CHAPTER 17

General planar motion of a single rigid object

The main goal here is to generate equations of motion for general planar motion of a (planar) rigid object that may roll, slide or be in free flight. Multi-object systems are also considered so long as they do not involve other kinematic constraints between the bodies. Features of the solution that can be obtained from analysis are discussed, as are numerical solutions.

Many machine and structural parts move in straight-lines (Chapter 14) or circles (Chapters 13). But other things have more general motions, like a plane in unsteady flight or a connecting rod in a car engine. Keeping track of such motion is a bit more difficult.

In this chapter we will use these two modeling approximations:

- The objects are planar, or symmetric with respect to a plane; and
- They have planar motions in that plane.

A *planar object* is one where the whole object is flat and all its matter is confined to one plane, say the xy plane. This is a palatable approximation for a piece cut out of flat sheet metal. For more substantial real objects, like a full car, the approximation seems at a glance to be terrible. But it turns out that so long as the *motion* is planar and the car is reasonably idealized as symmetrical (left to right) that treating the car as equivalent to it being squished into a plane does not introduce any more approximation. Thus, even in this 3-D world we live in with 3-D objects, it is fruitful to do 2-D analysis of the type you will learn in this chapter.

A *planar motion* is one where the velocities of all points are in the same constant plane, say a fixed xy plane, at all times and where points with, say, the same z coordinate have the same velocity. The positions of the points do not have to be in the same plane for a planar motion. Each point stays in a plane, but different points can be in different planes, with each plane parallel to the others.

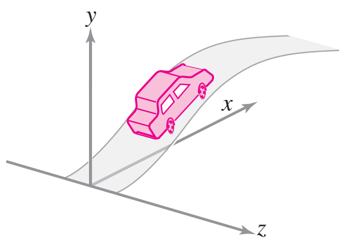

Figure 17.1: Planar motion of a 3D car. If the car is symmetrical it can be studied by the means of this chapter.

Example: A car going over a hill
Assume the road is straight in map view in, say, the x direction. Assume the whole width of the road has the same hump. Although the car is clearly not planar, the car motion is probably close to planar, with the velocities of all points in the car in the xy plane (see fig. 17.1)

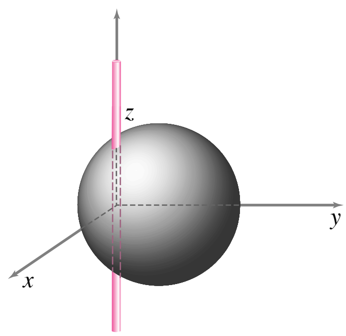

Figure 17.2: Planar motion of a skewered sphere. This can be studied by the means in this chapter.

Example: Skewered sphere
A sphere skewered and rotating about a fixed axis in the $\hat{\boldsymbol{k}}$ direction has a planar motion (see fig. 17.2). The points on the object do not all lie in a common plane. But all of the velocities are orthogonal to $\hat{\boldsymbol{k}}$ and thus in the xy plane. This problem does fit in with the methods of this chapter. The symmetry of the sphere with respect to the xy plane makes it so that the three-dimensional mass distribution does not invalidate the two-dimensional analysis.

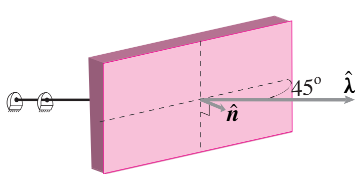

Figure 17.3: Planar motion of a planar object. But the plane of the motion is *not* the plane of the object. This is not a natural topic for this chapter.

Example: Skewered plate
A flat rectangular plate with normal $\hat{\boldsymbol{n}}$ has a fixed axis of rotation in the direction $\hat{\boldsymbol{\lambda}}$ that makes a 45° angle to $\hat{\boldsymbol{n}}$ (see fig. 17.3). This is a planar object (a plane normal to $\hat{\boldsymbol{n}}$) in planar motion (all velocities are in the plane normal to $\hat{\boldsymbol{\lambda}}$). But the plane of motion is not the plane of the mass distribution, the object is not symmetric with respect to a motion plane, so this example does not fit into the discussion of this chapter(1).

(1) Actually, a two-dimensional analysis of the plate in this example would be legitimate in this sense. Project all the plate's mass into the plane normal to the $\hat{\boldsymbol{\lambda}}$ direction. The projections of the forces on this plane would be correctly predicted, but three-dimensional effects, like those associated with dynamic imbalance, would be lost in this projection.

No real object is exactly planar and no real motion is exactly a planar motion. But many objects are relatively flat and thin or symmetrical and many motions are approximately planar motions. Thus many, if not most, simple engineering analyzes assume planar motion. For bodies that are approximately symmetric about the xy plane of motion (such as a car, if the asymmetrically placed driver's mass *etc.* is neglected), there is no loss in doing a two-dimensional planar rather than full three dimensional analysis.

The plan of this chapter. We start with planar kinematics. Then we evaluate and use expressions for the rates of change of linear and angular momentum for planar bodies. Finally we discuss rolling, sliding and collisions.