CHAPTER 14

# Constrained straight-line motion

*Here is an introduction to kinematic constraint in its simplest context, systems that are constrained to move without rotation in a straight line. In one dimension, pulley problems provide the main example. Two- and three-dimensional problems are covered, such as finding structural support forces in accelerating vehicles and the slowing or incipient capsize of a braking car or bicycle. Angular momentum balance is introduced as a needed tool but without the complexities of rotational kinematics.*

In the previous chapters you learned to write the equations of motion for a particle or collection of a few particles, assuming you have a model for the forces on the particles in terms of their positions, velocities, and time. Some caveats to using that approach for engineering systems that don't seem to behave like isolated particles, but rather are composed of many particles were listed at the start of chapter 13 on page 656. One way to finesse these problems is to make kinematic assumptions about how

Figure 14.1: A truck or car running on straight level road is in straight-line motion, neglecting, of course, the wheel rotation, the bouncing, the moving engine parts, and the wandering eyes of the passengers.

particles and collections of particles move. Why? Sometimes, often actually, the simplest model of mechanical interaction is not a law for force as a function of position, velocity and time, but just a geometric description of the relative positions or velocities of points. The reasons for this geometric, instead of force-based, approach are two-fold:

- **The minute details of the motion are often not of interest and therefore not worth tracking.** For example, the vibrations of a solid, or relative motions of atoms in a solid might be of a smaller scale than the overall motion of interest, and
- **Often one does not know an accurate force law.** For example, at the microscopic level one does not know the details of atomic interactions; or, at the machine level, one may not know exactly the relations between the small motions of one part relative to another with which it makes contact. For example, even though one knows that the axle being in a hole restricts the relative motion of the axle with the train one may not know in detail how the contact forces depend on the exact position of the axle in its hole.

## Kinematic constraints

Much mechanical modeling involves the replacement of force-interaction rules with assumptions about the geometry of the motions. Idealizing an interaction force as causing a definite geometric restriction on motion is called imposing a *kinematic constraint*.

A *kinematic constraint* is an equation that describes a restriction on allowed positions, velocities or accelerations of parts in a system. Kinematic constraints are always accompanied by one or more *a priori* unknown 'constraint' forces that maintain the geometric constraint relations.

The basic laws of forces and mechanics apply to all systems, no matter how they are or are not constrained. But, if objects are treated as kinematically constrained the methods in mechanics have a slightly different flavor. To get the idea we start with simple systems that have simple constraints and that move in simple ways. In this short chapter, we will discuss the mechanics of things where every point in each object has the same velocity and acceleration as every other point (so called *parallel motion*) and with the further restriction that every point moves in a straight line.

**Example: Train on Straight Level Tracks**
Consider a train on straight level tracks. If we focus on the body of the train, we can approximate the motion as parallel straight-line motion. All parts move the same amount, with the same velocities and accelerations in the same fixed direction.

We start with 1-D mechanics and constraint with string and pulleys, and then move on to 2-D and 3-D mechanics (of systems in 1D motion).