16.2 Angular velocity

Preparatory Problems

16.2.1 Give the definition of each term below in words and, but for the first, with an equation.

a) angular velocity *

b) angular acceleration *

16.2.2 Assume that in the reference configuration, when $\theta = 0$, that the $x'y'$ axes attached to an object are aligned with the xy axes. Consider a point P attached to the moving frame (object) that has coordinates (x', y'). Find each of the quantities below in terms of some or all of $x', y', \hat{\imath}, \hat{\jmath}, \theta, \dot{\theta}$ and $\ddot{\theta}$.

a) $\vec{r}_{\mathrm{P}}$ *

b) $\vec{v}_{\mathrm{P}}$ *

c) $\vec{a}_{\mathrm{P}}$ *

16.2.3 Assume that in the reference configuration, when $\theta = 0$, the $x'y'$ axes are aligned with the xy axes. Consider two points P_1 and P_2 attached to the moving frame. P_1 and P_2 have coordinates (x'_1, y'_1) (x'_2, y'_2). Find each of the quantities below in terms of some or all of $x'_1, y'_1, x'_2, y'_2, \hat{\imath}, \hat{\jmath}, \theta, \dot{\theta}$ and $\ddot{\theta}$.

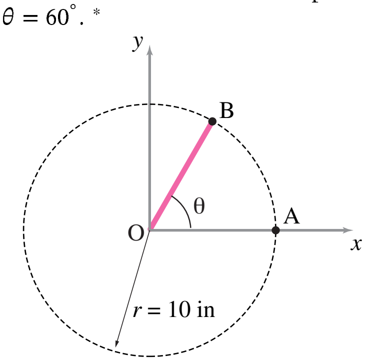
a) $\vec{r}_{\mathrm{P}_1/\mathrm{P}_2}$ *

b) $\vec{v}_{\mathrm{P}_1/\mathrm{P}_2}$ *

c) $\vec{a}_{\mathrm{P}_1/\mathrm{P}_2}$ *

16.2.4 Find $\vec{v} = \vec{\omega} \times \vec{r}$, if $\vec{\omega} = 1.5\,\mathrm{rad/s}\hat{k}$ and $\vec{r} = 2\,\mathrm{m}\hat{\imath} - 3\,\mathrm{m}\hat{\jmath}$. *

16.2.5 A rod OB rotates with its end O fixed as shown in the figure with angular velocity $\vec{\omega} = 5\,\mathrm{rad/s}\hat{k}$ and angular acceleration $\vec{\alpha} = 2\,\mathrm{rad/s^2}\hat{k}$ at the instant of interest. Find, draw, and label the tangential and normal acceleration of point B at $\theta = 60^\circ$. *



Problem 16.2.5

16.2.6 A disc rotates at 15 rpm. How many seconds does it take to rotate by 180 degrees? What is the angular speed of the disc in rad/s? *

16.2.7 A motor turns a uniform disc of radius R counter-clockwise about its mass center at a constant rate ω. The disc lies in the xy-plane and its angular displacement θ is measured (positive counter-clockwise) from the x-axis. What is the angular displacement $\theta(t)$ of the disc if it starts at $\theta(0) = \theta_0$ and $\dot{\theta}(0) = \omega$? What are the velocity and acceleration of a point P at position $\vec{r} = x\hat{\imath} + y\hat{\jmath}$? *

16.2.8 Find the angular velocities of the second, minute, and hour hands of a clock. *

16.2.9 A disc $\mathcal{C}$ spins at a constant rate of two revolutions per second counter-clockwise about its geometric center, G, which is fixed. A point P is marked on the disk at a radius of one meter. At the instant of interest, point P is on the x-axis of an xy-coordinate system centered at point G.

a) Draw a neat diagram showing the disk, the particle, and the coordinate axes.

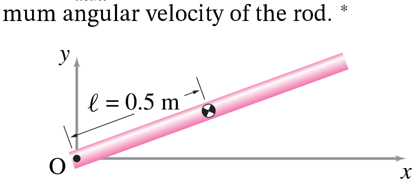
b) What is the angular velocity of the disk, $\vec{\omega}_{\mathcal{C}}$? *

c) What is the angular acceleration of the disk, $\dot{\vec{\omega}}_{\mathrm{C}}$? *

d) What is the velocity $\vec{v}_P$ of point P? *

e) What is the acceleration $\vec{a}_P$ of point P? *

16.2.10 A uniform rigid rod rotates at constant speed in the xy-plane about a peg at point O. The center of mass of the rod may not exceed a specified acceleration $a_{max} = 0.5\,\mathrm{m/s^2}$. Find the maximum angular velocity of the rod. *

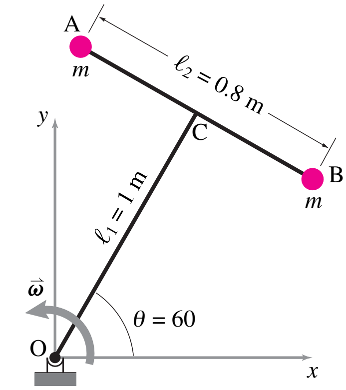


Problem 16.2.10

16.2.11 A dumbbell AB is welded to a rigid arm OC such that OC is perpendicular to AB. Arm OC rotates about O at a constant angular velocity $\vec{\omega} = 10\,\mathrm{rad/s}\hat{k}$. At the instant when $\theta = 60^\circ$,

a) Find the relative velocity of B with respect to A. *

b) Find the acceleration of point B relative to point A. *



Problem 16.2.11

More-Involved Problems

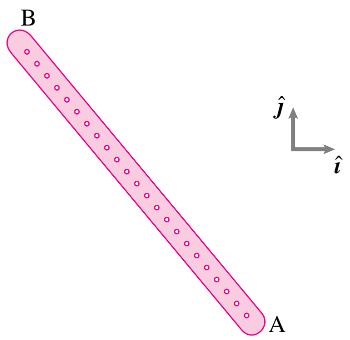
16.2.12 Two discs $\mathcal{A}$ and $\mathcal{B}$ rotate at constant speeds about their centers. Disc $\mathcal{A}$ rotates at 100 rpm and disc $\mathcal{B}$ rotates at 10 rad/s. Which is rotating faster? *

16.2.13 A motor turns a uniform disc of radius R counter-clockwise about its mass center at a constant rate ω. The disc lies in the xy-plane and its angular displacement θ is measured (positive counter-clockwise) from the x-axis. What are the velocity and acceleration of a point P at position $\vec{r}_P = c\hat{\imath} + d\hat{\jmath}$ relative to the velocity and acceleration of a point Q at position $\vec{r}_Q = 0.5(-d\hat{\imath} + c\hat{\jmath})$ on the disk? $(c^2 + d^2 < R^2.)$ *

16.2.14 A 0.4 m long rod AB has many holes along its length such that it can be pegged at any of the various locations. It rotates counter-clockwise at a constant angular speed about a peg whose location is not known. At some instant t, the velocity of end B is $\vec{v}_B = -3\,\mathrm{m/s}\hat{\jmath}$. After $\frac{\pi}{20}$ s, the velocity of end B is $\vec{v}_B = -3\,\mathrm{m/s}\hat{\imath}$. If the rod has not completed one revolution during this period,

a) find the angular velocity of the rod, and *

b) find the location of the peg along the length of the rod. *



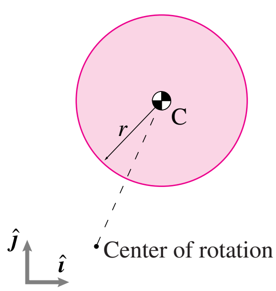
Problem 16.2.14

16.2.15 A circular disc of radius $r = 250\,\mathrm{mm}$ rotates in the xy-plane about a point which is at a distance $d = 2r$ away from the center of the disk. At the instant of interest, the linear speed of the center C is $0.60\,\mathrm{m/s}$ and the magnitude of its centripetal acceleration is $0.72\,\mathrm{m/s^2}$.

a) Find the rotational speed of the disk. *

b) Is the given information enough to locate the center of rotation of the disk? *

c) If the acceleration of the center has no component in the $\hat{\jmath}$ direction at the instant of interest, can you locate the center of rotation? If yes, is the point you locate unique? If not, what other information is required to make the point unique? *

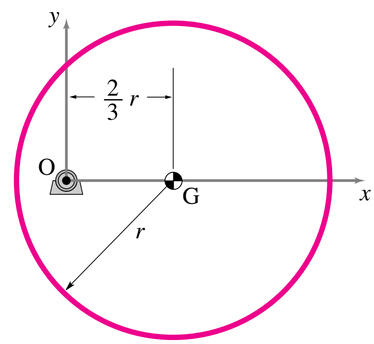


Problem 16.2.15

16.2.16 A uniform disc of radius $r = 200\,\mathrm{mm}$ is mounted eccentrically on a motor shaft at point O. The motor rotates the disc at a constant angular speed. At the instant shown, the velocity of the center of mass is $\vec{v}_G = -1.5\,\mathrm{m/s}\,\hat{\jmath}$.

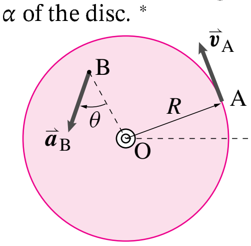
a) Find the angular velocity of the disc. *

b) Find the point with the highest linear speed on the disc. What is its velocity? *



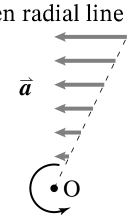
Problem 16.2.16

16.2.17 The circular disc of radius $R = 100\,\mathrm{mm}$ rotates about its center O. At a given instant, point A on the disk has a velocity $v_A = 0.8\,\mathrm{m/s}$ in the direction shown. At the same instant, the tangent of the angle θ made by the total acceleration vector of any point B with its radial line to O is 0.6. Compute the angular acceleration α of the disc. *



Problem 16.2.17

16.2.18 Show that, for non-constant rate circular motion, the acceleration of all points in a given radial line are parallel.



Problem 16.2.18

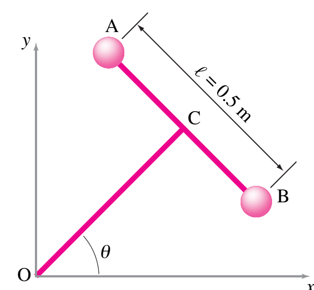
16.2.19 A motor turns a uniform disc of radius R about its mass center at a variable angular rate ω with rate of change $\dot{\omega}$, counter-clockwise. The disc lies in the xy-plane and its angular displacement θ is measured from the x-axis, positive counter-clockwise. What are the velocity and acceleration of a point P at position $\vec{r}_P = c\hat{\imath} + d\hat{\jmath}$ relative to the velocity and acceleration of a point Q at position $\vec{r}_Q = 0.5(-d\hat{\imath} + c\hat{\jmath})$ on the disk? ($c^2 + d^2 < R^2$.) *

16.2.20 The dumbbell AB shown in the figure rotates counterclockwise about point O with angular acceleration $3\,\mathrm{rad/s^2}$. Bar AB is perpendicular to bar OC. At the instant of interest, $\theta = 45^\circ$ and the angular speed is 2 rad/s.

a) Find the velocity of point B relative to point A. Will this relative velocity be different if the dumbbell were rotating at a *constant* rate of 2 rad/s? *

b) Without calculations, draw a vector approximately representing the acceleration of B relative to A.

c) Find the acceleration of point B relative to A. What can you say about the direction of this vector as the motion progresses in time? *



Problem 16.2.20

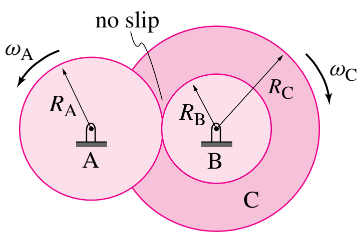
16.2.21 Bit-stream kinematics of a CD. A Compact Disk (CD) has bits of data etched on concentric circular tracks. The data from a track is read by a beam of light from a head that is positioned under the track. The angular speed of the disk remains constant as long as the head is positioned over a particular track. As the head moves to the next track, the angular speed of the disk changes, so that the *linear speed* at any track is always the same. The data stream comes out at a constant rate 4.32×10^6 bits/second. When the head is positioned on the outermost track, for which $r = 56\,\mathrm{mm}$, the disk rotates at 200 rpm.

a) What is the number of bits of data on the outermost track? *

b) find the angular speed of the disk when the head is on the innermost track (r = 22 mm), and *

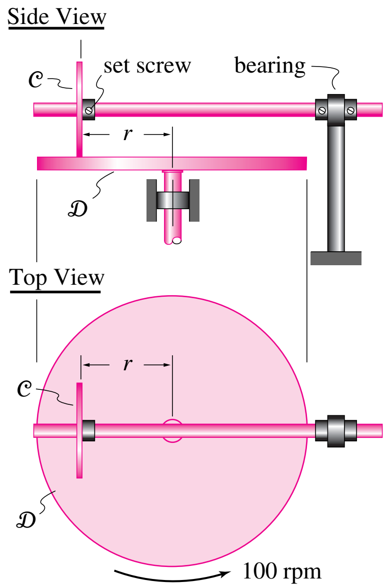
c) find the number of bits on the innermost track. *

16.2.22 2-D constant rate gear train. The angular velocity of the input shaft (driven by a motor not shown) is a constant, $\omega_{\text{input}} = \omega_A$. What is the angular velocity $\omega_{\text{output}} = \omega_C$ of the output shaft and the speed of a point on the outer edge of disc C, in terms of R_A, R_B, R_C, and ω_A? *



Problem 16.2.22: Gear B is welded to C and engages with A.

16.2.23 A horizontal disk $\mathcal{D}$ of diameter d = 500 mm is driven at a constant speed of 100 rpm. A small disk $\mathcal{C}$ can be positioned anywhere between r = 10 mm and r = 240 mm on disk $\mathcal{D}$ by sliding it along the overhead shaft and then fixing it at the desired position with a set screw (see the figure). Disk $\mathcal{C}$ rolls without slip on disk $\mathcal{D}$. The overhead shaft rotates with disk $\mathcal{C}$ and, therefore, its rotational speed can be varied by varying the position of disk $\mathcal{C}$. This gear system is called *brush gearing*. Find the maximum and minimum rotational speeds of the overhead shaft. *



Problem 16.2.23

16.2.24 Two points A and B are on the same machine part that is hinged at an as yet unknown location C. Assume you are given that points at positions $\vec{r}_A$ and $\vec{r}_B$ are supposed to move in given directions, indicated by unit vectors $\hat{\lambda}_A$ and $\hat{\lambda}_B$. For each of the problem parts below, illustrate your results with two numerical examples (in consistent units): i) $\vec{r}_A = 1\hat{\imath}$, $\vec{r}_B = 1\hat{\jmath}$, $\hat{\lambda}_A = 1\hat{\jmath}$, and $\hat{\lambda}_B = -1\hat{\imath}$ (thus $\vec{r}_C = \vec{0}$), and ii) a more complex example of your choosing.

a) Describe in detail what equations must be satisfied by the point $\vec{r}_C$.

b) Write a computer program that takes as input the 4 pairs of numbers $[\vec{r}_A]$, $[\vec{r}_B]$, $[\hat{\lambda}_A]$ and $[\hat{\lambda}_B]$ and gives as output the pair of numbers $[\vec{r}_C]$.

c) Find a formula of the form $\vec{r}_C$ = ... that explicitly gives the position vector for point C in terms of the 4 given vectors. *