

## 12.2 Momentum and energy for particle motion

### Preparatory Problems

**12.2.1** What symbols do we use for the following quantities? What are the definitions of these quantities? Which are vectors and which are scalars? What are the SI and US standard units for the following quantities?

- linear momentum
- rate of change of linear momentum
- angular momentum
- rate of change of angular momentum
- kinetic energy
- rate of change of kinetic energy
- moment
- work
- power

**12.2.2** Does angular momentum depend on reference point? (Assume that all candidate points are fixed in the same Newtonian reference frame.)

**12.2.3** Does kinetic energy depend on reference point? (Assume that all candidate points are fixed in the same Newtonian reference frame.)

**12.2.4** What is the relation between the dynamics 'Linear Momentum Balance' equation and the statics 'Force Balance' equation?

**12.2.5** What is the relation between the dynamics 'Angular Momentum Balance' equation and the statics 'Moment Balance' equation?

**12.2.6** A ball of mass  $m = 0.1$  kg is thrown from a height of  $h = 10$  m above the ground with velocity  $\vec{v} = 120 \text{ km/h} \hat{i} - 120 \text{ km/h} \hat{j}$ . What is the kinetic energy of the ball at its release?

**12.2.7** A ball of mass  $m = 0.2$  kg is thrown from a height of  $h = 20$  m above the ground with velocity  $\vec{v} = 120 \text{ km/h} \hat{i} - 120 \text{ km/h} \hat{j} - 10 \text{ km/h} \hat{k}$ . What is the kinetic energy of the ball at its release?

**12.2.8** How do you calculate  $P$ , the power of all external forces acting on a particle, from the forces  $\vec{F}_i$  and the velocity  $\vec{v}$  of the particle?

**12.2.9** A particle  $A$  has velocity  $\vec{v}_A$  and mass  $m_A$ . A particle  $B$  has velocity  $\vec{v}_B = 2 \vec{v}_A$  and mass equal to the other  $m_B = m_A$ . What is the relationship between:

- $\vec{L}_A$  and  $\vec{L}_B$ ,
- $\vec{H}_{A/C}$  and  $\vec{H}_{B/C}$ , and
- $E_{KA}$  and  $E_{KB}$ ?

**12.2.10** A bullet of mass 50 g travels with a velocity  $\vec{v} = 0.8 \text{ km/s} \hat{i} + 0.6 \text{ km/s} \hat{j}$ . (a) What is the linear momentum of the bullet? (Answer in consistent units.)

**12.2.11** A particle has position  $\vec{r} = 4 \text{ m} \hat{i} + 7 \text{ m} \hat{j}$ , velocity  $\vec{v} = 6 \text{ m/s} \hat{i} - 3 \text{ m/s} \hat{j}$ , and acceleration  $\vec{a} = -2 \text{ m/s}^2 \hat{i} + 9 \text{ m/s}^2 \hat{j}$ . For each position of a point  $P$  defined below, find  $\vec{H}_P$ , the angular momentum of the particle with respect to the point  $P$ .

- $\vec{r}_P = 4 \text{ m} \hat{i} + 7 \text{ m} \hat{j}$ ,
- $\vec{r}_P = -2 \text{ m} \hat{i} + 7 \text{ m} \hat{j}$ , and
- $\vec{r}_P = 0 \text{ m} \hat{i} + 7 \text{ m} \hat{j}$ ,
- $\vec{r}_P = \vec{0}$

**12.2.12** The position vector of a particle of mass 1 kg at an instant  $t$  is  $\vec{r} = 2 \text{ m} \hat{i} - 0.5 \text{ m} \hat{j}$ . If the velocity of the particle at this instant is  $\vec{v} = -4 \text{ m/s} \hat{i} + 3 \text{ m/s} \hat{j}$ , compute (a) the linear momentum  $\vec{L} = m\vec{v}$  and (b) the angular momentum ( $\vec{H}_O = \vec{r}_O \times (m\vec{v})$ ).

**12.2.13** The position of a particle of mass  $m = 0.5$  kg is  $\vec{r}(t) = \ell \sin(\omega t) \hat{i} + h \hat{j}$ ; where  $\omega = 2 \text{ rad/s}$ ,  $h = 2 \text{ m}$ ,  $\ell = 2 \text{ m}$ , and  $\vec{r}$  is measured from the origin.

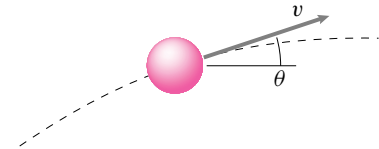
- Find the kinetic energy of the particle at  $t = 0$  s and  $t = 5$  s.
- Find the rate of change of kinetic energy at  $t = 0$  s and  $t = 5$  s.

**12.2.14** For a particle

$$E_K = \frac{1}{2} m v^2.$$

Why does it follow that  $\dot{E}_K = m \vec{v} \cdot \vec{a}$ ? [hint: write  $v^2$  as  $\vec{v} \cdot \vec{v}$  and then use the product rule of differentiation.]

**12.2.15** Consider a projectile of mass  $m$  at some instant in time  $t$  during its flight. Let  $\vec{v}$  be the velocity of the projectile at this instant (see the figure). In addition to the force of gravity, a drag force acts on the projectile. The drag force is proportional to the square of the speed (speed  $\equiv |\vec{v}| = v$ ) and acts in the opposite direction. Find an expression for the net power of these forces ( $P = \sum \vec{F} \cdot \vec{v}$ ) on the particle.



Problem 12.2.15

**12.2.16** A 10 gm wad of paper is tossed into the air. At some instant, the position, velocity, and acceleration of its center of mass are  $\vec{r} = 3 \text{ m} \hat{i} + 3 \text{ m} \hat{j} + 6 \text{ m} \hat{k}$ ,  $\vec{v} = -9 \text{ m/s} \hat{i} + 24 \text{ m/s} \hat{j} + 30 \text{ m/s} \hat{k}$ , and  $\vec{a} = -10 \text{ m/s}^2 \hat{i} + 24 \text{ m/s}^2 \hat{j} + 32 \text{ m/s}^2 \hat{k}$ , respectively. What is the translational kinetic energy of the wad at the instant of interest?

**12.2.17** A 2 kg particle moves so that its position  $\vec{r}$  is given by

$$\vec{r}(t) = [5 \sin(at) \hat{i} + bt^2 \hat{j} + ct \hat{k}] \text{ meters}$$

where  $a = \pi / \text{sec}$ ,  $b = .25 / \text{sec}^2$ ,  $c = 2 / \text{sec}$ .

- What is the linear momentum of the particle at  $t = 1$  sec?

- b) What is the force acting on the particle at  $t = 1$  sec?

**12.2.18** A particle A has mass  $m_A$  and velocity  $\vec{v}_A$ . A particle B at the same location has mass  $m_B = 2m_A$  and velocity equal to the other  $\vec{v}_B = \vec{v}_A$ . Point C is a reference point. What is the relationship between:

- $\vec{L}_A$  and  $\vec{L}_B$ ,
- $\vec{H}_{A/C}$  and  $\vec{H}_{B/C}$ , and
- $E_{KA}$  and  $E_{KB}$ ?

**12.2.19** A particle of mass  $m = 3$  kg moves in space. Its position, velocity, and acceleration at a given time are  $\vec{r} = 2m\hat{i} + 3m\hat{j} + 5m\hat{k}$ ,  $\vec{v} = -3m/s\hat{i} + 8m/s\hat{j} + 10m/s\hat{k}$ , and  $\vec{a} = -5m/s^2\hat{i} + 12m/s^2\hat{j} + 16m/s^2\hat{k}$ , respectively. For this particle at the instant of interest, find its:

- linear momentum  $\vec{L}$ ,
- rate of change of linear momentum  $\dot{\vec{L}}$ ,
- angular momentum about the origin  $\vec{H}_O$ ,
- rate of change of angular momentum about the origin  $\dot{\vec{H}}_O$ ,
- kinetic energy  $E_K$ , and
- rate of change of kinetic energy  $\dot{E}_K$ .

**12.2.20** A particle has position  $\vec{r} = 3m\hat{i} - 2m\hat{j} + 4m\hat{k}$ , velocity  $\vec{v} = 2m/s\hat{i} - 3m/s\hat{j} + 7m/s\hat{k}$ , and acceleration  $\vec{a} = 1m/s^2\hat{i} - 8m/s^2\hat{j} + 3m/s^2\hat{k}$ . For each position of a point  $P$  defined below, find the rate of change of angular momentum,  $\dot{\vec{H}}_P$ , of the particle with respect to the point  $P$ .

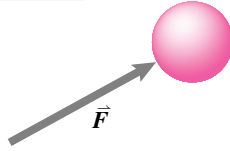
- $\vec{r}_P = 3m\hat{i} - 2m\hat{j} + 4m\hat{k}$ ,
- $\vec{r}_P = 6m\hat{i} - 4m\hat{j} + 8m\hat{k}$ ,
- $\vec{r}_P = -9m\hat{i} + 6m\hat{j} - 12m\hat{k}$ , and
- $\vec{r}_P = \vec{0}$

## More-Involved Problems

**12.2.21** A particle of mass  $m = 6$  kg is moving in space. Its position, velocity, and acceleration at some instant are  $\vec{r} = 1m\hat{i} - 2m\hat{j} + 4m\hat{k}$ ,  $\vec{v} = 3m/s\hat{i} + 4m/s\hat{j} - 7m/s\hat{k}$ , and  $\vec{a} = 5m/s^2\hat{i} + 11m/s^2\hat{j} - 9m/s^2\hat{k}$ , respectively. At this instant, find:

- the net force  $\sum \vec{F}$  on the particle,
- the net moment on the particle about the origin  $\sum \vec{M}_O$  due to the applied forces, and
- the power  $P$  of the applied forces.

Particle FBD



Problem 12.2.21: FBD of the particle

**12.2.22** At a time of interest, a particle with mass  $m_1 = 5$  kg has position, velocity, and acceleration  $\vec{r}_1 = 3m\hat{i}$ ,  $\vec{v}_1 = -4m/s\hat{j}$ , and  $\vec{a}_1 = 6m/s^2\hat{j}$ , respectively. Another particle with mass  $m_2 = 5$  kg has position, velocity, and acceleration  $\vec{r}_2 = -6m\hat{i}$ ,  $\vec{v}_2 = 5m/s\hat{j}$ , and  $\vec{a}_2 = -4m/s^2\hat{j}$ , respectively. For this system of two particles, and at this time, find its

- linear momentum  $\vec{L}$ ,
- rate of change of linear momentum  $\dot{\vec{L}}$
- angular momentum about the origin  $\vec{H}_O$ ,
- rate of change of angular momentum about the origin  $\dot{\vec{H}}_O$ ,
- kinetic energy  $E_K$ , and
- rate of change of kinetic energy  $\dot{E}_K$ .

**12.2.23** A particle of mass  $m = 250$  gm is shot straight up (parallel to the  $y$ -axis) from the  $x$ -axis at a distance  $d = 2$  m from the origin. The velocity of the particle is given by  $\vec{v} = v\hat{j}$  where  $v^2 = v_0^2 - 2ah$ ,  $v_0 = 100$  m/s,  $a = 10$  m/s<sup>2</sup> and  $h$  is the height of the particle from the  $x$ -axis.

- Find the linear momentum of the particle at the outset of motion ( $h = 0$ ).
- Find the angular momentum of the particle about the origin at the outset of motion ( $h = 0$ ).
- Find the linear momentum of the particle when the particle is 20 m above the  $x$ -axis.
- Find the angular momentum of the particle about the origin when the particle is 20 m above the  $x$ -axis.