CHAPTER 16

Circular motion of a rigid object

Here we extend the idea of circular motion from particles to objects. A particle can go in circles. On a rigid object in 2D, all pairs of particles go in circles around each other. The key theoretical idea is of rigid-object rotation. The primary applications are pendulums, gear trains, and rotationally accelerating motors or brakes.

We covered the special case of circular motion of a particle in the previous chapter. Now we consider the kinematics and mechanics of rotating rigid objects in 2D. The key new conceptual idea is that of rotation of a rigid object.

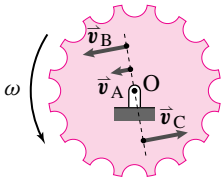

Figure 16.1: All the points on a gear move in circles, assuming the axle is not translating.

Figure 16.2: All points on a flywheel move in circles. The points further from the center move faster, so they move further while the camera shutter is open, so they show a bigger motion blur (the constant-thickness spokes look fatter near the rim).

Mechanics of circular motion

For the systems in this chapter, for every system we show in a free-body diagram we have, as always,

$$\text{linear momentum balance,} \qquad \sum \vec{\boldsymbol{F}}_i = \dot{\vec{\boldsymbol{L}}}$$

$$\text{angular momentum balance,} \qquad \sum \vec{\boldsymbol{M}}_{i/C} = \dot{\vec{\boldsymbol{H}}}_{/C},$$

$$\text{and power balance:} \qquad P = \dot{E}_K + \dot{E}_P + \dot{E}_{int}.$$

Because you already know how to work with forces and moments (the left sides of the top two equations), the primary new skill in this chapter is the evaluation of $\dot{\vec{\boldsymbol{L}}}, \dot{\vec{\boldsymbol{H}}}_{/C}$, and $\dot{E}_K$ for a rotating rigid object. That is, you need to understand the position, velocity and acceleration of points moving in circles. The rest of the skills used are universal, for example solving algebraic or differential equations, plotting, etc.