CHAPTER 13

Two or more particles in space (unconstrained)

This more advanced chapter concerns the motion of two or more particles in space. We will use $\vec{F} = m\vec{a}$ for each particle. We will use Cartesian coordinates only. The start is the set up of "two-body" type problems which are easily generalized to 3 or more particles. The first section concerns smooth motions due to forces from gravity, springs, smoothly applied forces and friction. The second section concerns the sudden change in velocities when impulsive forces are applied.

In the previous chapter you saw that once you know the forces on a particle, or how to find those forces given a particle's position, velocity and time, you can easily set up the equations of motion. That is, the linear momentum balance equation for a particle

$$\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}},$$

with initial conditions, gives a well defined mathematical problem. The solution of this math problem gives the position and velocity of the particle as a function of time. The solution may be hard or impossible to find with pencil and paper, but can usually be found quite directly using numerical integration.

Now we generalize this idea to two, three or more particles. In one model of the universe every one of its parts is made of particles, and each particle obeys Newton's laws. We could think of all materials as made of atoms, and of all the atoms moving in deterministic ways governed by Newton's laws and known force laws. If we knew the initial positions and velocities accurately enough, then we could accurately predict the motions of all things for all time. ①

①The mathematician and mechanician Laplace (1749-1827) imagined a 'vast intellect' that could solve the differential equations that describe the universe. "Laplace's demon" was a hyper mega super computer with access to perfect data: *"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes."*

To put it in other words, given a simple atomic view of the world and a big computer, we could end a course on dynamics here. You know how to use $\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ for each atom, so you could then simulate anything by simulating the motions of the atoms which make it up.

Of course there are some serious limitations to this point of view, so before proceeding, we list some serious caveats:

- there are no computers big enough to keep track of the 10^{23} or so atoms needed to describe macroscopic objects or the 10^{79} or so atoms in the universe;
- the laws of interaction between the most fundamental particles are not given by Newton's laws but by quantum chromodynamics, or whatever;
- one feature of the rules of the world, as physicists now understand them, is that they are not deterministic, quantum mechanics says that you *cannot* know the state of the world perfectly;
- the state of the world (the positions and velocities of all the bits is not that well known);
- the solutions of dynamics equations are often unstable in that the smallest of errors in the initial conditions propagates into a large error in the predicted motion (so called "chaos theory");
- some common descriptions of mechanical interactions, particularly those for contact between nominally rigid objects, are genuinely non-deterministic in that the governing equations do not have unique solutions; and finally
- massive simulations, even if accurate, are not always the best way to understand how things work.

Despite these limitations, in this chapter we look at the nature of systems of interacting particles. Using this particle model we can, for example, derive some results about angular momentum that turn out to be reliable, despite the questionable microscopic physics. Also, the multi-particle model of the systems is good for intuition and is also useful for modeling machines with many parts as well as of galaxies.