{x}(t) = \frac{t^2}{2}\hat{i}m$$

b) Find) $\theta(t)$
Here we are observing the plate with respect to the Center of Mass (i.e. pretend G is fixed)



AMB:

$$\sum \vec{M}_G = \dot{\vec{H}}$$
$$\vec{r}_{\frac{E}{G}} \times \vec{F}_\gamma = I_G\alpha\hat{k}$$
$$\vec{r}_{\frac{E}{G}} = (.5\hat{e}_r + 0\hat{e}_\theta) = .5\hat{e}_r; r_{\frac{E}{G}} = ||\vec{r}_{\frac{E}{G}}|| = .5m$$
$$\vec{r}_{\frac{E}{G}} \times \vec{F}_\gamma = (\vec{r}_{\frac{E}{G}} \cdot \hat{e}_r)(\vec{F}_\gamma \cdot \hat{e}_\theta\hat{k}) = I_G\ddot{\theta}$$

	$\hat{e}_r$	$\hat{e}_\theta$
$\hat{i}$	$sin(\theta)$	$cos(\theta)$
$\hat{j}$	$-cos(\theta)$	$sin(\theta)$

$$I_G = \frac{m}{12}(a^2 + b^2) = \frac{mL^2}{6}$$

$$F_\gamma = ||\vec{F}_{gamma}||$$

$$r_{\frac{E}{G}} F_\gamma cos(\theta)\hat{k} = I_G \ddot{\theta}\hat{k}$$

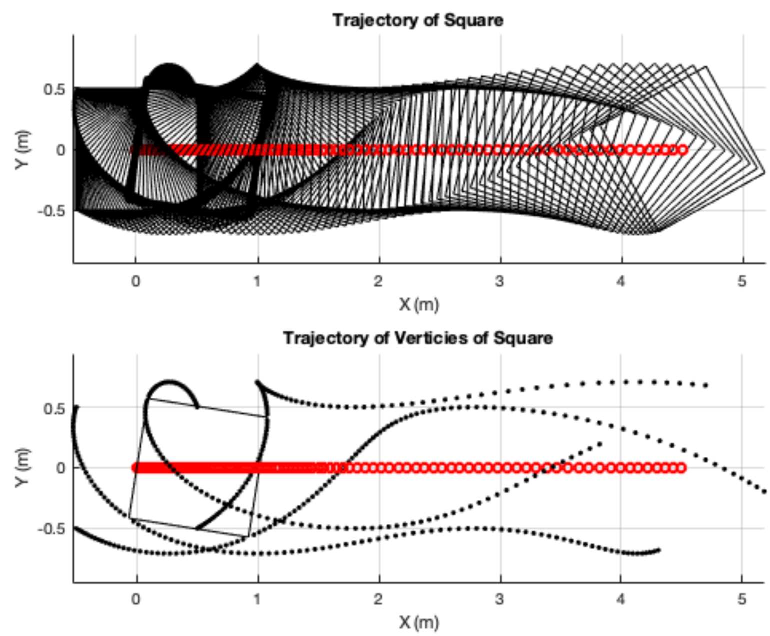
$$\ddot{\theta}\hat{k} = \frac{6r_{\frac{E}{G}} F_\gamma cos(\theta)}{mL^2}\hat{k}$$

dot with $\hat{k}$:

$$\ddot{\theta} = 3cos(\theta)\frac{rad}{s^2}$$

```
function P_16_2_7()
close all;clear all;clc;
% Problem 16.2.7 Part C & D

%Define Parameters
P = struct('L',1,'m',1,'F',1);
        %Length, Mass, Force

%Set time
tspan = [0 3];

%Initial Conditions
z0 = [0 0];

%Options
Opts = odeset('AbsTol',1e-7,'RelTol',1e-7);

%Solve the ODE:
f = @(t,z) myrhs(t,z,P);
[t,z] = ode45(f,tspan,z0,Opts);

%unpack variables
th = z(:,1);
thd = z(:,2);

%Calculate COM Positions over time
X = (P.F/P.m)*.5*t.^2;
Y = 0;

%Initial Square
squareX0 = [-P.L P.L P.L -P.L -P.L]/2;
squareY0 = [-P.L -P.L P.L P.L -P.L]/2;
Square0 = [squareX0;squareY0];

%Find index at which t is closest to 1 (t will be >= 1)
idx = find(t>=1,1);

%Format Figure
subplot(2,1,1)
title('Trajectory of Square')
xlabel('X (m)')
ylabel('Y (m)')
grid on
axis equal
subplot(2,1,2)
title('Trajectory of Verticies of Square')
xlabel('X (m)')
ylabel('Y (m)')
grid on
axis equal
for i = 1:length(t)
    %Create Rotation Matrix:
```

```
        R = [cos(th(i)), -sin(th(i));...
             sin(th(i)), cos(th(i))];

        %Determine Square EndPoint Locations
        RotatedSquare = R*Square0 + X(i)*[ones(1,5);zeros(1,5)];

        %Highlight the square at t=1 s
        if i ~= idx
            %Plot Trajetory of Square and Trajetory of Vertices
            subplot(2,1,1)
            hold on
            plot(RotatedSquare(1,:), RotatedSquare(2,:),'k') %Plot Plate
            plot(X(i),Y,'ro','LineWidth',1) %Plot G
            subplot(2,1,2)
            hold on
            plot(RotatedSquare(1,:), RotatedSquare(2,:),'k.') %Plot Plate
            plot(X(i),Y,'ro','LineWidth',1) %Plot G
        else
            %Highlight the square at t=1
            subplot(2,1,1)
            hold on
            %Plot Plate at t = 1s
            plot(RotatedSquare(1,:),
   RotatedSquare(2,:),'k','LineWidth',5);
            %Plot G
            plot(X(i),Y,'ro','LineWidth',1)
            subplot(2,1,2)
            hold on
            plot(RotatedSquare(1,:), RotatedSquare(2,:),'k')
            plot(X(i),Y,'ro','LineWidth',1)
        end
        %pause(.03) %uncomment to animate
    end

    function zdot = myrhs(t,z,P)
        %unpack z
        theta = z(1);
        thetadot = z(2);

        %unpack parameters
        r = P.L/2;
        I_G = (P.m/12)*(2*P.L^2);
        F = P.F;

        %RHS
        omega = thetadot;
        omegadot = (r/I_G)*F*cos(theta);

        %Pack up the derviatives
        zdot = [omega;omegadot];
    end
    end
```



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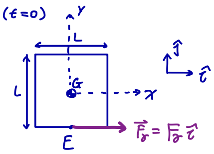
HW 10 17.2.7

Teaya Yang

Given: $(t=0)$



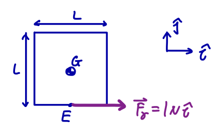
$L = 1m$

$m = 1\ kg$

$F_r = 1N$

a) Find $\vec{r}_G(t)$.

FBD:



LMB:

$$\Sigma \vec{F} = m\vec{a}$$
$$\{\vec{F}_r = m(\ddot{x}_G \hat{\imath} + \ddot{y}_G \hat{\jmath})\}$$
$$\{\ \} \cdot \hat{\imath} : \quad F_r = m\ddot{x}_G$$
$$\{\ \} \cdot \hat{\jmath} : \quad 0 = m\ddot{y}_G$$
$$\Rightarrow \begin{cases} \ddot{x}_G = \frac{F_r}{m} \\ \ddot{y}_G = 0 \end{cases}$$

Solving for x_G, y_G:

$$\ddot{x}_G = \frac{F_r}{m} = \frac{d}{dt}\dot{x}_G$$
$$\dot{x}_G = \frac{F_r}{m}t + C_1 = \frac{d}{dt}x_G$$
$$x_G = \frac{F_r}{2m}t^2 + C_1 t + C_2$$

ICs: $x_G(0) = 0 \rightarrow \frac{F_r}{2m}(0)^2 + C_1(0) + C_2 = 0$

$$C_2 = 0$$

$\dot{x}_G(0) = 0 \rightarrow \frac{F_r}{m}(0) + C_1 = 0$

$$C_1 = 0$$

$$\Rightarrow x_G = \frac{F_r}{2m}t^2 = \frac{(1N)}{2(1kg)}t^2$$
$$x_G = (\tfrac{1}{2}\ m/s^2)\ t^2$$

$$\ddot{y}_G = 0 = \frac{d}{dt}\dot{y}_G$$
$$\dot{y}_G = C_3 = \frac{d}{dt}y_G$$
$$y_G = C_3 t + C_4$$

ICs: $y_G(0) = 0 \rightarrow C_3(0) + C_4 = 0$

$$C_4 = 0$$

$\dot{y}_G(0) = 0 \rightarrow C_3 = 0$

$$\Rightarrow y_G = 0\ m$$

Then find $\vec{r}_G$:

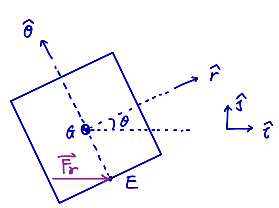
$$\vec{r}_G = x_G\hat{\imath} + y_G\hat{\jmath}$$
$$\boxed{\vec{r}_G(t) = (\tfrac{1}{2}\ m/s^2)\ t^2\ \hat{\imath}}$$

b) Find diff. eq. and ICs that can be used to solve for $\theta(t)$.

θ is ccw rotation of plate from $t=0$ configuration.

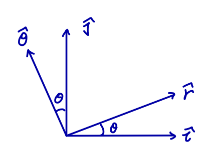
* $\hat{r}, \hat{\theta}$ used in this problem are unit vectors for polar coordinates (same as $\hat{e}_r, \hat{e}_\theta$).

At some $t > 0$:



AMB$_{/G}$:

$$\vec{M}_{/G} = \cancelto{0}{\vec{r}_{G/G}} \times m\vec{a}_G + I^G\ddot{\theta}\hat{k}$$
$$\vec{r}_{E/G} \times \vec{F}_r = I^G\ddot{\theta}\hat{k}$$
$$(-\tfrac{L}{2}\hat{\theta}) \times (F_r\hat{\imath}) = (\tfrac{mL^2}{6})\ddot{\theta}\hat{k}$$
$$\{\tfrac{L}{2}F_r\cos\theta\,\hat{k} = \tfrac{mL^2}{6}\ddot{\theta}\,\hat{k}\}$$
$$\{\ \}\cdot\hat{k} : \quad \tfrac{L}{2}F_r\cos\theta = \tfrac{mL^2}{6}\ddot{\theta}$$
$$\boxed{\ddot{\theta} = \tfrac{3}{L}\,m\,F_r\cos\theta}$$



where $m = 1kg$, $L = 1m$, $F_r = 1N$

$$\boxed{\text{ICs}: \quad \theta(0) = 0, \quad \dot{\theta}(0) = 0}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

c) d) Write code to generate drawing of plate at $t = 1s$.

horizontal translation : $x_G = \frac{F_x}{2m} t^2$

vertical translation : $y_G = 0$

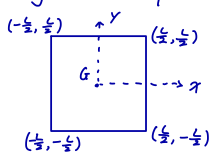
rotation : $z = [\theta, \dot{\theta}]'$;

rhs : $\dot{\theta} = z(2)$;

$\ddot{\theta} = \frac{3}{L} m F_x \cos\theta$;

$zdot = [\dot{\theta}, \ddot{\theta}]'$;

Plotting initial square :

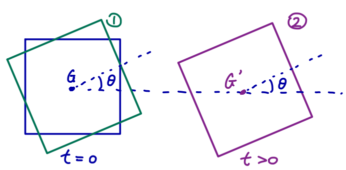


$x_0 = [-\frac{L}{2} \;\; \frac{L}{2} \;\; \frac{L}{2} \;\; -\frac{L}{2} \;\; -\frac{L}{2}]$;

$y_0 = [-\frac{L}{2} \;\; -\frac{L}{2} \;\; \frac{L}{2} \;\; \frac{L}{2} \;\; -\frac{L}{2}]$;

$sq_0 = [x_0 ; y_0]$;

Generating plot for $t > 0$:



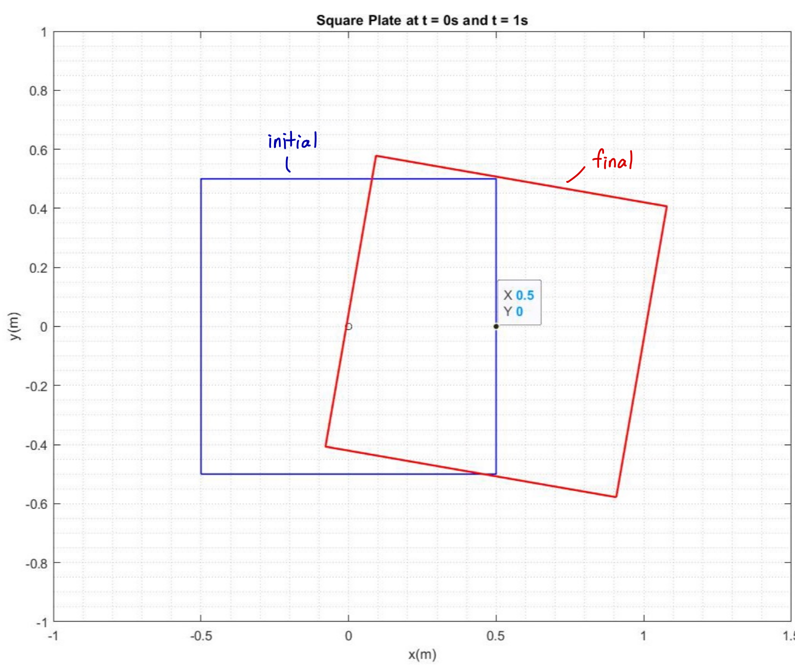
① use rotation matrix to find position of rotated square

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

② Translate square to final location

↳ add r_G' to each vertex.

$$\Rightarrow sq = R * sq + \begin{bmatrix} x_G' \\ y_G' \end{bmatrix}$$



$\theta(t=1s) \approx 1.4$ rad $\approx 80°$.

as seen from the plot.

$x_G\ (t=1s) = \frac{1}{2}(m/s^2)(1s)^2 = 0.5m$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

```
% HW10 P55 Q17.2.7
% Generate plot for square plate with applied force at t=1s
% Teaya Yang

% Given Parameters
p.m = 1; % [kg]
p.L = 1; % [m]
p.F = 1; % [N]

% IC
z0 = [0 0]';

% Time
tend = 1; tspan = [0 tend]; n = 1000;

% Equation
eq = @(t,z) rhs(t,z,p);

% Accuracy
smallnumber = 1e-6;
options = odeset('AbsTol',smallnumber,'RelTol',smallnumber);

% Solve
solution = ode45(eq,tspan,z0,options);
t = linspace(0,tend,n);
z = deval(solution,t);

% Inital square location
x0 = [-p.L p.L p.L -p.L -p.L]/2;
y0 = [-p.L -p.L p.L p.L -p.L]/2;
sq0 = [x0;y0];

% Square center locations
xG = p.F/(2*p.m)*t.^2;
yG = t.*0;

% Plot inital square
figure(1)
hold on
box on
grid minor
axis equal
xlabel('x(m)')
ylabel('y(m)')
axis([-1 1.5 -1 1])
title('Square Plate at t = 0s and t = 1s')
plot(x0,y0,'b','LineWidth',1) % plate
plot(xG(1),yG(1),'ko','MarkerSize',5) % COM

% Location of final square
theta = z(1,end);
R = [cos(theta), -sin(theta);...
sin(theta), cos(theta)];
sqend = R*sq0 + [xG(end);yG(end)];

% Plot final square
plot(sqend(1,:),sqend(2,:),'r','LineWidth',1.5) % plate
plot(xG(end),yG(end),'ko','MarkerSize',5) % COM
```

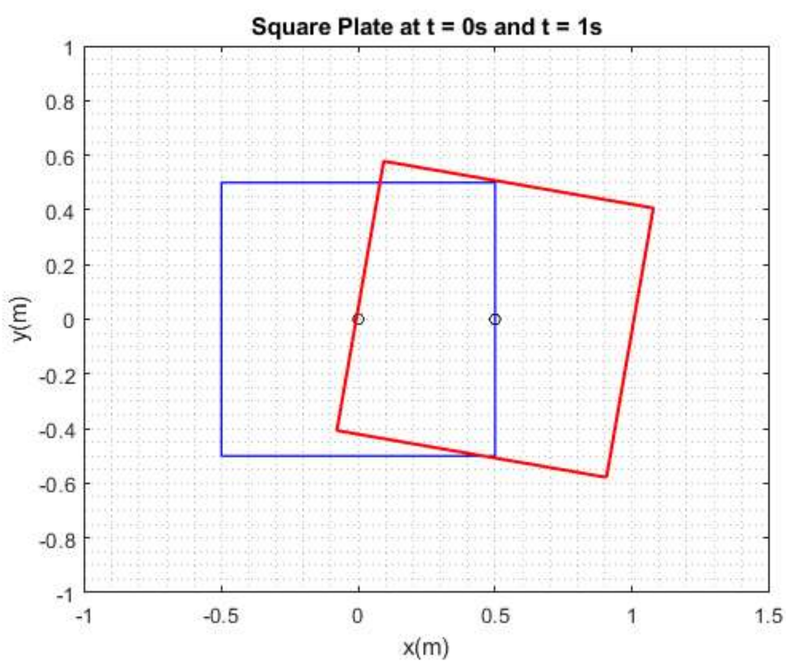
```
% Output configuration at t=1s
fprintf('COM at t = 1 s is located at x = %.1f m, y = %.1f m\n',xG(end),yG(end))
fprintf('Plate has been rotated by %.2f rad at t = 1 s\n',theta)

function zdot = rhs(t,z,p)
% Unpack
theta = z(1); thetad = z(2);
m = p.m; L = p.L; F = p.F;

% Equation
thetadd = 3*m*F*cos(theta)/L;

zdot = [thetad thetadd]';
end
```

```
COM at t = 1 s is located at x = 0.5 m, y = 0.0 m
Plate has been rotated by 1.40 rad at t = 1 s
```



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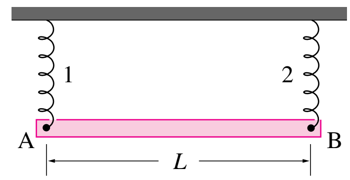
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.2.9 A uniform slender bar AB of mass m is suspended from two springs (each of spring constant K) as shown. Immediately after spring 2 breaks, determine

a) the angular acceleration of the bar,

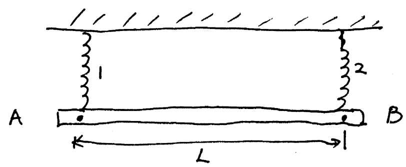
b) the acceleration of point A, and

c) the acceleration of point B.



Problem 17.9

14.21



1, 2 : two springs with spring const k

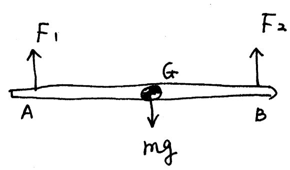
If spring 2 breaks, determine at that instant,

(a) angular acceleration of the bar

(b) the acceleration of point A

(c) the acceleration of point B.

Before "2" breaks, the bar is in equilibrium.



It's easy to get

$F_1 = F_2 = \frac{mg}{2}$ from LMB, AMB $/_G$.

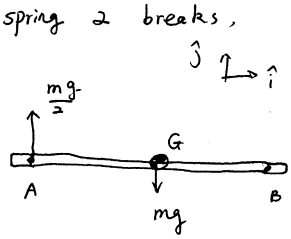
After "2" breaks,

The distance between A ~~bet~~ and the ceiling ~~is~~ at that instant is the same as that before "2" breaks.

$\therefore$ The stretch of spring 1 reemains unchanged. So the tension force on spring is still $\frac{mg}{2}$.

But spring 2 breaks, so $F_2 = 0$

$\therefore$



AMB $/_G$ $\Rightarrow$

$$\Sigma \vec{M}_{/G} = I_G \dot{\vec{\omega}}$$

$$\Rightarrow -\frac{mgL}{4}\hat{k} = \frac{mL^2}{12}\dot{\vec{\omega}}$$

$$\Rightarrow \boxed{\dot{\vec{\omega}} = -\frac{3g}{L}\hat{k}}$$

$\therefore$ angular acceleration of the bar is $-\frac{3g}{L}\hat{k}$ at that instant.

LMB $\Rightarrow$

$$-mg\hat{j} + \frac{mg}{2}\hat{j} = m\vec{a}_G$$

$$\Rightarrow \quad \vec{a}_G = -\frac{g}{2}\hat{j}$$

$$\therefore \quad \vec{a}_A = \vec{a}_G + \dot{\vec{\omega}} \times \vec{r}_{A/G} - \omega^2 \vec{r}_{A/G}$$

At this instant $\omega = 0$

$$\therefore \quad \vec{a}_A = -\frac{g}{2}\hat{j} + \left(-\frac{3g}{L}\hat{k}\right) \times \left(-\frac{L}{2}\hat{i}\right)$$

$$= -\frac{g}{2}\hat{j} + \frac{3g}{2}\hat{j} = g\hat{j}$$

Similarly

$$\vec{a}_B = \vec{a}_G + \dot{\vec{\omega}} \times \vec{r}_{B/G} - \cancelto{0}{\omega^2} \vec{r}_{B/G}$$

$$= -\frac{g}{2}\hat{j} + \left(-\frac{3g}{L}\hat{k}\right) \times \left(\frac{L}{2}\hat{i}\right)$$

$$= -\frac{g}{2}\hat{j} - \frac{3g}{2L}\hat{j}$$

$$= -2g\hat{j}$$

$\therefore$ accelerations of point A, B at that instant are

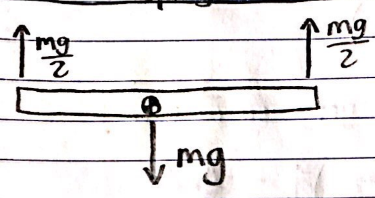
$$\boxed{\vec{a}_A = g\hat{j}}$$

$$\boxed{\vec{a}_B = -2g\hat{j}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

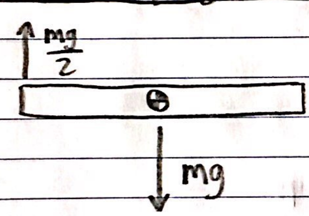
Ruina + Pratap 16.2.9, solution (4/10/19 by Walker Lee)

FBD before spring breaks:



The bar is in static equilibrium, so $\sum \vec{F} = \sum \vec{M} = \vec{0}$; it is easy to verify that both spring forces equal $mg/2\,\hat{j}$

FBD after spring breaks:



The bar hasn't started moving yet, so the length of the remaining spring, and therefore its force, is unchanged.

a. Find $\ddot{\theta}$ at the moment the spring snaps.

AMB$_{/G}$: $\sum \vec{M}_{/G} = \cancel{\vec{r}_{G/G}}^{\,0} \times m\vec{a}_G + I^G \ddot{\theta}\hat{k}$

$I^G = \int r^2 dm$, $dm = \frac{m}{\ell}dx$, $r = x \rightarrow I^G = 2\int_0^{\ell/2} x^2 \frac{m}{\ell}dx = \frac{m\ell^2}{12}$

$\rightarrow \vec{r}_{A/G} \times \frac{mg}{2}\hat{j} + \cancel{\vec{r}_{G/G}}^{\,0} \times -mg\hat{j} = \frac{m\ell^2}{12}\ddot{\theta}\hat{k}$

$\rightarrow -\frac{\ell}{2}\hat{i} \times \frac{mg}{2}\hat{j} = \frac{m\ell^2}{12}\ddot{\theta}\hat{k}$

$\rightarrow -\frac{\ell mg}{4} = \frac{m\ell^2}{12}\ddot{\theta}$

$\rightarrow \boxed{\ddot{\theta} = \frac{-3g}{\ell}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

b. Find $\vec{a}_A$ at the moment the spring snaps.

$$\vec{a}_A = \vec{a}_G + \vec{\alpha} \times \vec{r}_{A/G} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/G})$$

- The bar hasn't started rotating yet, so $\vec{\omega} = \vec{0}$;
- LMB gives $\left(\frac{mg}{2} - mg\right)\hat{j} = m\vec{a}_G \rightarrow \vec{a}_G = \frac{-g}{2}\hat{j}$

$$\rightarrow \vec{a}_A = \frac{-g}{2}\hat{j} + \left(\frac{-3g}{\ell}\hat{k}\right) \times -\frac{\ell}{2}\hat{i}$$

$$\rightarrow \vec{a}_A = \frac{-g}{2}\hat{j} + \frac{3g}{2}\hat{j}$$

$$\rightarrow \boxed{\vec{a}_A = g\hat{j}}$$

b. Find $\vec{a}_B$ at the moment the spring snaps.

$$\vec{a}_B = \vec{a}_G + \vec{\alpha} \times \vec{r}_{B/G} + \cancel{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/G})}^{\,0}$$

$$= \frac{-g}{2}\hat{j} + \left(\frac{-3g}{\ell}\hat{k}\right) \times \frac{\ell}{2}\hat{i}$$

$$= \frac{-g}{2}\hat{j} - \frac{3g}{2}\hat{j}$$

$$\rightarrow \boxed{\vec{a}_B = -2g\hat{j}}$$

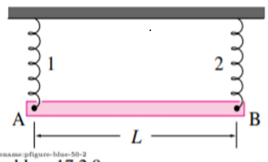
#56 Solution Michelle deSouza

17.2.9 A uniform slender bar AB of mass m is suspended from two springs (each of spring constant K) as shown. Immediately after spring 2 breaks, determine

a) the angular acceleration of the bar,

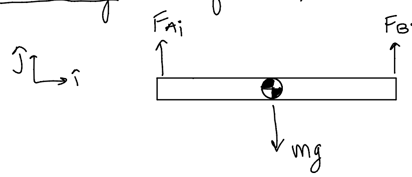
b) the acceleration of point A, and

c) the acceleration of point B.



Problem 17.2.9

Before breaking: static equilibrium, $\vec{a} = \vec{0}$



LMB: $\sum \vec{F} = -mg\hat{j} + F_{Ai}\hat{j} + F_{Bi}\hat{j} = 0$

$\{\}\cdot\hat{j}$ $\quad mg = F_{Ai} + F_{Bi}$

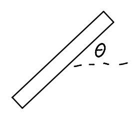
AMB: $\sum \vec{M}_{/G} = -\frac{L}{2}\hat{\imath} \times F_{Ai}\hat{j} + \frac{L}{2}\hat{\imath} \times F_{Bi}\hat{j} = 0$

$\{\}\cdot\hat{k}$ $\quad F_{Ai} = F_{Bi}$ plug into LMB

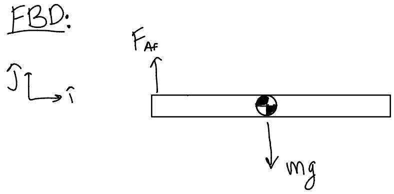
$mg = F_{Ai} + F_{Ai}$

$\frac{1}{2}mg = F_{Ai}$

Immediately after spring 2 breaks, the bar is in the same position, so spring 1 is stretched the same amount and $F_{Af} = F_{Ai} = \frac{1}{2}mg$



FBD:



$I^G = \int r^2 dm = 2\int_0^{L/2} x^2 \frac{m}{L}\, dx$

$= \frac{2m}{L}\left(\frac{1}{3}\frac{L^3}{8}\right) = \frac{1}{12}mL^2$

a) $\ddot{\theta}$ of bar after spring 2 snaps

AMB: $\sum \vec{M}_{/G} = -\frac{L}{2}\hat{\imath} \times \frac{1}{2}mg\hat{j} = I^G\ddot{\vec{\theta}}$

$\{\}\cdot\hat{k}$ $\quad -\frac{1}{4}mgL\hat{k} = \frac{1}{12}mL^2\ddot{\vec{\theta}}$

$-\frac{3g}{L}\hat{k} = \ddot{\vec{\theta}}$

$-\frac{3g}{L} = \ddot{\theta}$

b) $\vec{a}_A$ of bar after spring 2 snaps

Rigid object kinematics: $\vec{a}_A = \vec{a}_G + \ddot{\vec{\theta}} \times \vec{r}_{A/G} - \dot{\theta}^2\vec{r}_{A/G}$ ①

$\dot{\theta} = 0$ (object is motionless), $\ddot{\vec{\theta}} = -\frac{3g}{L}\hat{k}$

LMB: $\quad m\vec{a}_G = F_A\hat{j} - mg\hat{j} = -\frac{1}{2}mg\hat{j}$

$\vec{a}_G = -\frac{1}{2}g\hat{j} \approx -5\hat{j}\ m/s^2$

plug into ①

$\vec{a}_A = -\frac{1}{2}g\hat{j} + -\frac{3g}{L}\hat{k} \times -\frac{L}{2}\hat{\imath}$

$\vec{a}_A = -\frac{1}{2}g\hat{j} + \frac{3}{2}g\hat{j} = g\hat{j} \approx 10\hat{j}\ m/s^2$

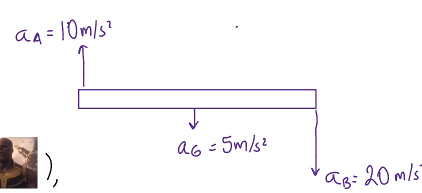
c) $\vec{a}_B$ of bar after spring 2 snaps

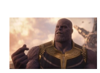
Rigid object kinematics: $\vec{a}_B = \vec{a}_G + \ddot{\vec{\theta}} \times \vec{r}_{B/G} - \dot{\theta}^2\vec{r}_{B/G}$

$\vec{a}_G, \ddot{\vec{\theta}}, \dot{\theta}$ all the same as in part b)

$\vec{a}_B = -\frac{1}{2}g\hat{j} + -\frac{3g}{L}\hat{k} \times \frac{L}{2}\hat{\imath}$

$\vec{a}_B = -\frac{1}{2}g\hat{j} - \frac{3}{2}g\hat{j} = -2g\hat{j} \approx -20\hat{j}\ m/s^2$



Intuition: After the snap (when half of the springs are gone ), the bar is rotating clockwise ($-\hat{k}$ direction). Therefore, point A must be accelerating upward relative to the CoM, and point B must be accelerating downward relative to the CoM.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

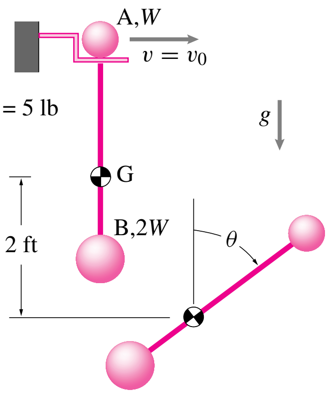
17.2.10 Two small spheres A and B are connected by a rigid rod of length $\ell = 1.0\,\text{ft}$ and negligible mass. The assembly is hung from a hook, as shown. Sphere A is struck, suddenly breaking its contact with the hook and giving it a horizontal velocity $v_0 = 3.0\,\text{ft/s}$ which sends the assembly into free fall. Determine the angular momentum of the assembly about its mass center at point G immediately after A is hit. After the center of mass has fallen two feet, determine:

a) the angle θ through which the rod has rotated,

b) the velocity of sphere A,

c) the total kinetic energy of the assembly of spheres A and B and the rod, and

d) the acceleration of sphere A.



Problem 17.10

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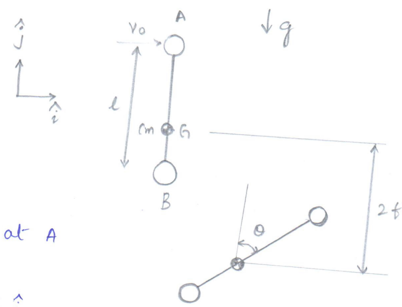
16.2.10



$W = 5\,lb$

$\ell = 1\,ft$

$V_0 = 3.0\,ft/sec$

If a force is applied at A

$$\vec a_{cm} = \frac{\vec F}{m} \; ; \; \frac{F}{m}\hat i$$

$$\vec N_{/cm} = \frac{2}{3}\ell\hat j \times F\hat i$$

$$= \left[m\left(\frac{2}{3}\ell\right)^2 + 2m\left(\frac{\ell}{3}\right)^2\right]\alpha\,(-\hat k)$$

$$\Rightarrow \alpha = -\left(\frac{2\ell F}{3}\times\frac{9}{6m\ell^2}\right)\hat k = \frac{-F}{m\ell}\hat k$$

$$\vec a_A = \frac{-F}{m\ell}\hat k \times \left(\frac{2}{3}\ell\hat j\right) + \vec a_{cm}$$

$$= \left(\frac{2F}{3m} + \frac{F}{m}\right)\hat i = \frac{5}{3}\cdot\frac{F}{m}\hat i$$

$$\vec a_A = \frac{5}{3}\vec a_{cm}$$

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At the end of the impulse

$\vec{V}_A = 3 \frac{ft}{sec}$; $\vec{V}_{cm} = \frac{9}{5}$ ft/sec

$\vec{V}_A - \vec{V}_{cm} = \vec{\omega} \times \vec{r}_{A/cm}$

$\frac{6}{5}\hat{i} = -\omega\hat{k} \times \left(\frac{2}{3}\ell\hat{j}\right)$

$\omega = 1.8$ rad/sec

$\vec{\omega} = -1.8\hat{k}$ rad/sec

Angular Momentum

$\vec{H}_{/cm} = I\vec{\omega} = \left(\frac{2}{3}m\ell^2 \times 1.8\right)(-\hat{k}) = -1.2\, m\ell^2\hat{k}$

$\vec{H}_{/cm} = -12\hat{k}\ \frac{lb\, ft^2}{sec}$

$g = 32.56$ ft/sec^2

$2 = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{4}{32.56}}$ sec

$\theta = \omega t = 1.8\sqrt{\frac{4}{32.56}}$ radians.

$\theta = 0.63$ rad

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b) Velocity of sphere A.

$$\vec{V}_{cm} = \left(\frac{9}{5}\right)\hat{i} - \sqrt{2\times g\times 2}\ \hat{j}$$

$$\vec{V}_A = \vec{V}_{cm} + \vec{\omega}\times\vec{r}_{A/cm}$$

$$= \frac{9}{5}\hat{i} - \sqrt{4g}\ \hat{j} + (-1.8\hat{k})\times\left(\frac{2}{3}l\sin\theta\,\hat{i} + \frac{2}{3}l\cos\theta\,\hat{j}\right)$$

$$\vec{V}_A = 9/5\,\hat{i} - 2\sqrt{g}\ \hat{j} - 1.2\sin\theta\,\hat{j} + 1.2\cos\,\hat{i}$$

c) Total Kinetic Energy

$$= \frac{1}{2}(3m)V_{cm}^2 + (3m)g(h) + \frac{1}{2}I_{cm}(\omega)^2$$

$$= \left(\frac{15}{2}\right)\left(\frac{9}{5}\right)^2 + \left(\frac{1}{2}\right)\left(\frac{2}{3}ml^2\right)(1.8)^2 + 3mg(h)$$

$$= \frac{243}{10} + \frac{5}{3}\times(1.8)^2 + 15\cdot 32.66\times 2$$

$$KE = 24.3 + 5.4 + 30\times 32.66$$

$$K.E = 1009.5\ \frac{lb\,ft^2}{sec^2}$$

The next several problems concern Work, power and energy

17.2.11 Verify that the expressions for work done by a force F, $W = F\Delta S$, and by a moment M, $W = M\Delta\theta$, are dimensionally correct if ΔS and $\Delta\theta$ are linear and angular displacements respectively.

(i) $W = F\Delta S$

$$LHS = \left[\frac{Kg\ M^2}{sec^2}\right]$$

$$RHS = \left[\frac{Kg\ M}{sec^2}\right] \cdot [M] = \left[\frac{Kg \cdot M^2}{sec^2}\right]$$

(ii) $W = M\Delta\theta$

$$LHS = \left[\frac{Kg\ M^2}{sec^2}\right]$$

$$\left[\begin{array}{l} \because [M][rad] = [M] \\ \because r\theta^c = s \end{array}\right]$$

$$RHS = \left[\frac{Kg\ M^2}{sec^2}\right][rad] = \left[\frac{Kg\ M^2}{sec^2}\right]$$

17.2.12 A uniform disc of mass m and radius r rotates with angular velocity $\omega\hat{\boldsymbol{k}}$. Its center of mass translates with velocity $\vec{\boldsymbol{v}} = \dot{x}\hat{\boldsymbol{\imath}} + \dot{y}\hat{\boldsymbol{\jmath}}$ in the xy-plane. What is the total kinetic energy of the disk?

$$K.E = \frac{1}{2} m \vec{V} \cdot \vec{V} + \frac{1}{2} I \vec{\omega} \cdot \vec{\omega}$$

$$K.E = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I \omega^2$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

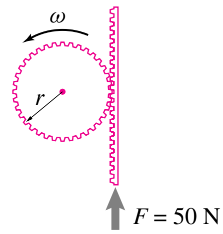
17.2.13 Calculate the energy stored in a spring using the expression $E_\text{P} = \frac{1}{2}k\delta^2$ if the spring is compressed by 100 mm and the spring constant is 100 N/m.

$$E = \frac{1}{2} K \delta^2 = \frac{1}{2} \cdot 100 \cdot \left(\frac{100}{1000}\right)^2$$

$$= 0.5 \text{ Joules}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.2.14 In a rack and pinion system, the rack is acted upon by a constant force $F = 50\,\text{N}$ and has speed $v = 2\,\text{m/s}$ in the direction of the force. Find the input power to the system.



Problem 17.14

Input power

$= \vec{F} \cdot \vec{V}$

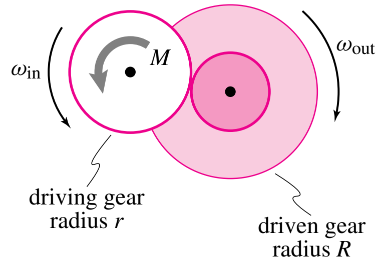
$= 50\,\hat{j} \cdot 2\,\hat{j}$

$= 100$ Joules

17.2.15 The driving gear in a compound gear train rotates at constant speed ω_0. The driving torque is M_{in}. If the driven gear rotates at a constant speed ω_{out}, find:

a) the input power to the system, and

b) the output torque of the system assuming there is no power loss in the system; i.e., power in = power out.



Problem 17.15

a) Input power = $\vec{M} \cdot \vec{\omega} = M_{in} \omega_{in}$

b) output torque $\vec{M}_{out} \cdot \vec{\omega}_{out} = \vec{M}_{in} \cdot \vec{\omega}_{in}$

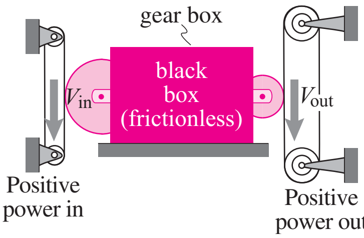
$$M_{out} = \frac{M_{in}\,\omega_{in}}{\omega_{out}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

17.2.16 An elaborate frictionless gear box has an input and output roller with $V_{in} = const$. Assuming that $V_{out} = 7V_{in}$ and the force between the left belt and roller is $F_{in} = 3\,\text{lb}$:

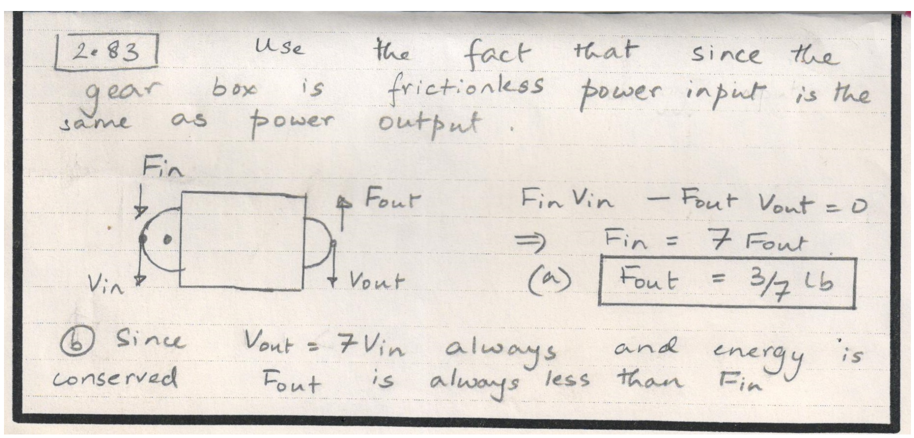
a) What is F_{out} (draw a picture defining the signs of F_{in} and F_{out})?.

b) Is F_{out} greater or less than the F_{in}? (Assume $F_{in} > 0$.) Why?



Problem 17.16

Answer:
(a) $F_{out} = \frac{3}{7}$ lb.
(b) F_{out} is always less than the F_{in}.



$\boxed{2.83}$ Use the fact that since the gear box is frictionless power input is the same as power output.

$F_{in} V_{in} - F_{out} V_{out} = 0$

$\Rightarrow F_{in} = 7 F_{out}$

(a) $\boxed{F_{out} = 3/7 \text{ lb}}$

(b) Since $V_{out} = 7V_{in}$ always and energy is conserved F_{out} is always less than F_{in}

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*