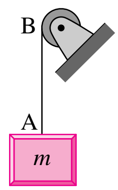
For all problems, unless stated otherwise, treat all strings as inextensible, flexible and massless. Treat all pulleys and wheels as round, frictionless and massless. Assume all massive objects are prevented from rotating (e.g., wheels stay on the ground, *etc.*). When numbers are called for use $g = 10\,\mathrm{m/s^2}$ or $g = 32\,\mathrm{ft/s^2}$.

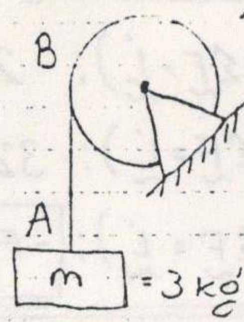
14.1.1 A motor at B allows the block of mass $m = 3$ kg shown in the figure to accelerate downwards at $2\,\mathrm{m/s^2}$. There is gravity. What is the tension in the string AB?



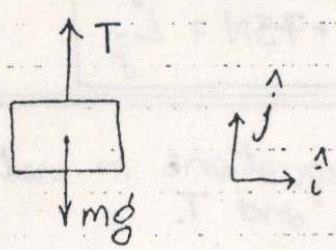
Problem 14.1

\# 1.20. 2

The block at A is accelerating downwards at $2\,\mathrm{m/s^2}$. What is the tension in line AB?



FBD of mass:



use $\sum \underline{F} = \dot{\underline{L}}$:

$$\sum \underline{F} = T\hat{j} - mg\hat{j}\ , \quad \dot{\underline{L}} = m\underline{a}$$

and $\underline{a} = -2\,\mathrm{m/s^2}\,\hat{j}$

$$\Rightarrow \quad \hat{j}\cdot(\sum \underline{F} = \dot{\underline{L}}) = T - mg = (-2\,\mathrm{m/s^2})m$$

$$T = m(-2\,\mathrm{m/s^2} + 9.8\,\mathrm{m/s^2}) = 3\,\mathrm{kg}(7.8\,\mathrm{m/s^2}) \Rightarrow$$

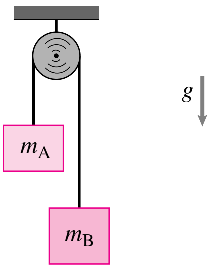
$$\boxed{T = 23.4\ \mathrm{N}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.2 Two masses connected by an inextensible string hang from an ideal pulley.

a) Find the downward acceleration of mass B. Answer in terms of any or all of m_A, m_B, g, and the present velocities of the blocks. As a check, your answer should give $a_B = g$ when $m_A = 0$ and $a_B = 0$ when $m_A = m_B$. .

b) Find the tension in the string. As a check, your answer should give $T = m_B g = m_A g$ when $m_A = m_B$ and $T = 0$ when $m_A = 0$.



Problem 14.2

Answer:

(a) $a_B = \left(\frac{m_B - m_A}{m_A + m_B}\right) g$

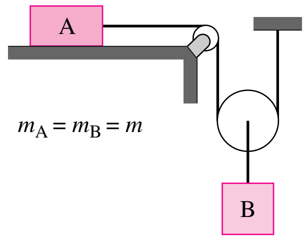
(b) $T = 2\frac{m_A m_B}{m_A + m_B} g.$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.3 The blocks shown are released from rest.

a) What is the acceleration of block A at $t = 0^+$ (just after release)?

b) What is the speed of block B after it has fallen 2 meters?

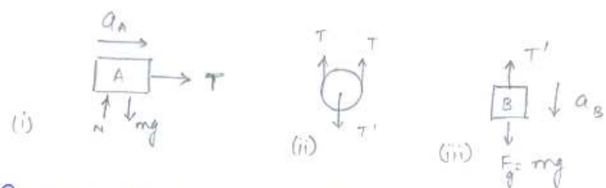


Problem 14.3

1

13.1.3

FBD



a) Dynamic equilibrium equation

$$m a_A = T \longrightarrow (i)$$

$$2T = T'$$

$$m a_B = mg - T' = mg - 2T \longrightarrow (ii)$$

Kinematic constraint ($\Delta x_A = 2\Delta y_B$)

$$a_A = 2a_B \longrightarrow (iii)$$

Solving (i), (ii), (iii) ; ~~According~~ ~~of~~

Acc. of Block 'A' just after release

$$\frac{m a_A}{2} = mg - 2 m a_A$$

$$a_A = \frac{2g}{5} = 4 \text{ m/sec}^2$$

b) $a_B = \frac{a_A}{2} = \frac{g}{5}$

Speed of block B after it has fallen 2m.

$u_B = 0$, $V_B = ?$, $a_B = g/5$, $S = 2m$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

2

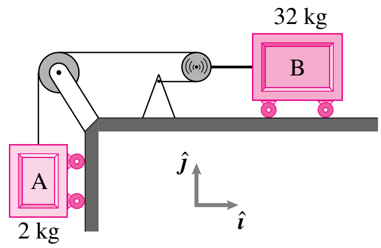
$V_B^2 - u_B^2 = 2 a_B s$

$V_B = \sqrt{2 \times \frac{9}{5} \times 2}$

$V_B = \sqrt{4 \times 4} = 4 \, m/sec$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.4 What is the acceleration of block A? Use $g = 10\,\mathrm{m/s^2}$.

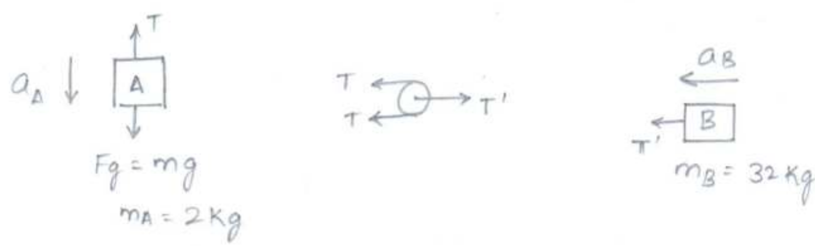


Problem 14.4

Answer:
$\vec{a}_A = -2\,\mathrm{m/s^2}\,\hat{\jmath}$

13.1.4

FBD



Dynamic equilibrium equation

$$T' = 2T$$

$$m_B a_B = T' = 2T \longrightarrow (i)$$

$$m_A a_A = m_A g - T \longrightarrow (ii)$$

Kinematic constraint

$$\Delta y_A = 2\Delta x_B$$

$$a_A = 2a_B \longrightarrow (iii)$$

Solving (i), (ii) and (iii) we get

$$m_A a_A = m_A g - \frac{m_B a_B}{2} \Rightarrow 2a_A + \frac{32}{2}\left(\frac{a_A}{2}\right) = 20$$

$$2a_A + 8a_A = 20$$

$$a_A = 2\ m/sec^2$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.5 For the system shown in problem 14.1.2, find the acceleration of mass B using energy balance ($P = \dot{E}_K$).

3

13.1.5

a_B using Energy Balance

$a_A = 2a_B$, $V_A = 2V_B$

$P_{in} = m_A g V_A$

$\dot{E}_K = m_A a_A V_A + m_B a_B V_B$

Power balance requires

$m_A g V_A = m_A a_A V_A + m_B a_B V_B$

$\Rightarrow m_A g V_A = m_A (2a_B) V_A + m_B a_B \left(\frac{V_A}{2}\right)$

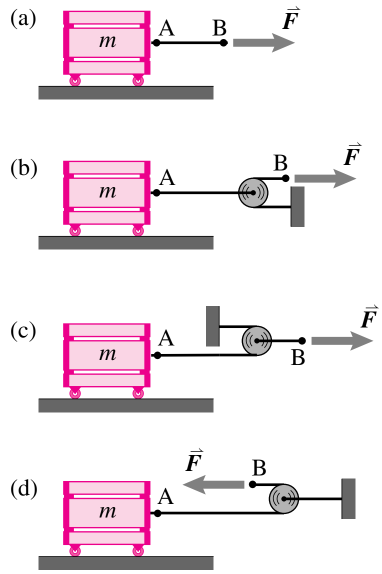
$$a_B = \frac{m_A g}{\left(2m_A + \frac{m_B}{2}\right)}$$

$$a_B = \frac{2 \times 10}{2 \times 2 + \frac{32}{2}} = \frac{20}{4 + 16}$$

$a_B = 1 \text{ m/sec}^2$

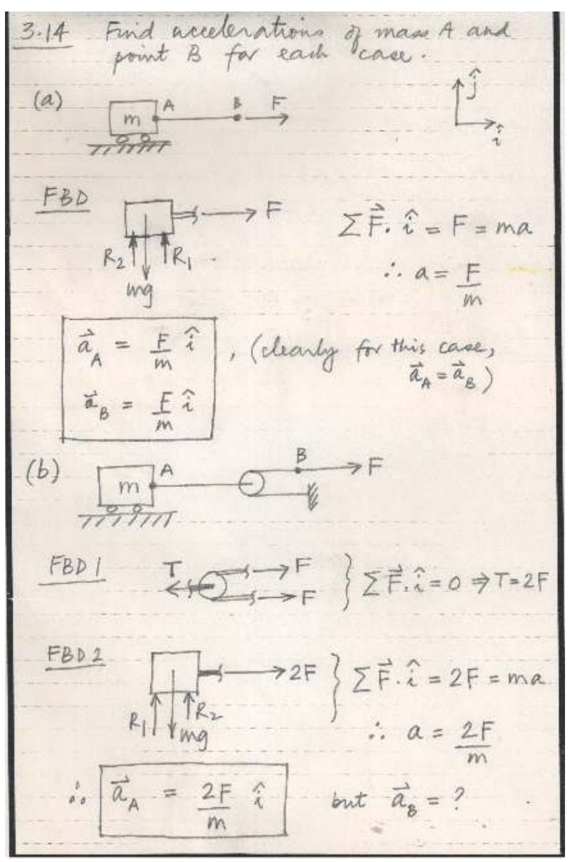
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.6 For the various situations pictured, find the acceleration of mass A and point B. Clearly define any variables, coordinates or sign conventions that you use.



Problem 14.6: Four different ways to pull a mass.

Answer:
(a) $\vec{a}_A = \vec{a}_B = \frac{F}{m}\hat{\imath}$, where $\hat{\imath}$ is parallel to the ground and pointing to the right., (b) $\vec{a}_A = \frac{2F}{m}\hat{\imath}$, $\vec{a}_B = \frac{4F}{m}\hat{\imath}$, (c) $\vec{a}_A = \frac{F}{2m}\hat{\imath}$, $\vec{a}_B = \frac{F}{4m}\hat{\imath}$, (d) $\vec{a}_A = \frac{F}{m}\hat{\imath}$, $\vec{a}_B = -\frac{F}{m}\hat{\imath}$.



3.14 Find accelerations of mass A and point B for each case.

(a)

FBD

$\sum \vec{F}\cdot\hat{\imath} = F = ma$

$\therefore a = \frac{F}{m}$

$\boxed{\vec{a}_A = \frac{F}{m}\hat{\imath} \quad \vec{a}_B = \frac{F}{m}\hat{\imath}}$, (clearly for this case, $\vec{a}_A = \vec{a}_B$)

(b)

FBD 1 $\quad \sum \vec{F}\cdot\hat{\imath} = 0 \Rightarrow T = 2F$

FBD 2 $\quad \sum \vec{F}\cdot\hat{\imath} = 2F = ma$

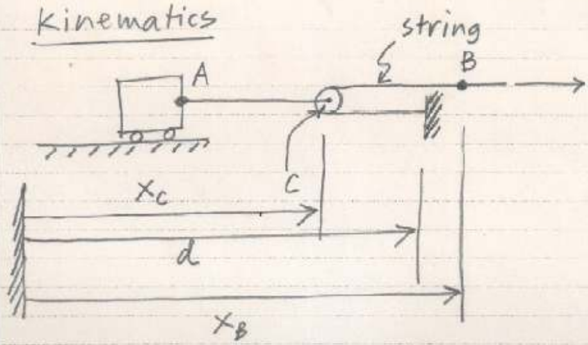
$\therefore a = \frac{2F}{m}$

$\therefore \boxed{\vec{a}_A = \frac{2F}{m}\hat{\imath}}$ but $\vec{a}_B = ?$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Page 4

Kinematics



C is a point on pulley;

$a_A = \ddot{x}_C$

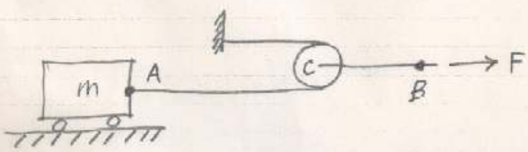
Length of string $= d - x_C + x_B - x_C =$ const.

diff w.r.t. $\Rightarrow \dot{x}_B - 2\dot{x}_C = 0$

again $\Rightarrow \ddot{x}_B - 2\ddot{x}_C = 0, \ \ddot{x}_B = 2\ddot{x}_C = 2a_A$

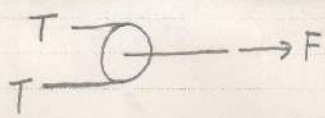
$\therefore \boxed{\vec{a}_B = 2\vec{a}_A = \frac{4F}{m}\hat{\imath}}$

(c)



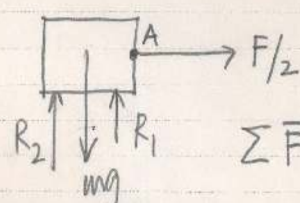
(again, note intermediate point C)

FBD 1



$\therefore 2T = F, \quad T = F/2$

FBD 2



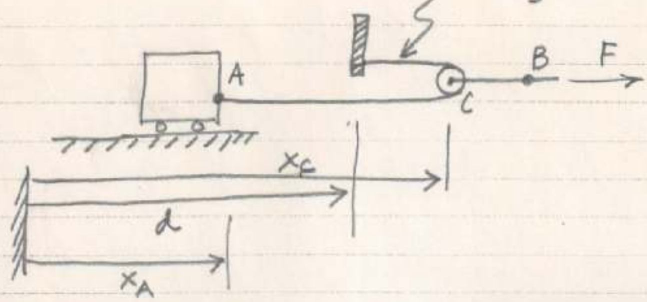
$\sum \vec{F} \cdot \hat{\imath} = \frac{F}{2} = ma$

$\therefore \boxed{\vec{a}_A = \frac{F}{2m}\hat{\imath}}$ but $\vec{a}_B = ?$.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Kinematics

Page 5



Length of string $= x_C - x_A + x_C - d = \text{const.}$

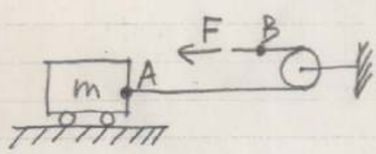
diff wrt time $\Rightarrow \quad 2\dot{x}_C - \dot{x}_A = 0$

again $\Rightarrow \quad 2\ddot{x}_C - \ddot{x}_A = 0$

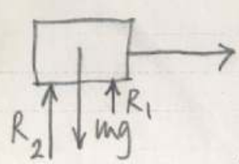
Note, $\ddot{x}_C = a_B$

$$\Rightarrow \boxed{\vec{a}_B = \frac{\vec{a}_A}{2} = \frac{F}{4m}\hat{\imath}}$$

(d)



FBD



$\Rightarrow \sum \vec{F}\cdot\hat{\imath} = F = ma$

$$\therefore \boxed{\vec{a}_A = \frac{F}{m}\hat{\imath}}$$

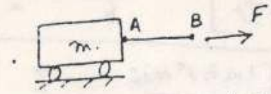
By inspection, (and lecture, etc.) the pulley doesn't do anything other than change the direction of the force, and

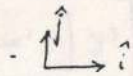
$$\boxed{\vec{a}_B = -\vec{a}_A = \frac{-F}{m}\hat{\imath}}$$

Homework Solutions # 4 Page 1

3.55

(a)





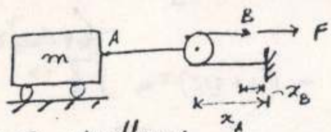
$\sum \underline{F} = m\underline{a}$ (linear momentum balance)

the acceleration of the mass at A and at point B are equal.

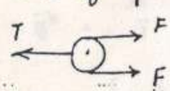
$F\hat{\imath} = m\underline{a}_A = m\underline{a}_B$

$\Rightarrow \boxed{\underline{a}_A = \underline{a}_B = \frac{F}{m}\hat{\imath}}$

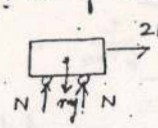
(b)



FBD of pulley:



FBD of cart:



$\sum F_H = 2F - T = 0$

$\Rightarrow 2F = T$

Hence LMB for the cart:

$\sum \underline{F} = m\underline{a}_A \Rightarrow 2F\hat{\imath} = m\underline{a}_A$

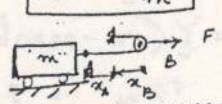
$\Rightarrow \boxed{\underline{a}_A = \frac{2F}{m}\hat{\imath}}$

Using the fact that the total length of string is constant, we obtain the relation:

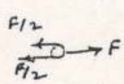
$2x_A - x_B = L \Rightarrow 2\dot{x}_A - \dot{x}_B = 0 \Rightarrow 2\ddot{x}_A = \ddot{x}_B = a_B$

$\Rightarrow \boxed{\underline{a}_B = \frac{4F}{m}\hat{\imath}}$

(c)



FBD of pulley:



FBD of mass:

Using LMB for mass:

$\sum \underline{F} = m\underline{a}_A \Rightarrow \frac{F}{2}\hat{\imath} = m\underline{a}_A$; from kinematics:

$2x_B - x_A = L$

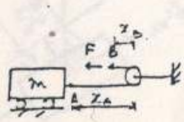
$\Rightarrow 2\dot{x}_B = \dot{x}_A \Rightarrow 2\ddot{x}_B = \ddot{x}_A$

$\Rightarrow \ddot{x}_B = \ddot{x}_A/2 = a_B$

$\Rightarrow \boxed{\underline{a}_A = \frac{F}{2m}\hat{\imath}}$

$\Rightarrow \boxed{\underline{a}_B = \frac{F}{4m}\hat{\imath}}$

(d)



FBD of mass:

from kinematics:

$x_A + x_B = L$

$\dot{x}_A = -\dot{x}_B \Rightarrow \ddot{x}_A = -\ddot{x}_B = -a_B$

$\therefore \boxed{\underline{a}_B = -\underline{a}_A = -\frac{F}{m}\hat{\imath}}$

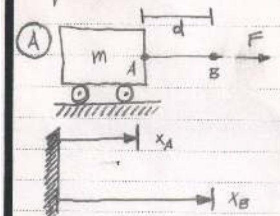
Using LMB: $\sum \underline{F} = m\underline{a}_A$

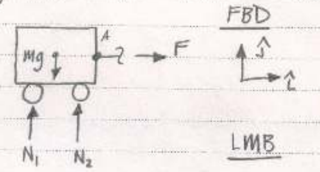
$\Rightarrow F\hat{\imath} = m\underline{a}_A \Rightarrow \boxed{\underline{a}_A = \frac{F}{m}\hat{\imath}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

3.60 Find the accelerations of mass A and point B in the following situations.

(A)





FBD

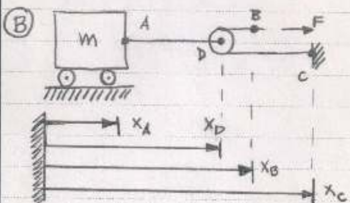
LMB

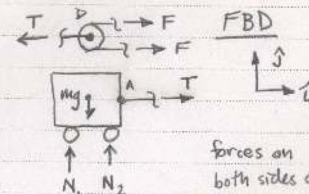
$\hat{\imath}\cdot\{N_1\hat{\jmath}+N_2\hat{\jmath}-mg\hat{\jmath}+F\hat{\imath}=m\ddot{x}_A\hat{\imath}\}$

- for inextensible string between A & B of length d
  $x_A + d = x_B$
- differentiating twice $\ddot{x}_B = \ddot{x}_A$ or

$\boxed{\ddot{x}_A = \frac{F}{m} \quad \ddot{x}_B = \frac{F}{m}}$ dotting LMB

(B)





FBD

forces on both sides of pulley equal, $m_{pulley} = 0$

- notice that $\ddot{x}_A = \ddot{x}_D$
  pulley is massless, string between A & D inextensible
- for string between B & C (ℓ) inextensible we have
  $x_C - x_D + x_B - x_D = \ell$
  differentiating
  $2\ddot{x}_D = \ddot{x}_B$ or

LMB

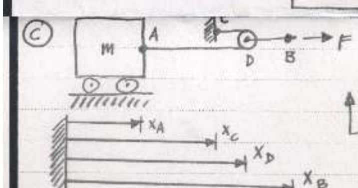
$\hat{\imath}\cdot\{2F\hat{\imath} - T\hat{\imath} = m_{pulley}\ddot{x}_D\hat{\imath}\}$

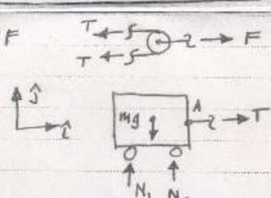
$T = 2F$

$\hat{\imath}\cdot\{N_1\hat{\jmath}+N_2\hat{\jmath}-mg\hat{\jmath}+T\hat{\imath} = m\ddot{x}_A\hat{\imath}\}$

$\boxed{\ddot{x}_B = \frac{4F}{m} \quad \ddot{x}_A = \frac{2F}{m}}$ $\quad \ddot{x}_A = \ddot{x}_D$

(C)





FBD

$m_{pulley} = 0$

LMB

- notice $\ddot{x}_D = \ddot{x}_B$ and
  $x_D - x_A + x_D - x_C = \ell$
  so $\ddot{x}_A = 2\ddot{x}_D$
  $\ddot{x}_A = 2\ddot{x}_B$

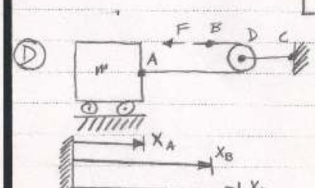
$\hat{\imath}\cdot\{-2T\hat{\imath} + F\hat{\imath} = m_{pulley}\ddot{x}_D\hat{\imath}\}$

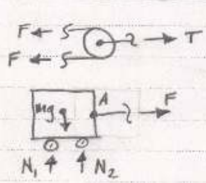
$T = \frac{F}{2}$

$\hat{\imath}\cdot\{N_1\hat{\jmath}+N_2\hat{\jmath}-mg\hat{\jmath}+T\hat{\imath} = m\ddot{x}_A\hat{\imath}\}$

$\boxed{\ddot{x}_B = \frac{F}{4m} \quad \ddot{x}_A = \frac{F}{2m}}$

(D)





FBD

- notice $\ddot{x}_D = 0$
  $x_D - x_A + x_D - x_B = 0$
  $\ddot{x}_A = -\ddot{x}_B$ or

$\hat{\imath}\cdot\{N_1\hat{\jmath}+N_2\hat{\jmath}-mg\hat{\jmath}+F\hat{\imath} = m\ddot{x}_A\hat{\imath}\}$

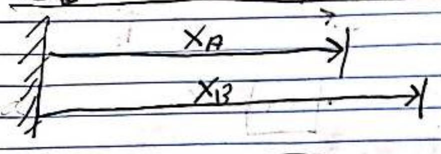
$\boxed{\ddot{x}_A = \frac{F}{m} \quad \ddot{x}_B = -\frac{F}{m}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1/3

13.1.6 Pulleys

Q: Find a_A & a_B in each case.



Kinematics

$\ell = x_B - x_A = \text{const}$

$\Rightarrow a_B = a_A$

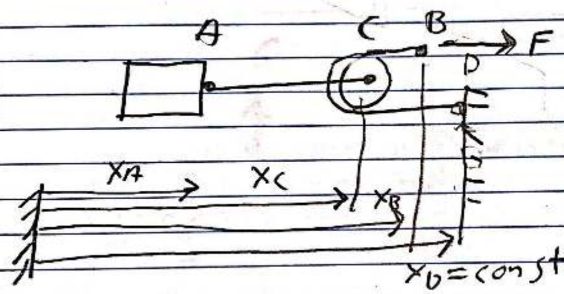
a)

No Pulley

LMB! $F = ma$

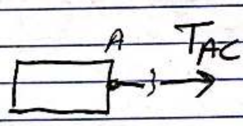
$\Rightarrow \boxed{a_A = a_B = F/m}$

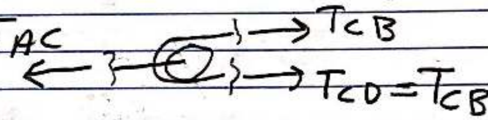
b)



twice the force, half the motion

FBDs





$T_{AC} = 2T_{CB} = 2F$

Kinematics

$\text{const} = \ell_{AC} = x_C - x_A$

$\Rightarrow a_C = a_A$

$\text{const} = \ell_{BCD}$

$= (x_B - x_C) + (x_D - x_C)$

$+ \pi R$

$\Rightarrow a_B = 2a_C = 2a_B$

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Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

2/3

13.1.6 (cont'd)

b) (cont'd)

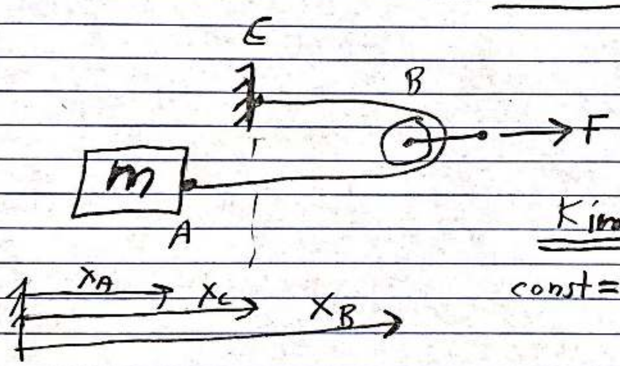
LMB

$$F = ma$$

$$\Rightarrow T_{AC} = m a_A$$

$$2F = m a_A \Rightarrow \boxed{a_A = \frac{2F}{m} \qquad a_B = 2a_A = \frac{4F}{m}}$$

c)



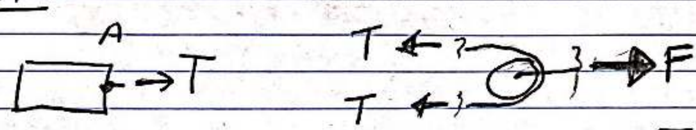
half the force, twice the motion

Kinematics

$$\text{const} = (x_C - x_B) + (x_B - x_A) + \pi R$$

$$\Rightarrow \ddot{x}_A = 2\ddot{x}_B$$

FBDs



$$\Rightarrow F = 2T$$

LMB $\overset{F/2}{T} = m a_A \Rightarrow \boxed{a_A = \frac{F}{2m} \Rightarrow a_B = \frac{a_A}{2} = \frac{F}{4m}}$

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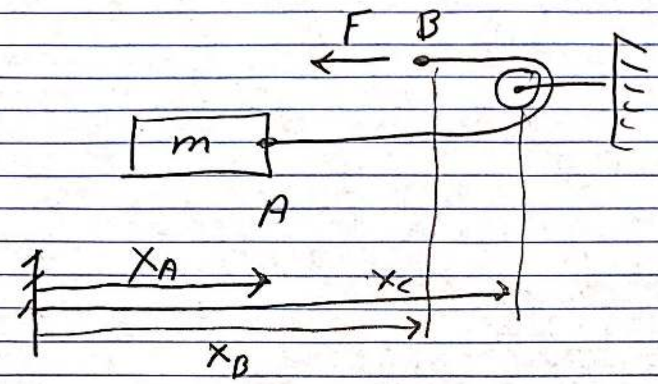
3/3

13.1.6 (cont'd)

d)

reverse force & motion

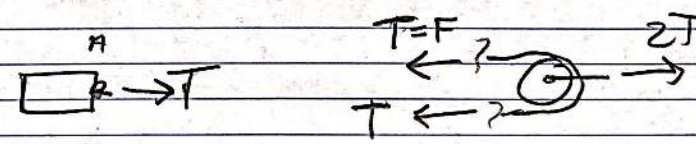


Kinematics

$$(x_c - x_A) + \pi R + (x_c - x_B) = const$$

$$\Rightarrow \ddot{x}_B = -\ddot{x}_A$$

FBDs



LMB

$$F = ma$$

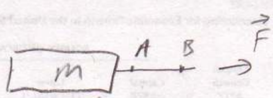
$$\Rightarrow ma_A = F$$

$\ddot{x}_B$

$$\boxed{a_A = \frac{F}{m} \Rightarrow a_B = \frac{-F}{m}}$$

12.6

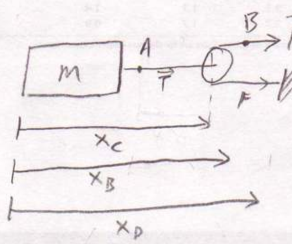
a)



$$F = ma$$

$$\boxed{a = F/m} \quad \text{at both } A \text{ an } B$$

b)



$$T = 2F$$

$$2F = m a_A$$

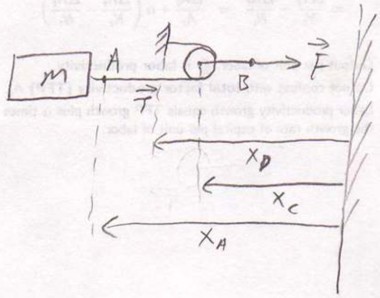
$$\boxed{a_A = \frac{2F}{m}}$$

$$l = (x_B - x_c) + (x_D - x_c) + \pi R$$

$$\{\ddot{\ }\} \to \ddot{x}_B = 2\ddot{x}_c = 2\ddot{x}_a$$

$$\boxed{a_B = 2a_A = \frac{4F}{m}}$$

c)



$$T = m a_A$$

$$T = \frac{F}{2}$$

$$\boxed{a_A = \frac{F}{2m}}$$

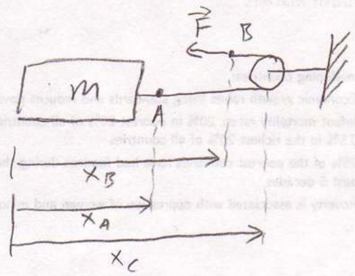
$$l_{tot} = (x_A - x_c) + (x_D - x_c) + \pi R$$

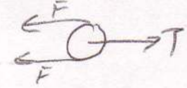
$$\{\ddot{\ }\} = \ddot{x}_A = 2\ddot{x}_c = 2\ddot{x}_B$$

$$\boxed{a_B = \frac{F}{4m}}$$

12.6 continued

d)



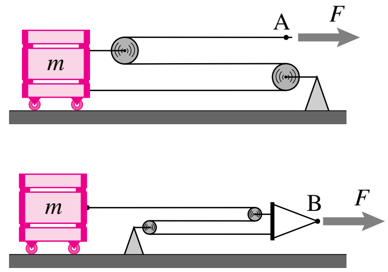


$F = m a_A$

$$\boxed{a_A = \frac{F}{m} \qquad a_B = \frac{F}{m}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.8 What is the ratio of the acceleration of point A to that of point B in each configuration? $m = m$ and $F = F$.



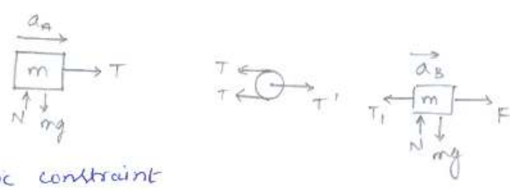
Problem 14.8

Answer:
$\frac{a_A}{a_B} = 81.$

4

13.1.9

F.B.D



Kinematic constraint

$\Delta x_A = 2\Delta x_B$

$a_A = 2a_B \longrightarrow$ (i)

Dynamic Equilibrium

$T' = 2T$

$ma_A = T \longrightarrow$ (ii)

$ma_B = F - T' = F - 2T \longrightarrow$ (iii)

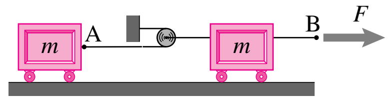
Solving (i) (ii) and (iii)

$ma_B = F - 2m(2a_B)$

$a_B = \frac{F}{5m} \Rightarrow a_A = \frac{2F}{5m}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.9 Find the acceleration of points A and B in terms of F and m.



Problem 14.9

Answer:
$a_A = 2F/(5m)$, $a_B = F/(5m)$ to the right.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.10 For the situation pictured in problem 14.1.9 find the accelerations of the two masses using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

5

13.1.10

Power input by force

$P_{in} = F v_B$

Kinematic requires:

$\Delta x_A = 2\Delta x_B$

$v_A = 2 v_B$

$a_A = 2 a_B$

$\dot{E}_K = m\vec{v}_B \cdot \vec{a}_B + m\vec{v}_A \cdot \vec{a}_A$

$F v_B = m v_B a_B + m(2v_B)(2a_B)$

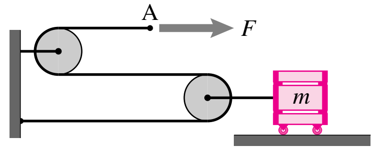
$F = 5 m a_B$

$a_B = \frac{F}{5m}$

$a_A = \frac{2F}{5m}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.11 The point of application A of the force moves twice as fast as the mass. At some instant in time t, the speed of the mass is $\dot{x}$ to the left. Find the input power to the system at time t.



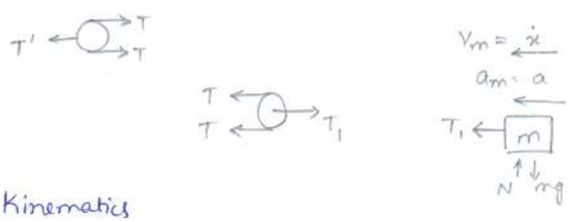
Problem 14.11

Answer:
$P = 2F\dot{x}$

6

13·1·11

FBD



Kinematics

$\Delta x_A = 2\Delta x_m$

$V_A = 2V_m$

$a_A = 2a_m$

Input power to the system at any instant

$P_{in} = FV_A$

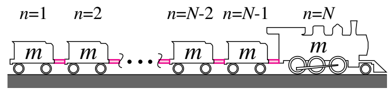
$= 2FV_m$

$V_m = \dot{x}$

$P_{in} = 2F\dot{x}$

$P_{in} = 2F\dot{x}$

14.1.12 A train engine of mass m pulls and accelerates N cars, each of mass m. The power of the engine is P_t and its speed is v_t. Find the tension T_n between car n and car $n+1$. Assume there is no resistance and the ground is level. Assume the cars are connected with rigid links.

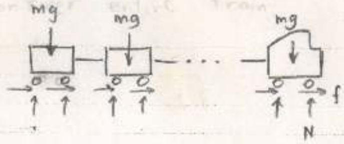


Problem 14.12

Answer:
$T_n = \frac{P_t}{v_t}\frac{n}{N}$.

3.59

Consider entire train:



NOTATION

$(\cdot\cdot)_N$ refers to system of N cars

$(\cdot\cdot)_n$ ... system of n cars

$\underset{\sim}{v}_t = v_t \hat{\imath}, \quad \underset{\sim}{a}_t = a_t \hat{\imath}$

None of the forces do work

$-mg\hat{\jmath} \cdot \underset{\sim}{v}_t = 0$

$N\hat{\jmath} \cdot \underset{\sim}{v}_t = 0$

$f\hat{\imath} \cdot \underset{\sim}{v}_c = 0$ because $\underset{\sim}{v}_c = 0$

instantaneous velocity of contact point of wheel on ground.

$\therefore \quad \dot{W}_N = (\dot{E}_K)_N + (\dot{E}_P)_N + (\dot{D})_N$

$0 = M v_t a_t + 0 + (-P_t)$

net work on system because all external forces do 0 work.

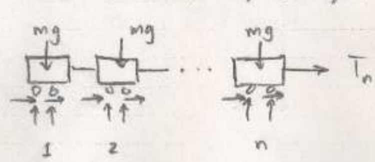
total mass

power supplied by engine is negative dissipation.

$\dot{E}_P = 0$ by assumption,

$\therefore \boxed{P_t = M v_t a_t = (Nm) v_t a_t} \Rightarrow \frac{P_t}{v_t} = Nm a_t$

Cut FBD of cars 1, 2, ..., n.



only force that does work is T_n.

$\dot{W}_n = T_n v_t = (\dot{E}_K)_n + (\dot{E}_P)_n + (\dot{D})_n$

$= (nm) v_t a_t + 0 + 0$

$T_n = (nm) a_t = \frac{n}{N}(Nm) a_t = \boxed{\frac{n}{N}\frac{P_t}{v_t}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

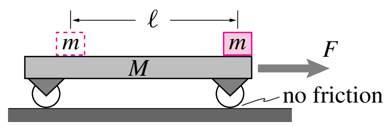
14.1.13 A cart of mass M, initially at rest, can move horizontally along a frictionless track. When $t = 0$, a force F is applied as shown to the cart. During the acceleration of M by the force F, a small box of mass m slides along the cart from the front to the rear. The coefficient of friction between the cart and the box is μ, and it is assumed that the acceleration of the cart is sufficient to cause sliding.

a) Draw free-body diagrams of the cart, the box, and the cart and box together.

b) Write the equation of linear momentum balance for the cart, the box, and the system of cart and box.

c) Show that the equations of motion for the cart and box can be combined to give the equation of motion of the mass center of the system of two bodies.

d) Find the displacement of the cart at the time when the box has moved a distance ℓ along the cart.



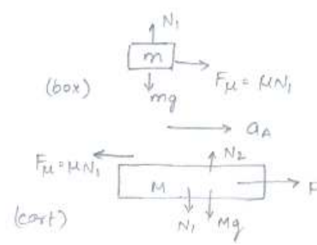
Problem 14.13

Answer:
displacement of cart $= \frac{F-\mu mg}{F-\mu(m+M)g}\ell$

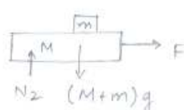
7

13.1.13

a)



Together



b) Linear momentum Balance

$N_1 = mg$

box $\quad m a_B = \mu N_1 = \mu mg \Rightarrow a_B = \mu g \longrightarrow (i)$

cart $\quad m a_A = F - \mu N_1 = F - \mu mg$

$$a_A = \frac{F - \mu mg}{M} \longrightarrow (ii)$$

combined

$(M+m)\, a_{cm} = F$

$$a_{cm} = \frac{F}{M+m} \longrightarrow (iii)$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

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c) Equation of motion of centre of Mass

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{mx_1 + Mx_2}{m+M}$$

$$\Rightarrow \quad a_{cm} = \frac{ma_B + Ma_A}{m+M}$$

$$a_{cm} = \frac{m(\mu g) + M\left(\frac{F-\mu mg}{M}\right)}{M+m}$$

$$a_{cm} = \frac{(\mu mg + F - \mu mg)}{M+m} = \frac{F}{M+m}$$

This is same as (iii) Hence proved

d) At $t=0$, $u_A = u_B = 0$

Acc of the Box when seen from cart frame

$$a_{B/A} = a_B - a_A = \mu g - \frac{F-\mu mg}{M} = \frac{\mu g(m+M) - F}{M}$$

Displacement 's' $= -l$

initial velocity $u_{B/A} = 0$

$$-l = -\frac{1}{2}\left(\frac{F-\mu(M+m)}{M}\right)t^2$$

9

$$t = \sqrt{\frac{2Ml}{F-\mu(M+m)}}$$

Displacement of Cart in this time

$$x_A = \frac{1}{2}\left(\frac{F-\mu mg}{M}\right)t^2$$

$$x_A = \frac{1}{2}\left(\frac{F-\mu mg}{M}\right)\left(\frac{2Ml}{F-\mu(M+m)}\right)$$

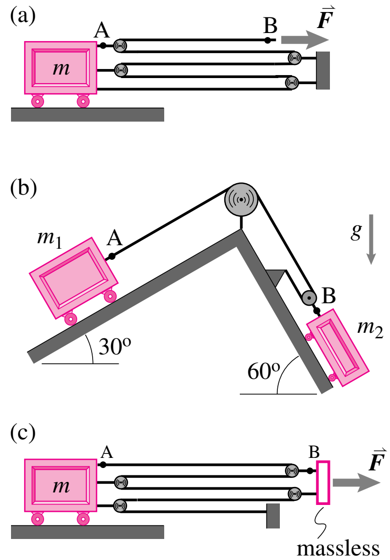
$$x_A = \frac{l(F-\mu mg)}{F-\mu(M+m)}$$

14.1.14 For the situations pictured, find the accelerations of mass A and of point B. Clearly define any variables, coordinates or sign conventions that you use.

a) A single mass and four pulleys.

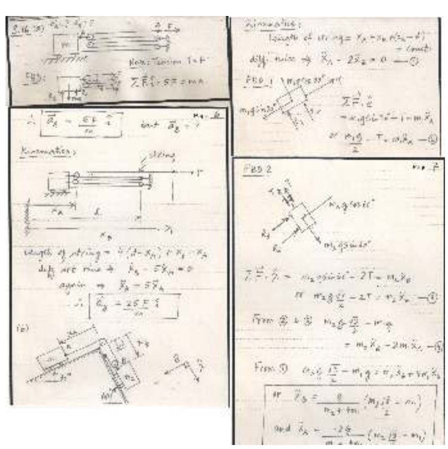
b) Two masses and two pulleys.

c) A single mass and four pulleys.



Problem 14.14: Various pulley arrangements.

Answer:
(a) $\vec{a}_A = \frac{5F}{m}\hat{\imath}$, $\vec{a}_B = \frac{25F}{m}\hat{\imath}$, where $\hat{\imath}$ is parallel to the ground and points to the right.
(b) $\vec{a}_A = \frac{g}{(4m_1+m_2)}(2m_2 - \sqrt{3}m_2)\hat{\lambda}_1$, $\vec{a}_B = -\frac{g}{2(4m_1+m_2)}(2m_1 - \sqrt{3}m_2)\hat{\lambda}_2$, where $\hat{\lambda}_1$ is parallel to the slope that mass m_1 travels along, pointing down and to the left, and $\hat{\lambda}_2$ is parallel to the slope that mass m_2 travels along, pointing down and to the right.
$a_A = \frac{5F}{4m}$ and $a_B = \frac{25F}{16m}$ in the direction of F.



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3.66. from 3.62 we know that: Page 4

$\boxed{\ddot{x}_A = 2\ddot{x}_B}$ and $\boxed{\dot{x}_A = 2\dot{x}_B}$

Using energy balance for the whole system:

$\sum \underline{F}\cdot\underline{v} = m v a$

$F v_B = m v_A a_A + m v_B a_B$

from kinematics, we know that $v_B = \frac{v_A}{2}$ and $a_B = \frac{a_A}{2}$

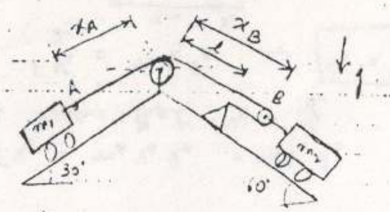
$\therefore F\frac{v_A}{2} = m v_A a_A + m\frac{v_A}{2}\cdot\frac{a_A}{2}$

$\therefore \frac{F}{2} = m\left(a_A + \frac{a_A}{4}\right) = m\left(\frac{5a_A}{4}\right)$

$\Rightarrow \boxed{a_A = \frac{2}{5}\frac{F}{m}}$

$\Rightarrow \boxed{a_B = \frac{a_A}{2} = \frac{F}{5m}}$

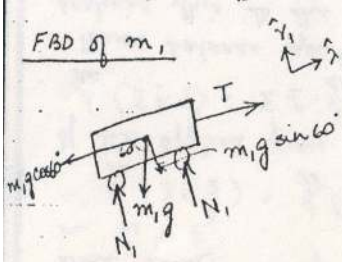
3.67
(b)



Kinematics: $-x_A + x_B + x_B - \ell = L$

$\therefore -\dot{x}_A + 2\dot{x}_B = 0 \Rightarrow \boxed{\ddot{x}_A = 2\ddot{x}_B}$

FBD of m_1



Using LMB:

$\sum \underline{F} = m\,\underline{a}_A$

$\left[T - m_1 g\cos 60°\right]\hat{\lambda}_1 = m_1\,\underline{a}_A$

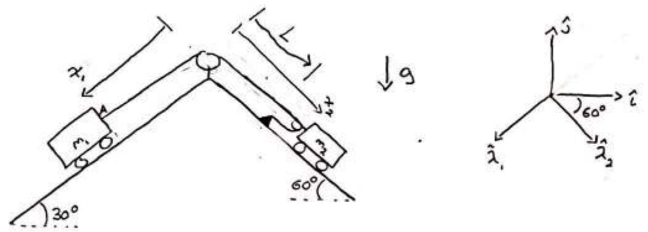
$\left[T - \frac{m_1 g}{2}\right]\hat{\lambda}_1 = m_1\,\underline{a}_A$

$\Rightarrow \underline{a}_A = \frac{1}{m_1}\left[T - \frac{m_1 g}{2}\right]\hat{\lambda}_1$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

13.1.14:

Find: Acceleration of point A and point B



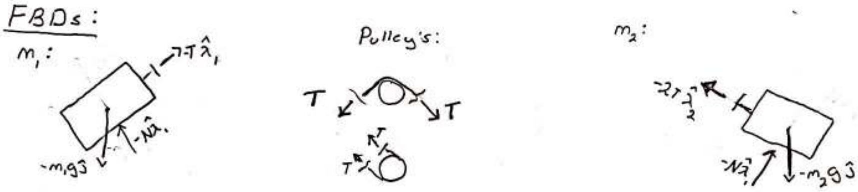
★ Kinematic relation between ropes:

$x_1 + x_2 + (x_2 - L) = \text{constant}$

$\dot{x}_1 + \dot{x}_2 + \dot{x}_2 = 0$

$\ddot{x}_1 + \ddot{x}_2 + \ddot{x}_2 = 0 \rightarrow \ddot{x}_1 = -2\ddot{x}_2$

FBDs:



LMB:

$m_1\ddot{x}_1\hat{\lambda}_1 = -N\hat{\lambda} + T\hat{\lambda}_1 - m_1 g\hat{\jmath}$

$= -T\hat{\lambda}_1 - m_1 g\hat{\jmath}\} \quad \{\}\cdot\hat{\lambda}_1$

$= -T + m_1 g\sin(30°)$

LMB:

$m_2\ddot{x}_2\hat{\lambda}_2 = -N\hat{\lambda}_1 - 2T\hat{\lambda}_2 - m_2 g\hat{\jmath}$

$= -2T\hat{\lambda}_2 - m_2 g\hat{\jmath}\} \quad \{\}\cdot\hat{\lambda}$

$= -2T + m_2 g\sin(60°)$

Kinematics:

Rope is inextensible, therefore a change in length of one unit corresponds to a -2 change in length in x_2. By kinematics★: (If confused look at top right)

$\ddot{x}_1 = -2\ddot{x}_2$

Solution:

Unknowns: $\ddot{x}_1, \ddot{x}_2, T$

Our three equations: ① $m_1\ddot{x}_1 = -T + m_1 g\sin(30°)$, ② $m_2\ddot{x}_2 = -2T + m_2 g\sin(60°)$, ③ $\ddot{x}_1 = -2\ddot{x}_2$

Solve equation 1 for T:

$T = -m_1\ddot{x}_1 + m_1 g\sin(30°)$

Substitute T into 2:

$m_2\ddot{x}_2 = -2(-m_1\ddot{x}_1 + m_1 g\sin(30°)) + m_2 g\sin(60°)$

Use $\ddot{x}_1 = -2\ddot{x}_2$

$m_2\ddot{x}_2 = -2(2m_1\ddot{x}_2 + m_1 g\sin(30°)) + m_2 g\sin(60°)$

$= -4m_1\ddot{x}_2 - 2m_1 g\sin(30°) + m_2 g\sin(60°)$

$(4m_1 + m_2)\ddot{x}_2 = m_2 g\sin(60°) - 2m_1 g\sin(30) \rightarrow \ddot{x}_2 = \dfrac{m_2 g\sin(60°) - 2m_1 g\sin(30°)}{4m_1 + m_2} = \boxed{\dfrac{\sqrt{3}\,m_2 g - 2m_1 g}{8m_1 + 2m_2}}$

$\ddot{x}_1 = -2\ddot{x}_2 \rightarrow \ddot{x}_1 = \boxed{\dfrac{2m_1 g - \sqrt{3}\,m_2 g}{4m_1 + m_2}}$

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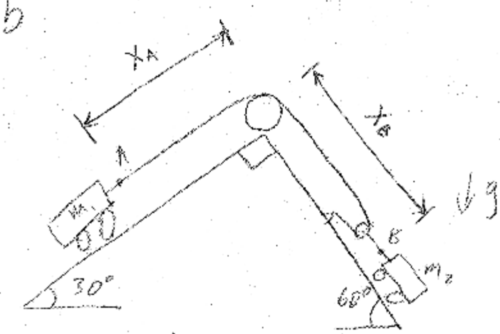
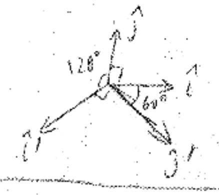
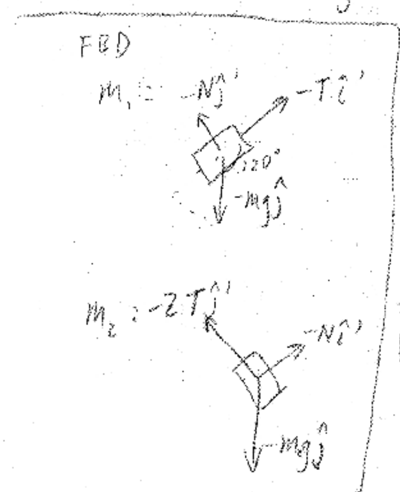
12.14 b





T is the tension force in the cable.



$$m_1\ddot{x}_A = -m_1 g\hat{\jmath} - T\hat{\imath}' - N\hat{\jmath}'$$

$\{\} \cdot \hat{\imath}'$:

$$m_1\ddot{x}_A = -m_1 g(\hat{\jmath}\cdot\hat{\imath}') - T$$

$$\hat{\jmath}\cdot\hat{\imath}' = \cos(120) = -\sin(30)$$

$$\boxed{m_1\ddot{x}_A = m_1 g\sin 30 - T} \quad ①$$

$$m_2\ddot{x}_B = -2T\hat{\jmath}' - N\hat{\imath}' - mg\hat{\jmath}$$

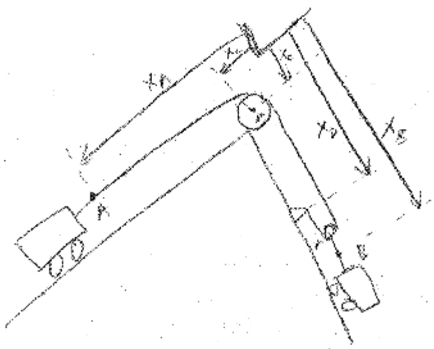
$\{\} \cdot \hat{\jmath}'$:

$$m_2\ddot{x}_B = -2T - m_2 g(\hat{\jmath}\cdot\hat{\jmath}')$$

$$\hat{\jmath}\cdot\hat{\jmath}' = \cos(150) = -\sin 60$$

$$\boxed{m_2\ddot{x}_B = -2T + m_2 g\sin 60} \quad ②$$

12.14 b continued



$$\ell_{tot} = (x_A - x_c) + \frac{\pi R}{2} + (x_B - x_c) + (x_B - x_D) + \pi r$$

$$\{\}^{\cdot\cdot}: \quad \ddot{x}_A + 2\ddot{x}_B = 0$$

$$\boxed{\ddot{x}_A = -2\ddot{x}_B} \quad (3)$$

We have 3 unknowns : $\ddot{x}_A$, $\ddot{x}_B$, T

(1) $\Rightarrow$ $T = m_1 g \sin 30 - m_1 \ddot{x}_A$

substitute into (2)

$$m_2 \ddot{x}_B = -2(m_1 g \sin 30 - m_1 \ddot{x}_A) + m_2 g \sin 60$$

(3) $\Rightarrow$ $\ddot{x}_A = -2\ddot{x}_B$ substitute into above

$$m_2 \ddot{x}_B = -2m_1 g \sin 30 - 4m_1 \ddot{x}_B + m_2 g \sin 60$$

$$(m_2 + 4m_1)\ddot{x}_B = -2m_1 g \sin 30 + m_2 g \sin 60$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

$$\boxed{\begin{aligned}\ddot{x}_B &= \frac{-2m_1 g\sin 30^\circ + m_2 g\sin 60^\circ}{m_2 + 4m_1} \\ &= \frac{-2m_1 + \sqrt{3}m_2}{2m_2 + 8m_1}\, g \\ \ddot{x}_A &= -2\ddot{x}_B = \frac{2m_1 - \sqrt{3}m_2}{m_2 + 4m_1}\, g\end{aligned}}$$

∴ Acceleration of point A is

$$\vec{a}_A = \ddot{x}_A \hat{\imath}' = \frac{2m_1 - \sqrt{3}m_2}{m_2 + 4m_1}\, g\, \hat{\imath}'$$

$$= \frac{2m_1 - \sqrt{3}m_2}{m_2 + 4m_1}\, g \left(-\frac{\sqrt{3}}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}\right)$$

Acceleration of point B is

$$\vec{a}_B = \ddot{x}_B \hat{\jmath}' = \frac{\sqrt{3}m_2 - 2m_1}{2m_2 + 8m_1}\, g\, \hat{\jmath}'$$

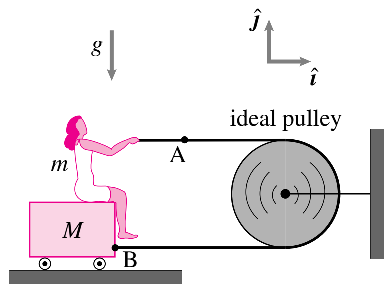
$$= \frac{\sqrt{3}m_2 - 2m_1}{2m_2 + 8m_1}\, g \left(\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right)$$

14.1.16 A person of mass m, modeled as a rigid object, is sitting on a cart of mass $M > m$ and pulling the string towards herself. The coefficient of friction between her seat and the cart is μ. Point B is attached to the cart and point A is attached to the rope.

a) If you are given that she is pulling rope in with acceleration a_0 relative to herself (that is, $\vec{\boldsymbol{a}}_{A/B} \equiv \vec{\boldsymbol{a}}_A - \vec{\boldsymbol{a}}_B = -a_0\hat{\boldsymbol{\imath}}$) and that she is not slipping relative to the cart, find $\vec{\boldsymbol{a}}_A$. (Answer in terms of some or all of $m, M, g, \mu, \hat{\boldsymbol{\imath}}$ and a_0.)

b) Find the largest possible value of a_0 without the person slipping off the cart? (Answer in terms of some or all of m, M, g and μ. You may assume her legs get out of the way if she slips backwards.)

c) If instead, $M < m$, what is the largest possible value of a_0 without the person slipping off the cart? (Answer in terms of some or all of m, M, g and μ. You may assume her legs get out of the way if she slips backwards.)

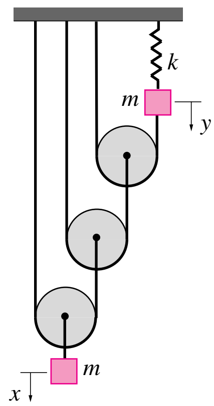


Problem 14.16: **Pulley.**

Answer:
$\vec{\boldsymbol{a}}_A = -\frac{a_0}{2}\hat{\boldsymbol{\imath}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.18 What is the natural frequency of vibration of this system? Include gravity. x measures the vertical position of the lower mass from equilibrium. y measures the vertical position of the upper mass from equilibrium.



Problem 14.18

Answer:
angular frequency of vibration $\equiv \lambda = \sqrt{\frac{64k}{65m}}$.

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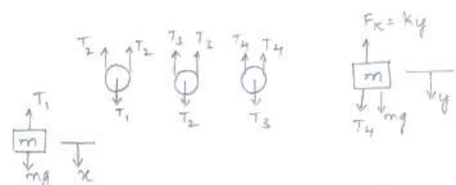
13.1.18



Kinematic constraint

$\Delta x = \frac{\Delta y}{8}$

$\dot{x} = \frac{\dot{y}}{8}$ & $\ddot{x} = \frac{\ddot{y}}{8}$ $\longrightarrow$ (i)

Dynamical equilibrium

$m\ddot{x} = mg - T_1 \longrightarrow$ (ii)

$m\ddot{y} = mg + T_4 - ky' \longrightarrow$ (iii)

$T_1 = 2T_2$; $T_2 = 2T_3$; $T_3 = 2T_4$; $T_1 = 8T_4$

$\Rightarrow m\ddot{y} = mg + \frac{1}{8}(mg - m\ddot{x}) - ky'$

$\Rightarrow m\ddot{y} = mg + \frac{1}{8}\left(mg - \frac{m\ddot{y}}{8}\right) - ky'$

Now Assume $y' = y_0 + y$; y = position from equilibrium

$\ddot{y}' = \ddot{y}$

Such that

$ky_0 = \frac{9mg}{8}$

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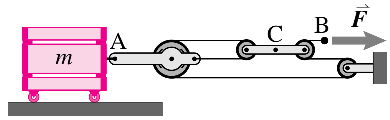
$$m\ddot{y}'\left(1+\frac{1}{64}\right) = mg\left(\frac{9}{8}\right) - k(y_0+y')$$

$$m\ddot{y}'\left(\frac{65}{64}\right) = -ky'$$

$$\ddot{y}' = -\frac{64k}{65m}y' = -\omega^2 y'$$

Natural frequency of vibration

$$\omega = \sqrt{\frac{64k}{65m}}$$



Problem 14.19

14.1.19 For the situation pictured, find the acceleration of mass A and points B and C shown. [Hint: the situation with point C is subtle.]

Answer:
The assembly at C has, with the idealizations given, no mass and no net force acting on it. The equation $\vec{F} = m\vec{a}$ says $0 = 0$ and, within the assumptions made, the acceleration is indeterminate. If you built such a machine, the acceleration of C would be determined by the effects which are neglected here, such as the mass of the central assembly and the friction in the pulley bearings. If you built such an assembly you would see that only small additional forces are needed to move point C most any which way.

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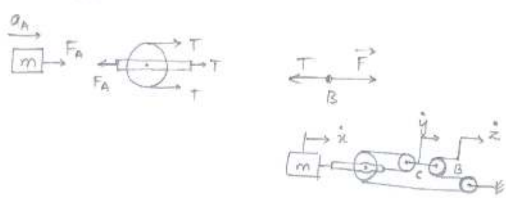
13.1.19

FBD



The rate of change of length of rope is

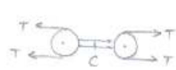
$$\dot L = -\dot x + 2(\dot y - \dot x) - \dot y + (\dot z - \dot y) = 0$$

$$\Rightarrow \quad -3\dot x + \dot z = 0$$

$$\dot z = 3\dot x \quad \& \quad \ddot z = 3\ddot x \longrightarrow (i)$$

Dynamic equilibrium

$$m a_A = F_A = 3T = 3F$$



$$a_A = \frac{3F}{m} \longrightarrow (ii)$$

using relation (i) we get

$$a_B = 3a_A = \frac{9F}{m} \longrightarrow (iii)$$

14.1.20 For the situation pictured in problem 14.1.19, find the acceleration of point A using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

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13.1.20

using Energy Methods

as derived already

$\dot{Z} = 3\dot{x}$

$P_{in} = F\dot{Z}$

$\dot{E}_K = m a_A \dot{x}$

$m a_A \dot{x} = F\dot{Z}$

$m a_A = 3F$

$a_A = \frac{3F}{m}$

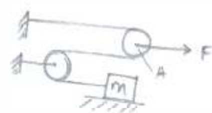
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.21 Design a pulley system. You are to design a pulley system to move a mass. There is no gravity. Point A has a force $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{\imath}}$ pulling it to the right. Mass B has mass m_B. You can connect point A to the mass with any number of ideal strings and ideal pulleys. You can make use of rigid walls or supports anywhere you like (say, to the right or left of the mass). You must design the system so that mass B accelerates to the left with $\frac{F}{2m_B}$ (i.e., $\vec{\boldsymbol{a}}_B = -\frac{F}{2m_B}\hat{\boldsymbol{\imath}}$).

a) Draw the system clearly. Justify your answer with enough words or equations so that a reasonable person, say a grader, can tell that you understand your solution.

b) Find the acceleration of point A.

(14)



FBD

Kinematics

$F\dot{x}_A = T v_B$

$\dot{x}_A = \frac{v_B}{2} \Rightarrow a_A = \frac{a_B}{2}$

$F = 2T \longrightarrow (i)$

$m_B a_B = T$

$\Rightarrow a_B = \frac{T}{m_B} = \frac{F}{2m_B}$

$\vec{a}_B = -\frac{\vec{F}}{2m_B}$

$\vec{a}_A = -\frac{\vec{a}_B}{2} = -\frac{\vec{F}}{4m_B}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.22 Design a pulley system. You are to design a pulley system to move a mass. There is no gravity. Point A has a force $\vec{F} = F\hat{\imath}$ pulling it to the right. Mass B has mass m_B. You can connect the point A to the mass with any number of ideal strings and ideal pulleys. You can make use of rigid walls or supports anywhere you like (say, to the right or left of the mass). Draw the system clearly. Justify your answer with enough words or equations to convince a skeptical person that your solution is correct. You must design the system so that the mass B accelerates .

a) to the left with $\frac{F}{m_B}$ (i.e., $\vec{a}_B = -\frac{F}{m_B}\hat{\imath}$)

b) to the left with $\frac{2F}{m_B}$

c) to the left with $\frac{F}{2m_B}$

d) to the right with $\frac{2F}{m_B}$

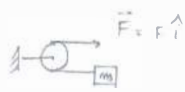
e) to the right with $\frac{F}{2m_B}$

f) to the left with $\frac{8F}{m_B}$

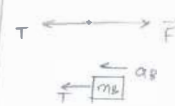
g) to the right with $\frac{F}{5m_B}$

$\vec{F} = F\hat{i}$

a) $\vec{a}_B = -\frac{F}{m_B}\hat{i}$



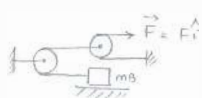
FBD



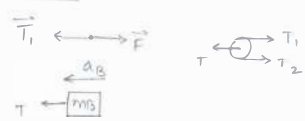
$\vec{T} = \vec{F}$

$\vec{a}_B = \frac{\vec{T}}{m_B} = -\frac{\vec{F}}{m_B}$

b) $\vec{a}_B = -\frac{2F\hat{i}}{m_B}$



F.B.D



equation

$F = T_1$

$T = 2T_1$

$m_B a_B = T = 2T_1 = 2F$

$a_B = \frac{2F}{m_B}$

Taking direction

$\vec{a}_B = -\frac{2\vec{F}}{m_B}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

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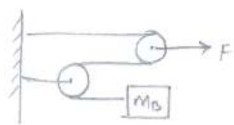
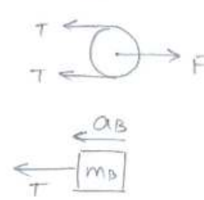
c) $\vec{a}_B = -\dfrac{F\hat{i}}{2m_B}$



FBD



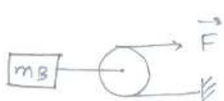
Equation

$m_B a_B = T = \dfrac{F}{2}$

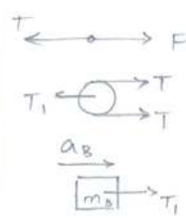
$a_B = \dfrac{F}{2m}$

$\vec{a}_B = \dfrac{-\vec{F}}{2m}$

d) $\vec{a}_B = \dfrac{2\vec{F}}{m_B}$



FBD



Equation

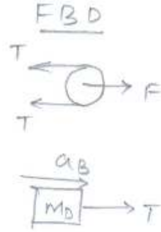
$m_B a_B = T_1 = 2T = 2F$

$a_B = \dfrac{2F}{m_B}$

$\vec{a}_B = \dfrac{2\vec{F}}{m_B}$

e) $\vec{a}_B = \dfrac{\vec{F}}{2m_B}$

FBD



Equation

$F = 2T$

$m_B a_B = T = \dfrac{F}{2}$

$a_B = \dfrac{F}{2m_B}$

$a_B = \dfrac{\vec{F}}{2m_B}$

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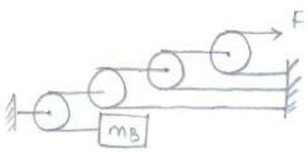
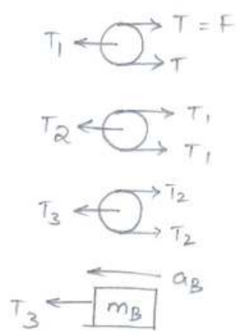
f) $\vec{a}_B = \dfrac{-8\vec{F}}{m_B}$



F.B.D



Equation

$T_1 = 2T \; ; \; T_2 = 2T_1 \; ; \; T_3 = 2T_2$

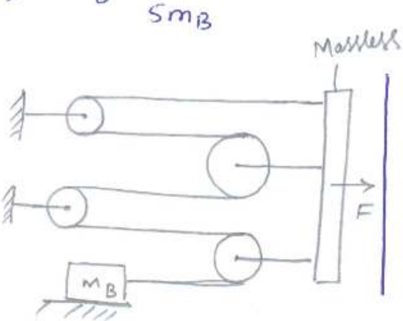
$T_3 = 8T = 8F$

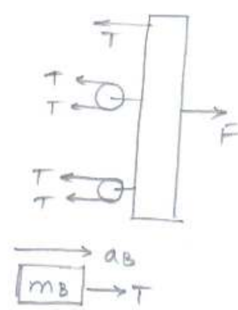
$m_B a_B = T_3$

$a_B = \dfrac{T_3}{m_B} = \dfrac{8F}{m_B}$

$\vec{a}_B = -\dfrac{8\vec{F}}{m_B}.$

g) $\vec{a}_B = \dfrac{\vec{F}}{5m_B}$





Equation

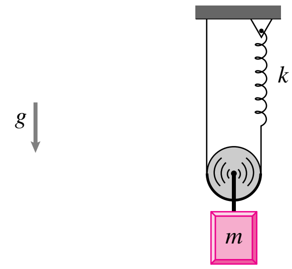
$5T = F$

$m_B a_B = T = \dfrac{F}{5}$

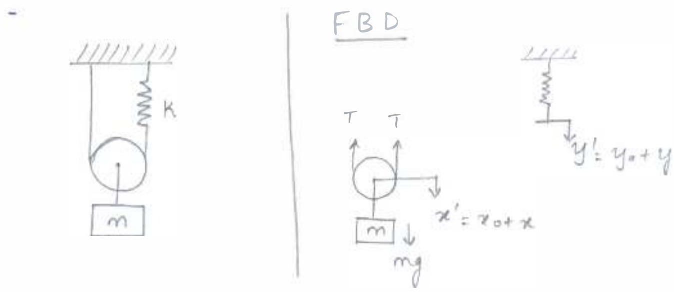
$\vec{a}_B = \dfrac{\vec{F}}{5m_B}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.23 Pulley and spring. For the hanging mass find the period of oscillation. Only vertical motion is of interest. There is gravity.



Problem 14.23



Equations;

Kinematics

$$\dot{y}' = 2\dot{x}'$$

$$\dot{y} = 2\dot{x}$$

$$\ddot{y} = 2\ddot{x}$$

$$m\ddot{x} = mg - 2T \qquad T = ky' \quad \Rightarrow \quad m\ddot{x} = mg - 2ky'$$

At equilibrium $\ddot{x} = 0 \Rightarrow mg = 2ky_0 \Rightarrow y_0 = \frac{mg}{2k}$

$$\Rightarrow m\ddot{x} = mg - k\left[\frac{mg}{2k} + y\right] \Rightarrow m\ddot{x} = -2ky$$

$$m\ddot{y} = -4ky$$

and

$$m\ddot{x} = -4kx$$

Angular frequency $\omega = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$

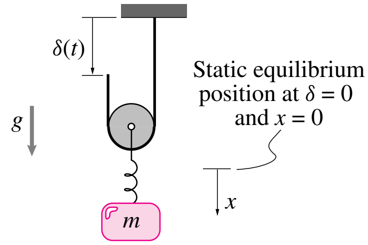
$$T = \pi\sqrt{\frac{m}{k}}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.24 The spring-mass system shown ($m = 10$ slugs ($\equiv$ lb $\cdot$ sec^2/ft), $k = 10\,\text{lb}/ft$) is excited by moving the free end of the cable vertically according to $\delta(t) = 4\sin(\omega t)$ ∈, as shown in the figure.

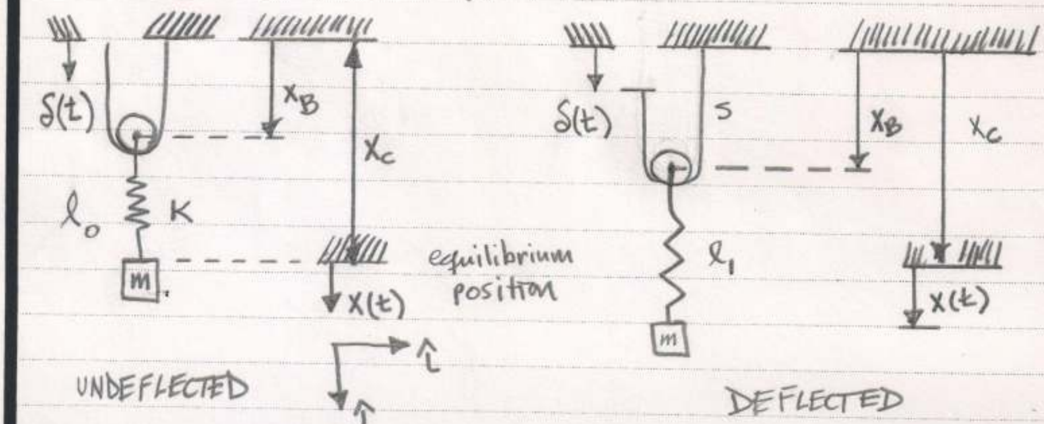
a) Derive the equation of motion for the block in terms of the displacement x from the static equilibrium position, as shown in the figure.

b) If $\omega = 0.9$ rad/s, check to see if the pulley is always in contact with the cable (ignore the transient solution).



Problem 14.24

\# 10.6 PART A)



FBD

$F = K\Delta l$

$s = 2x_B - \delta(t)$ length of string

$l_0 = x_c - s/2$

$l_1 = x(t) + x_c - x_B$

$\quad = x(t) + x_c - \frac{s}{2} - \delta(t)/2$

$\Delta l = x(t) - \delta(t)/2$

LMB $\quad \sum \underline{F} = m\underline{a} \qquad \underline{a} = \ddot{x}\hat{\jmath}$

$-K\Delta l\,\hat{\jmath} + mg\,\hat{\jmath} = m\ddot{x}\,\hat{\jmath}$ dot w/ $\hat{\jmath}$

$-Kx + \frac{K\delta}{2} + mg = m\ddot{x}$

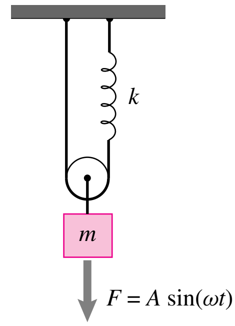
$$\boxed{\ddot{x} + \frac{K}{m}x = +\frac{K\delta}{2m} + g}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.25 The block of mass m hanging on the spring with constant k and a string shown in the figure is forced by $F = A\sin(\omega t)$. Do not neglect gravity. The pulley has negligible mass.

a) What is the differential equation governing the motion of the block? You may assume that the only motion is vertical motion.

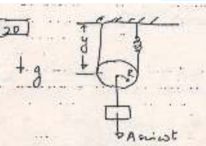
b) Given A, m and k, for what values of ω would the string go slack at some point in the cyclical motion? (The common assumption in such problems, which you can use, is to neglect the homogeneous solution to the differential equation. It is assumed that the damping, small enough to be neglected in the governing equations is large enough so that the particular solution will have damped out at the time of observation.)



Problem 14.25

Answer:
(a) $m\ddot{x} + 4kx = A\sin\omega t + mg$, where x is the distance measured from the mass position when the spring is unstretched.
(b) The string will go slack if $\omega > \sqrt{\frac{4k}{m}\left(1 - \frac{A}{mg}\right)}$.

1D.20



Let the initial position of the system (when there are no forces acting) be y_0

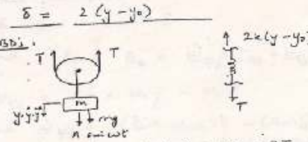
Kinematics: Page 9

$2y_0 + \pi R$ = unstretched length of spring + string

$2y + \pi R$ = stretched length of spring + length of string

$\therefore\ \delta = 2(y - y_0)$

FBD's:



LMB: $\therefore\ m\ddot{y} = A\sin\omega t + mg - 2T$

$\therefore\ T = 2k(y - y_0)$

$\Rightarrow m\ddot{y} + 4k(y - y_0) = A\sin\omega t + mg$

If we measure distances from $y_0 \Rightarrow x = y - y_0$

then $\ddot{x} = \ddot{y}$, and the eqn of motion is:

$\boxed{m\ddot{x} + 4kx = A\sin\omega t + mg}$ — (*)

Solution: $x = \underbrace{(\text{homogeneous part})}_{\text{ignore (transient)}} + \underbrace{(\text{particular integral})}_{\text{we want to find this}}$

Guess $x = C_1\sin\omega t + C_2$ (C_1, C_2 to be found)

then, plugging into (*)

$\left(-C_1\omega^2 + \frac{4k}{m}C_1\right)\sin\omega t + \frac{4k}{m}C_2 = \frac{A}{m}\sin\omega t + g$

$\therefore\ C_1 = \left(\frac{A}{4k - m\omega^2}\right) \qquad C_2 = \left(\frac{mg}{4k}\right)$

$\therefore\ x = \left[\frac{A}{4k - m\omega^2}\sin\omega t + \frac{mg}{4k}\right]$

The string will go slack if $x = 0$ at some point in the cycle $\Rightarrow$ if Page 10

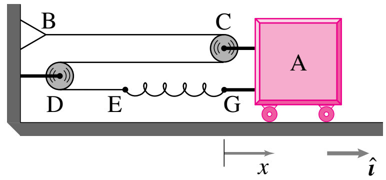
$\frac{A}{4k - m\omega^2} \geq \frac{mg}{4k} \Rightarrow \boxed{\omega > \sqrt{\frac{4k}{m}\left(1 - \frac{A}{mg}\right)}}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

14.1.26 Block A, with mass m_A, is pulled to the right a distance d from the position it would have if the spring were relaxed. It is then released from rest. Assume ideal string, pulleys and wheels. The spring has constant k.

a) What is the acceleration of block A just after it is released (in terms of k, m_A, and d)?

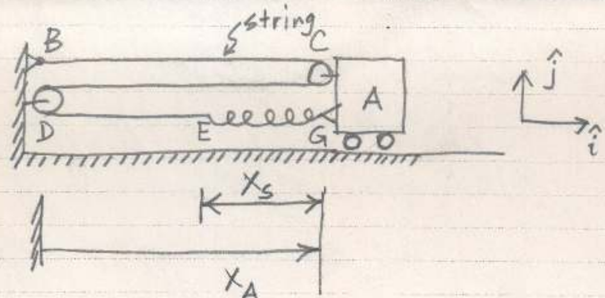
b) What is the speed of the mass when the mass passes through the position where the spring is relaxed? .



Problem 14.26

Answer:
(a) $\vec{a}_A = -\frac{9kd}{m_A}\hat{\imath}$.
(b) $v = 3d\sqrt{\frac{k}{m_A}}$

(3.34)



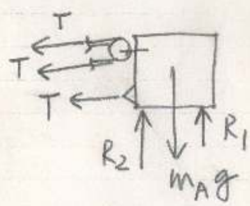
Mass pulled to right a distance 'd'.

Note: $3x_A = \text{Length}_{\text{string}} + x_s$

$\therefore 3\Delta x_A = \Delta x_s$; if $\Delta x_A = d$, $\Delta x_s = 3d$

(a) $\therefore T = 3kd$

FBD



$\sum \vec{F}\cdot\hat{\imath} = -3T = ma$

$\Rightarrow -9kd = ma$

$$\boxed{\vec{a} = -\frac{9kd}{m}\hat{\imath}}$$

(b) Energy Conservation

Speed of mass when spring is relaxed?

$(PE + KE)_1 = (PE + KE)_2$

$\frac{1}{2}k(3d)^2 + 0 = 0 + \frac{1}{2}m_A v^2$

$\therefore v = \sqrt{\frac{k}{m_A}}\cdot 3d$, $\boxed{\vec{v} = -3d\sqrt{\frac{k}{m_A}}\,\hat{\imath}}$

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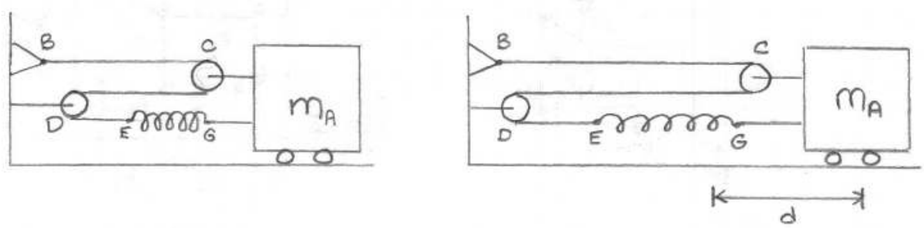
TAM 203
Homework Solution

Due 4/8/08

12.26

ORIGINAL: PULLED:

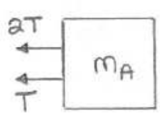


a) Find the stretching of the spring:

We know BC has increased by d, as well as CD and DG. Since the string is inextensible, the spring must have stretched by 3d.

$\therefore$ $F_{spring} = 3kd$, which must equal T_{cable}

FBD:



$T = 3kd$

$\Sigma F = ma$; so $-3T = m_A a$ $\therefore$ $\boxed{a = \frac{-9kd}{m_A}}$

b) We know $\ddot{x} = -\frac{9kx}{m_A}$, so $x(t) = c_1 \cos\left(3\sqrt{\frac{k}{m_A}}\,t\right) + c_2 \sin\left(3\sqrt{\frac{k}{m_A}}\,t\right)$

$\dot{x}(0) = 0 = -c_1\left(3\sqrt{\frac{k}{m_A}}\right)\sin(0) + c_2\left(3\sqrt{\frac{k}{m_A}}\right)\cos(0)$ $\therefore$ $c_2 = 0$

$x(0) = d = c_1\cos(0)$ $\therefore$ $c_1 = d$

$x(t) = d\cos\left(3\sqrt{\frac{k}{m_A}}\,t\right)$ AND $\dot{x}(t) = -3d\sqrt{\frac{k}{m_A}}\sin\left(3\sqrt{\frac{k}{m_A}}\,t\right)$

$x(t) = 0 = d\cos\left(3\sqrt{\frac{k}{m_A}}\,t\right)$

$\therefore$ $3t\sqrt{\frac{k}{m_A}} = \frac{\pi}{2}$

OR $t = \frac{\pi}{6}\sqrt{\frac{m_A}{k}}$

$\dot{x}\left(\frac{\pi}{6}\sqrt{\frac{m_A}{k}}\right) = -3d\sqrt{\frac{k}{m_A}}\sin\left(\frac{\pi}{2}\right)$

$\therefore$ when spring is relaxed,

$\boxed{\dot{x} = -3d\sqrt{k/m_A}}$

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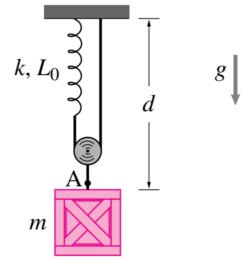
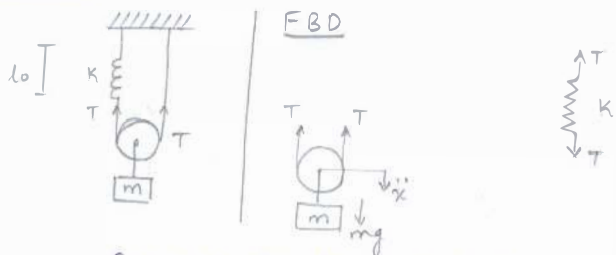
14.1.27 What is the static displacement of the mass from the position where the spring is just relaxed?



Problem 14.27

(19)



Say $x = 0$ when spring is just relaxed

$$m\ddot{x} = mg - 2T$$

where

$$T = K\delta$$

for equilibrium $\ddot{x} = 0$

$$mg - 2K\delta = 0$$

$$\delta = \frac{mg}{2K}$$

For deformation δ in spring mass deflects by

$$x = \frac{\delta}{2}$$

deflection in mass is

$$x = \frac{mg}{4K}$$

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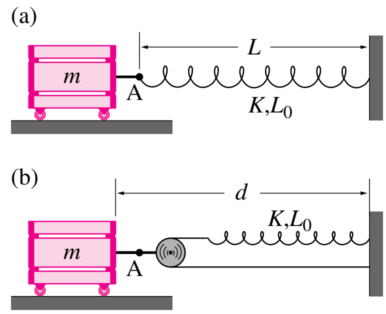
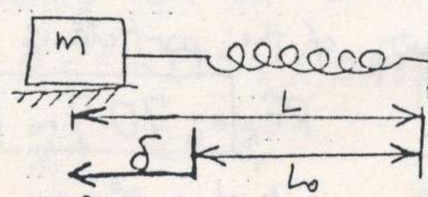
14.1.28 For the two situations pictured, find the acceleration of point A shown using balance of linear momentum ($\sum \vec{F} = m\vec{a}$). Assuming both masses are deflected an equal distance from the position where the spring is just relaxed, how much smaller or bigger is the acceleration of block (b) than that of block (a). Define any variables, coordinate system origins, coordinates or sign conventions that you need to do your calculations and to define your solution.



Problem 14.28

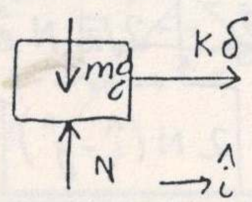
3.83. Find the acceleration of pt. A.

a)



define:
δ = displacement from rest = $L - L_0$

FBD:



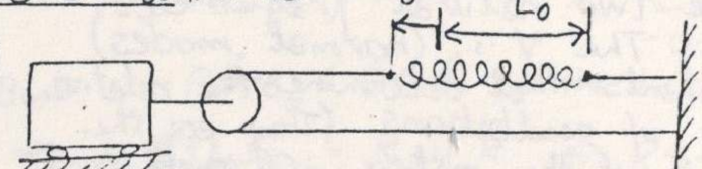
$\hat{\imath} \cdot (\sum \underset{\sim}{F} = \dot{\underset{\sim}{L}})$

$m a_x = K\delta$

$\boxed{a_x = \dfrac{K\delta}{m}}$

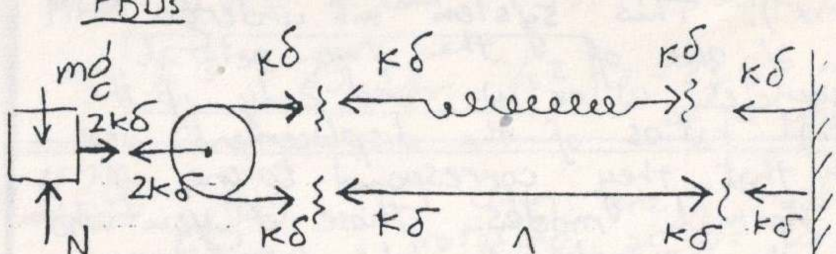
#3.83 continued. p.3

b)



δ = amount spring is stretched

FBDs



$\hat{\imath} \cdot (\sum \underset{\sim}{F} = \dot{\underset{\sim}{L}}) \Rightarrow 2K\delta = m a_x \Rightarrow \boxed{a_x = \dfrac{2K\delta}{m}}$

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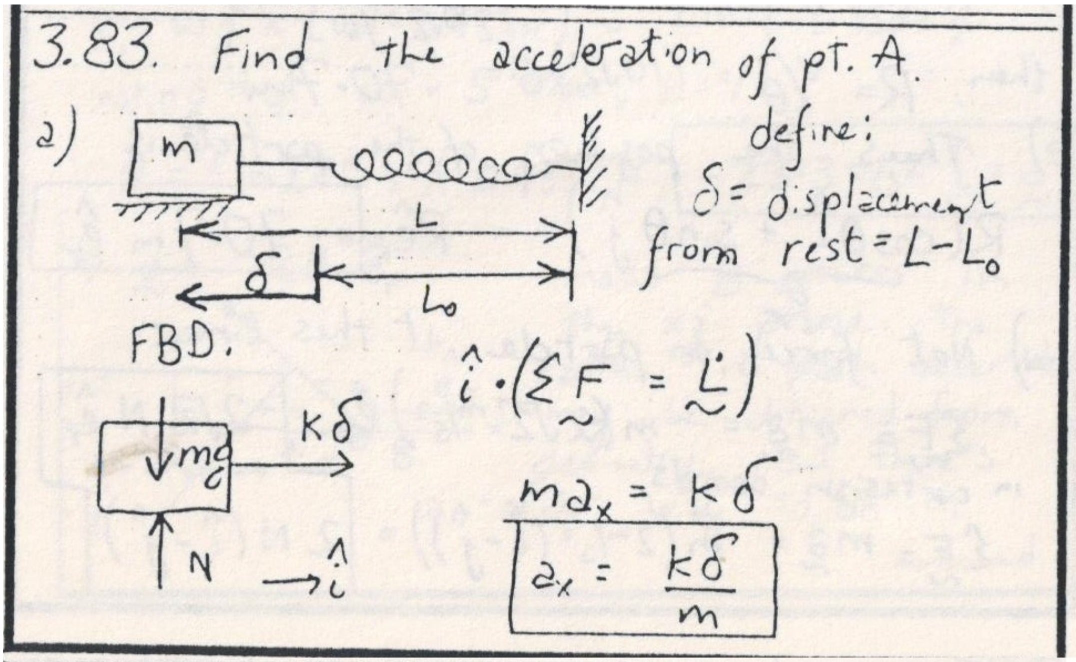
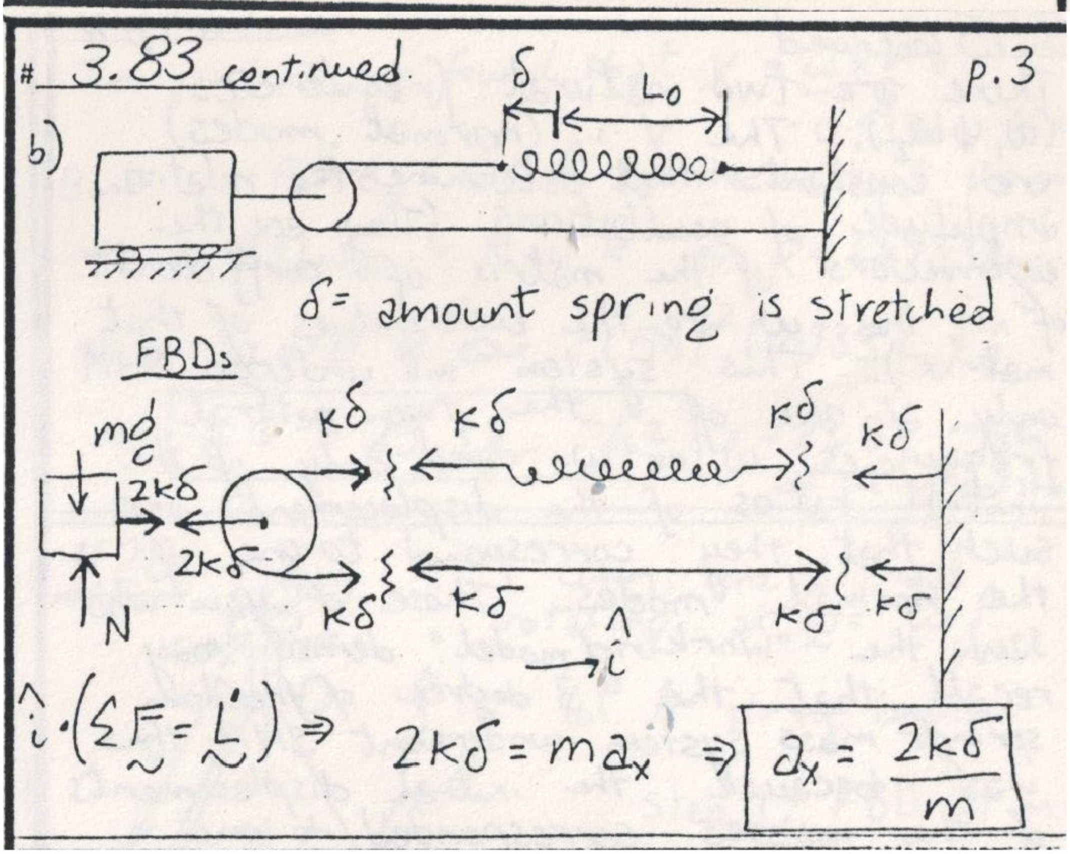
14.1.29 For each of the situations pictured in problem 14.1.28, find the acceleration of the mass using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.





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