Partial Solutions Manual
Ruina and Pratap
Introduction to Statics and Dynamics

This draft: January 30, 2011

Have a suggestion? Want to contribute a solution?
Contact ruina@cornell.edu with Subject: Solutions Manual

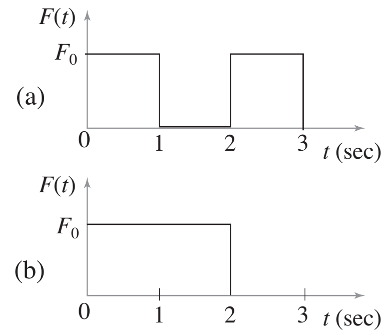
Note, the numbering of hand-written solutions is most-often wrong (corresponding to an old numbering scheme). The hand-written problem numbers should be ignored.

9.1.15 Consider a force $F(t)$ acting on a cart over a 3 second span. In case (a), the force acts in two impulses of one second duration each as shown in fig. 9.1.15. In case (b), the force acts continuously for two seconds and then goes to zero. Given that the mass of the cart is 10 kg, $v(0\,\text{s}) = 0$, and $F_0 = 10\,\text{N}$, for each force profile,

a) Find the speed of the cart at the end of 3 seconds, and

b) Find the distance travelled by the cart in 3 seconds.

Comment on your answers for the two cases.



Filename:pfigure9-1-fcompare
Problem 9.15

9.15

$m = 10$ kg $F = ma \;\therefore\; a_0 = F_0/m_0 = 10\,\text{N}/10\,\text{kg}$

$v(0) = 0$ $= 1\ \text{m/s}^2$

$F_0 = 10$ N

Force profile (a):

@ $t = 1\text{s}$: $a = 1\ \text{m/s}^2$, $v = v_0 + at = 0 + 1\,\text{m/s}^2(1\text{s}) = 1\ \text{m/s}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(1\,\text{m/s}^2)(1\text{s})^2 = 0.5\text{m}$

@ $t = 2\text{s}$: $a = 0$, $v = v_0 + at = 1\ \text{m/s} + 0 = 1\ \text{m/s}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0.5 + 1(1) + 0 = 1.5\text{m}$

@ $t = 3\text{s}$: $a = 1\,\text{m/s}^2$, $v = v_0 + at = 1 + 1(1) = \underline{2.0\ \text{m/s}}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 1.5 + 1(1) + \frac{1}{2}(1)(1)^2 = \underline{3.0\ \text{m}}$

Force profile (b):

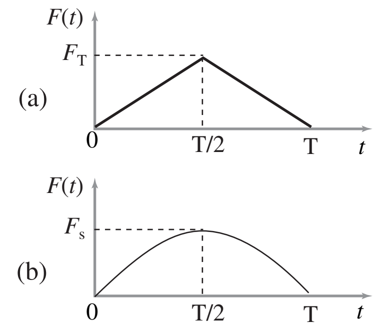
@ $t = 2\text{s}$: $a = 1\ \text{m/s}^2$, $v = v_0 + at = 0 + 1(2) = 2\ \text{m/s}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(1)(2)^2 = 2\ \text{m}$

@ $t = 3\text{s}$: $a = 0$, $v = v_0 + at = 2 + 0 = \underline{2.0\ \text{m/s}}$

$x = x_0 + v_0 t + \frac{1}{2}at^2 = 2 + 2(1) + 0 = \underline{4.0\ \text{m}}$

9.1.16 A car of mass m is accelerated by applying a triangular force profile shown in fig. 9.1.16(a). Find the speed of the car at $t = T$ seconds. If the same speed is to be achieved at $t = T$ seconds with a sinusoidal force profile, $F(t) = F_s \sin\frac{\pi t}{T}$, find the required force magnitude F_s. Is the peak higher or lower? Why?



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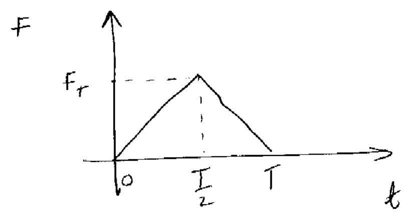
Problem 9.16

9.16



a) $V = \int a\,dt$

$= \frac{1}{m} \underbrace{\int F\,dt}_{\text{area under F-t curve}}$

$= \frac{1}{m}\left[\frac{1}{2} \times T \times F_T\right]$ {Area of triangle = $\frac{1}{2}$ x base x height}

$\therefore \boxed{V = \frac{TF_T}{2m}}$

b) $F(t) = F_s \sin\left(\frac{\pi t}{T}\right)$

$V = \frac{1}{m}\int_0^T F\,dt = \frac{F_s}{m}\int_0^T \sin\left(\frac{\pi t}{T}\right) dt$

$= \frac{F_s}{m} \frac{\left[-\cos\frac{\pi t}{T}\right]_0^T}{\frac{\pi}{T}}$

$= \frac{T F_s}{m\pi}\left[-\cos\pi + \cos 0\right] = \frac{2TF_s}{m\pi}$

But $\frac{TF_T}{2m} = \frac{2TF_s}{\pi m} \Rightarrow \boxed{F_s = \frac{\pi}{4} F_T}$

9.1.22 A grain of sugar falling through honey has a negative acceleration proportional to the difference between its velocity and its 'terminal' velocity, which is a known constant v_t. Write this sentence as a differential equation, defining any constants you need. Solve the equation assuming some given initial velocity v_0.

9.22. A grain of sugar falling through honey with negative acceleration $\propto (V - V_{terminal})$

Write a differential eqn, and solve using given initial velocity V_0

Kinematics only no FBD needed

let V = velocity

V_t = terminal velocity

V_0 = initial velocity

$\dot{V}$ = acceleration

$$\dot{V} = -\overset{\text{constant}}{k}(V - V_t)$$

$$\frac{dv}{dt} = -k(v - v_t)$$

$$\int_{v_0}^{v} \frac{dv}{v - v_t} = -\int k\, dt$$

$$\ln(v - v_t)\Big]_{v_0}^{v} = -kt$$

$$\ln(v - v_t) - \ln(v_0 - v_t) = -kt$$

$$\ln\left(\frac{V - V_t}{V_0 - V_t}\right) = -kt$$

$$\frac{V - V_t}{V_0 - V_t} = e^{-kt}$$

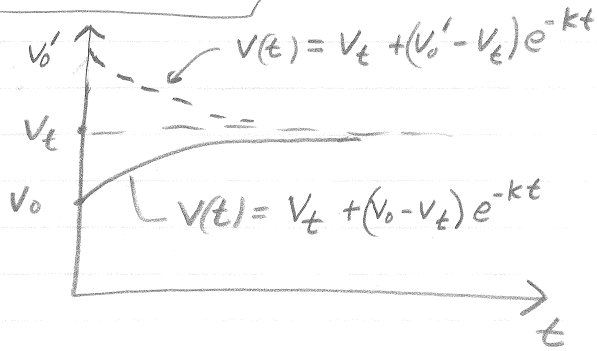
$$\boxed{V = V_t + (V_0 - V_t)e^{-kt}}$$

Two solns:

$V_0' > V_t$

$V_0 < V_t$

$\Rightarrow$



9.1.26 A bullet penetrating flesh slows approximately as it would if penetrating water. The drag on the bullet is about $F_D = c\rho_w v^2 A/2$ where ρ_w is the density of water, v is the instantaneous speed of the bullet, A is the cross sectional area of the bullet, and c is a drag coefficient which is about $c \approx 1$. Assume that the bullet has mass $m = \rho_l AL$ where ρ_l is the density of lead, A is the cross sectional area of the bullet and L is the length of the bullet (approximated as cylindrical). Assume $m = 2$ grams, entering velocity $v_0 = 400\,\mathrm{m/s}$, $\rho_l/\rho_w = 11.3$, and bullet diameter $d = 5.7\,\mathrm{mm}$.

a) Plot the bullet position vs time.

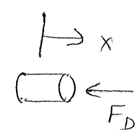
b) Assume the bullet has effectively stopped when its speed has dropped to $5\,\mathrm{m/s}$, what is its total penetration distance?

c) According to the equations implied above, what is the penetration distance in the limit $t \to \infty$?

d) How would you change the model to make it more reasonable in its predictions for long time?

9.26

FBD



$$\sum F_{ext} = \dot{L}$$

$$-F_D = m\frac{dv}{dt}$$

$$-\frac{c\rho_w v^2 A}{2} = m\frac{dv}{dt}$$

$$\therefore\quad m\frac{dv}{dt} + \frac{c\rho_w v^2 A}{2} = 0$$

But $m = \rho_l A l$

$$\therefore\quad \rho_l A l \frac{dv}{dt} + \frac{c\rho_w v^2 A}{2} = 0$$

$$\frac{dv}{dt} + \frac{1}{2}\cdot\left(\frac{c}{l}\right)\left(\frac{\rho_w}{\rho_l}\right)v^2 = 0$$

Let's calculate l.

$$m = \rho_l A l$$

$$\therefore\ l = \frac{m}{\rho_l A}$$

$m = 2\,gm$; $\rho_w = 1$ (known) $\rho_l = 11.3\,\rho_w = 11.3$

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4}(0.57)^2 = 0.26\,cm^2$$

$$\therefore\ l = \frac{2}{(11.3)(0.26)} = 0.68\,cm = 0.68\times10^{-2}\,m$$

Now set $\lambda = \left(\frac{1}{2}\right)\left(\frac{c}{l}\right)\left(\frac{\rho_w}{\rho_c}\right)$

$$\lambda = \left(\frac{1}{2}\right)\left(\frac{1}{0.68\times10^{-2}}\right)\left(\frac{1}{11.3}\right)$$

$$\lambda = 6.5$$

Thus $\frac{dv}{dt} + 6.5v^2 = 0$

$\Rightarrow$ setting equation for matlab: see bullet.m

$$\left.\begin{aligned}\dot{x} &= v\\ \dot{v} &= -6.5v^2\end{aligned}\right\}\text{ rhs for ode45}$$

$\Rightarrow$ Analytical solution

$$\dot{v} = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dx}(v)\frac{dx}{dt} = \frac{dv}{dx}\,v$$

$$\therefore\ v\frac{dv}{dx} = \dot{v} = -6.5v^2$$

$$\therefore\ v\frac{dv}{dx} + 6.5v^2 = 0$$

$$\therefore\ \frac{dv}{dx} + 6.5v = 0$$

$$v = C_1 e^{-6.5x} \quad \text{(I)}$$

{ On substituting $v = C_1 e^{sx}$ & solving for s }

Given $x = 0$; $V_0 = 400$

Solving for C_1 in I gives $C_1 = 400$

$$\therefore \quad V = 400\, e^{-6.5x}$$

$$\frac{dx}{dt} = 400\, e^{-6.5x}$$

$$\int_0^x e^{6.5x}\, dx = 400 \int_0^t dt$$

$$\left[\frac{e^{6.5x}}{6.5}\right]_0^x = 400\,[t]_0^t$$

$$\therefore \quad \frac{e^{6.5x}}{6.5} - \frac{1}{6.5} = 400t$$

$$\boxed{e^{6.5x} = 2600t + 1} \quad \text{— II}$$

a) See attached plot. Done using matlab.

b) Put $V = 5$ in I

$$5 = 400\, e^{-6.5x}$$

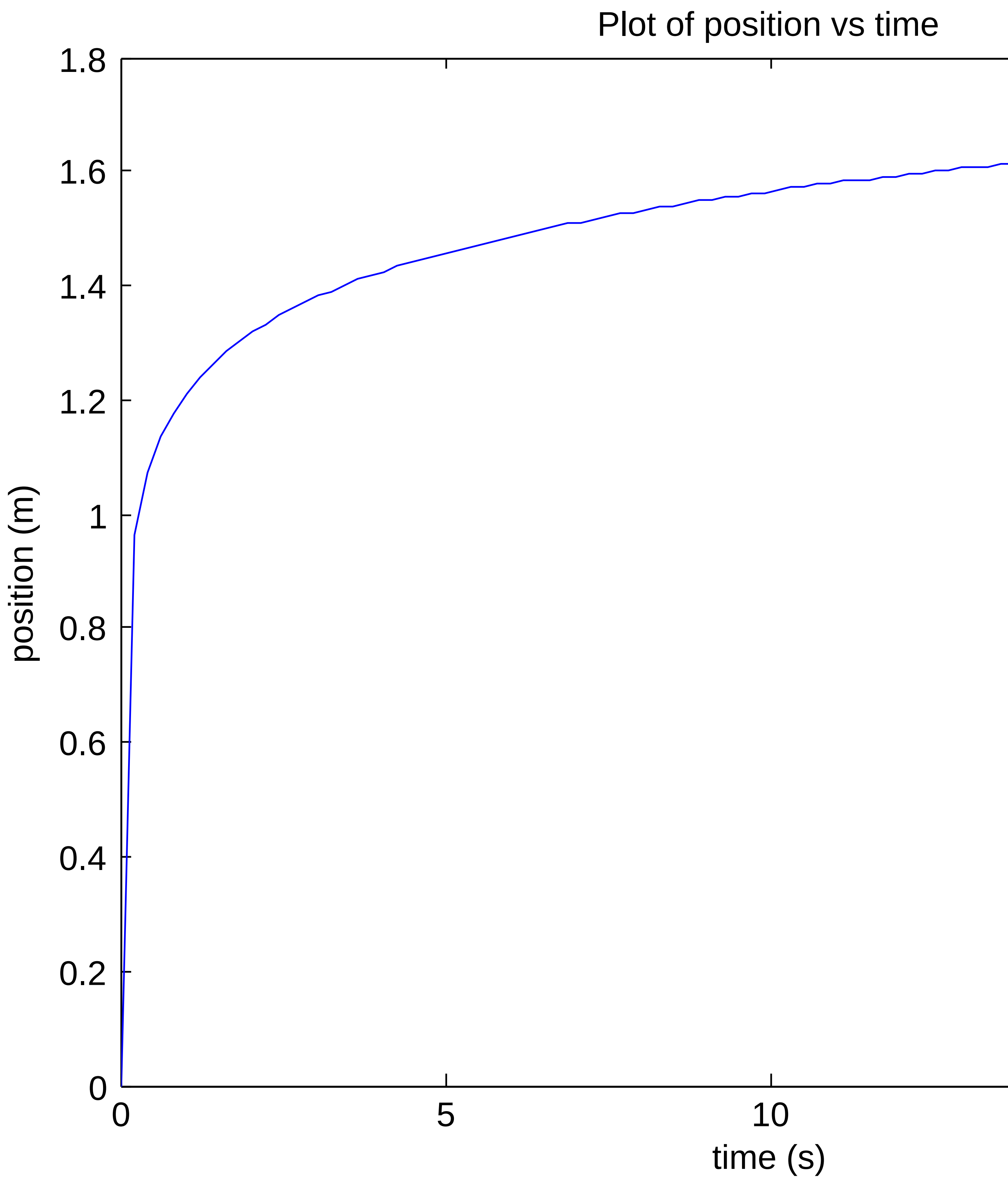
Take ln $\quad \ln\left(\frac{5}{400}\right) = -6.5x$

solving $\boxed{x = 0.67 \text{ m}}$

c) From II as $t \to \infty$ $x \to \infty$

Thus the bullet would penetrate infinite distance (clearly impossible in reality)

d) Add frictional resistance in addition to drag.


Plot of position vs time
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
position (m)
0
5
10
time (s)

9.2.3 A force $F = F_0 \sin(ct)$ acts on a particle with mass $m = 3\,\mathrm{kg}$ which has position $x = 3\,\mathrm{m}$, velocity $v = 5\,\mathrm{m/s}$ at $t = 2\,\mathrm{s}$. $F_0 = 4\,\mathrm{N}$ and $c = 2/\mathrm{s}$. At $t = 2\,\mathrm{s}$ evaluate (give numbers and units):

a) a,

b) E_K,

c) P,

d) $\dot{E}_K$,

e) the rate at which the force is doing work.

9.30

A force $F = F_0 \sin(ct)$ acts on a particle with $m = 3\,kg$. At $t = 2s$; $x = 3m$, $v = 5m/s$

$F_0 = 4N$, $c = 2/s$

a) Find a at $t = 2s$.

$$F = (4N) \sin(2/s \cdot t)$$

$$a = \frac{F}{m} = \frac{4N}{3kg} \sin(2/s \cdot t)$$

$$= \left(\frac{4}{3} \sin(2t)\right) m/s^2$$

$$\boxed{a(2s) = -1.01\, m/s^2}$$

b) Find E_K at $t = 2s$

$$E_K = \frac{1}{2} m v^2$$

At $t = 2s$, $v = 5m/s$

$$E_K = \frac{1}{2}(3kg)(5m/s)^2$$

$$\boxed{E_K = 37.5\, J}$$

c) Find P at $t = 2s$

$$P = Fv$$

$$F(2) = (4 \sin(2 \cdot 2))\, N$$

$$= -3.03\, N$$

$$P = (-3.03N)(5m/s)$$

$$\boxed{P \approx -15.14\, W}$$

9.30 continued.

d) Find $\dot{E}_K$ at $t=2s$

$$P = \dot{E}_K$$

$$\boxed{\dot{E}_K = -15.14\ W}$$

e) Find the rate at which force is doing work

$$W = \int P\,dt$$

$$\dot{W} = \frac{d}{dt}\left(\int P\,dt\right)$$

$$= P$$

$$\boxed{\dot{W} = -15.14\ W}$$

9.2.10 A kid ($m = 90\,\text{lbm}$) stands on a $h = 10\,\text{ft}$ wall and jumps down, accelerating with $g = 32\,\text{ft/s}^2$. Upon hitting the ground with straight legs, she bends them so her body slows to a stop over a distance $d = 1\,\text{ft}$. Neglect the mass of her legs. Assume constant deceleration as she brakes the fall.

a) What is the total distance her body falls?

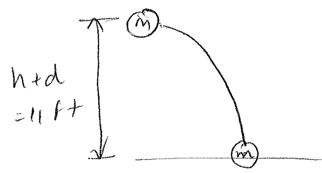
b) What is the potential energy lost?

c) How much work must be absorbed by her legs?

d) What is the force of her legs on her body? Answer in symbols, numbers and numbers of body weight (*i.e.,* find F/mg).

9.37

Given $m = 90$ lbm, $h = 10$ ft, $g = 32\,\text{ft/s}^2$, $d = 1$ ft



Treat the entire body as a particle concentrated at the center of mass

a) Total distance = 11 feet

b) $\Delta PE = mgh = 90\text{ lbm}\,(32\,\text{ft/s}^2)\,(11\text{ ft})$

$$\boxed{\Delta PE = 31{,}680\ \text{lb-ft}}$$

c) All work must be absorbed.

Thus $\boxed{W = 31{,}680\ \text{lb-ft}}$

d) $W = Fd$

so $F = \dfrac{W}{d} = \dfrac{31{,}680\ \text{lb-ft}}{1\ \text{ft}}$

$= \dfrac{mg(h+d)}{d}$

$= 11\,mg$

$$\boxed{F = 11\,mg = 11\,\text{wt}}$$

9.2.11 In traditional archery, when pulling an arrow back the force increases approximately linearly up to the peak 'draw force' F_{draw} that varies from about $F_{draw} = 25$ lbf for a bow made for a small person to about $F_{draw} = 75$ lbf for a bow made for a big strong person. The distance the arrow is pulled back, the draw length ℓ_{draw}, varies from about $\ell_{draw} = 2$ ft for a small adult to about 30 inch for a big adult. An arrow has mass of about 300 grain (1 grain $\approx$ 64.8milli gm, so an arrow has mass of about $19.44 \approx 20$ gm $\approx 3/4$ ounce). Give all answers in symbols and numbers.

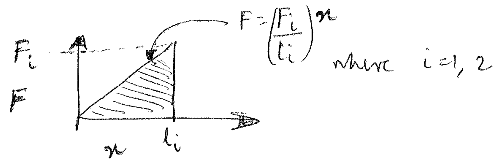
a) What is the range of speeds you can expect an arrow to fly?

b) What is the range of heights an arrow might go if shot straight up (it's a bad approximation, but for this problem neglect air friction)?

9.38

Given $F_1 = 25$ lbf $\quad l_1 = 2$ ft

$F_2 = 75$ lbf $\quad l_2 = 2.5$ ft



Energy stored in pulled arrow

$$W = \int F\,dx$$

$$= \int_0^{l_i} \left(\frac{F_i}{l_i}\right) x\,dx$$

$$W = \frac{F_i l_i}{2}$$ {which is area of triangle above}

$W_1 = \frac{1}{2} \times 25 \times 2 = 25$ lbf-ft

$= 25 \times 32.2$ lbm-ft/s²

$\therefore W_1 = 805$ lbm-ft/s²

$W_2 = \frac{1}{2} \times 75 \times 2.5 = 93.75$ lbf-ft

$= 93.75 \times 32.2$ lbm-ft/s²

$\therefore W_2 = 3018.75$ lbf-ft/s²

a) $W = \frac{1}{2} m v^2 \Rightarrow V = \sqrt{\frac{2W}{m}}$

Given $m = \frac{3}{4}$ ounce $= 0.047$ lbm

$\therefore V_1 = \sqrt{\frac{2W_1}{m}} = \sqrt{\frac{2 \times 805}{0.047}}$

$\therefore V_1 = 185.1$ ft/s

$V_2 = \sqrt{\frac{2W_2}{m}} = \sqrt{\frac{2 \times 3018.75}{0.047}}$

$\therefore V_2 = 358.2$ ft/s

Thus $\boxed{185.1 \text{ ft/s} \leq V \leq 358.2 \text{ ft/s}}$

b) $W = mgh \Rightarrow h = \frac{W}{mg}$

$h_1 = \frac{W_1}{mg} = \frac{805}{0.047 \times 32.2} = 532$ ft

$h_2 = \frac{W_2}{mg} = \frac{3018.75}{0.047 \times 32.2} = 1994.6$ ft

$\boxed{532 \text{ ft} \leq h \leq 1994.6 \text{ ft}}$

9.2.16 The power available to a very strong accelerating cyclist over short periods of time (up to, say, about 1 minute) is about 1 horsepower. Assume a rider starts from rest and uses this constant power. Assume a mass (bike + rider) of 150 lbm, a realistic drag force of $.006\,\text{lbf}/(\text{ft}/\text{s})^2 v^2$. Neglect other drag forces.

a) What is the peak (steady state) speed of the cyclist?

b) Using analytic or numerical methods make an accurate plot of speed vs. time.

c) What is the acceleration as $t \to \infty$ in this solution?

d) What is the acceleration as $t \to 0$ in your solution?

e) How would you improve the model to fix the problem with the answer above?

9.43

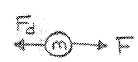
$P = 1\text{ HP (constant)} = 550\ \frac{\text{lbf}\cdot\text{ft}}{\text{s}}$

$v_0 = 0,\ m = 150\text{ lbm},\ F_d = 0.006v^2\ [\text{lbf}]$

1) The peak speed is the speed at which all power is resisted by drag.

$$550\ \frac{\text{lbf}\cdot\text{ft}}{\text{s}} = F_d v = 0.006v^3$$

$$\therefore\ \boxed{v_{max} = 45.1\ \text{ft}/\text{s}}$$

2) 

$\sum F = ma = F - F_d \quad \therefore\ m\dot{v} = \frac{P}{v} - 0.006v^2$

Solve numerically using Matlab.

9.43)

```
function homework943()
% Problem 9.43 Solution
% Feb 5, 2008

% CONSTANTS
P= 550  ; % power in lbf*ft/s
m= 150;  % lbm
g= 32.2;  % ft/s^2

% INTIAL CONDITIONS
v0= 0.001; % initial velocity, zero makes the solution explode

tspan =[0 1000]; %time interval of integration

error = 1e-4;
% Set error tolerance and use 'event detection'
options = odeset('abstol', error, 'reltol', error) ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Ask Matlab to SOLVE odes in function 'rhs'
[t v] = ode45(@rhs,tspan, v0, options, P, m, g)

%UNPACK the zarray (the solution) into sensible variables
plot (t,v)
title('Problem 9.43')
xlabel('Time, t (s)'); ylabel('Speed, v (ft/s)')
axis([0 inf -inf inf])  %inf self scales plot

end  % end of main function
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%
% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function vdot = rhs(t,v,P,m,g)

vdot = P/(m*v)-0.006*v^2/m;  %  F = m a

end % end of rhs
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

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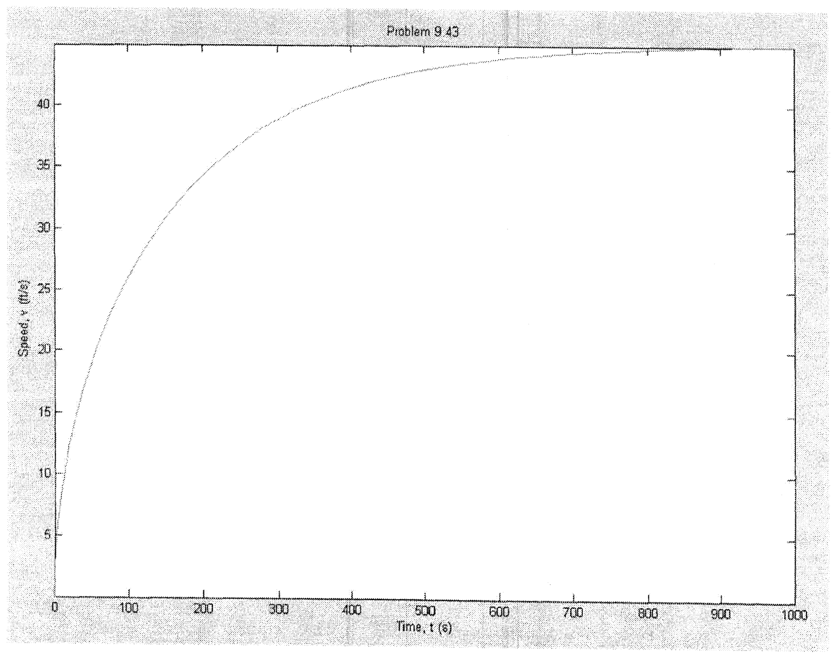
Results from Matlab Code



3) Acceleration is the slope of the velocity on the plot above. As time goes to infinity, the acceleration goes to zero.

4) As time goes to zero, the acceleration goes to infinity. This is why the initial velocity had to be inputted as a very small number (i.e. 0.001 ft/s) instead of zero.

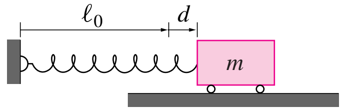
9.3.6 A spring k with rest length ℓ_0 is attached to a mass m which slides frictionlessly on a horizontal ground as shown. At time $t = 0$ the mass is released from rest with the spring stretched a distance d. Measure the mass position x relative to the wall.

a) What is the acceleration of the mass just after release?

b) Find a differential equation which describes the horizontal motion x of the mass.

c) What is the position of the mass at an arbitrary time t?

d) What is the speed of the mass when it passes through $x = \ell_0$ (the position where the spring is relaxed)?



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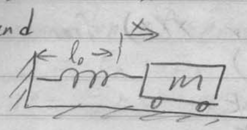
Problem 9.6

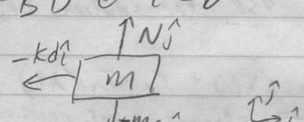
9.49

A spring with rest length l_0 attached to mass m slides frictionless on a horizontal ground



At $t=0$ mass is released with $v_0=0$ and spring stretch a distance d.

FBD @ $t=0$



$N\hat{\jmath} = mg\hat{\jmath}$

a) Find a of mass just after release

$\{m\ddot{x}\hat{\imath} = -kd\hat{\imath}\}$

$\{\}\cdot\hat{\imath}: \quad m\ddot{x} = -kd$

$$\boxed{\ddot{x} = \frac{-kd}{m}}$$

b) Find a differential eqn that describes horizontal motion of mass

$\{m\ddot{x}\hat{\imath} = -kx\hat{\imath}\}$

$\{\}\cdot\hat{\imath}: \quad \boxed{m\ddot{x} = -kx}$

9.49 continued

c) Find position of mass at arbitrary time t.

- we will solve for $x(t)$ from the eqn we find in (b)

$$m\ddot{x} = -kx$$
$$\ddot{x} = -\frac{k}{m}x$$
$$\ddot{x} + \frac{k}{m}x = 0$$

let $\lambda^2 = \frac{k}{m}$

$$\ddot{x} + \lambda^2 x = 0$$

according to Page 438, the solution to the above eqn is

$$x = C_1 \cos(\lambda t) + C_2 \sin(\lambda t)$$

apply initial conditions

At $t=0$, $x=d$ → letting $x=0$ at position spring relaxes

$$x(0) = C_1 \cos(\lambda \cdot 0) + C_2 \cancel{\sin(\lambda \cdot 0)}$$
$$d = C_1$$

At $t=0$, $v=0$

$$\dot{x} = -\lambda C_1 \sin(\lambda t) + \lambda C_2 \cos(\lambda t)$$
$$\dot{x}(0) = -\lambda C_1 \cancel{\sin(\lambda \cdot 0)} + \lambda C_2 \cos(\lambda \cdot 0)$$
$$0 = \lambda C_2$$
$$C_2 = 0$$

$$\boxed{x = d\cos\left(\sqrt{\frac{k}{m}}\,t\right)}$$

9.49 continued

d) Find speed of mass when it passes through the position where spring is relaxed

conservation of energy

$E_{total} = E_P + E_K$

$\frac{1}{2}kd^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

for $x = 0$

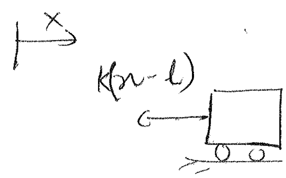
$\frac{1}{2}kd^2 = \frac{1}{2}mv^2$

$\rightarrow \boxed{v = \pm\sqrt{\frac{k}{m}}\,d}$

9.49 — additional note

If you assume x to be from the wall as stated in the problem, then

FBD



$$\Sigma F_{ext} = \dot{L}$$

$$-k\cdot(x-l_0) = m\ddot{x}$$

Part b:

$$\boxed{m\ddot{x} + kx = kl_0}$$

Part c:

$$\boxed{x = l_0 + d\cos\left(\sqrt{\frac{k}{m}}\,t\right)}$$

Part a, d: will have the same answer as above!

9.3.10 Mass m hangs from a spring with constant k and which has the length l_0 when it is relaxed (i.e., when no mass is attached). It only moves vertically.

a) Draw a Free Body Diagram of the mass.

b) Write the equation of linear momentum balance.

c) Reduce this equation to a standard differential equation in x, the position x of the mass.

d) Verify that one solution is that $x(t)$ is constant at $x = l_0 + mg/k$.

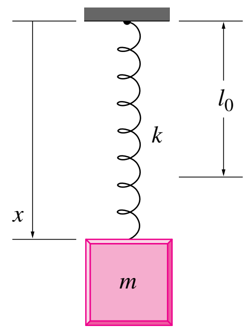
e) What is the meaning of that solution? (That is, describe in words what is going on.)

f) Define a new variable $\hat{x} = x - (l_0 + mg/k)$. Substitute $x = \hat{x} + (l_0 + mg/k)$ into your differential equation and note that the equation is simpler in terms of the variable $\hat{x}$.

g) Assume that the mass is released from an an initial position of $x = D$. What is the motion of the mass?

h) What is the period of oscillation of this oscillating mass?

i) Why might this solution not make physical sense for a long, soft spring if the initial stretch is large. In other words, what is wrong with this solution if $D > \ell_0 + 2mg/k$?



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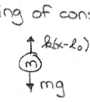
Problem 9.10

9.53

Mass m hanging from spring of constant k, ℓ_o

a) Free body diagram:



b) $\Sigma F_x = ma \Rightarrow mg - k(x-\ell_o) = ma$

c) $m\ddot{x} + k(x-\ell_o) = mg \Rightarrow \ddot{x} + \frac{k}{m}(x-\ell_o) = g$

d) Check $x(t) = \ell_o + \frac{mg}{k}$, $\ddot{x} = 0$

$\Rightarrow 0 + \frac{k}{m}(\ell_o + \frac{mg}{k} - \ell_o) = \frac{k}{m}\frac{mg}{k} = g$ ✓

e) This is the deformed equilibrium position of the system under gravity load.

f) Let $\hat{x} = x - (\ell_o + \frac{mg}{k}) \Rightarrow x = \hat{x} + (\ell_o + \frac{mg}{k})$

$\therefore$ we have $\ddot{\hat{x}} + \frac{k}{m}(\hat{x} + \ell_o - \ell_o + \frac{mg}{k}) = g$

OR $\ddot{\hat{x}} + \frac{k}{m}(\hat{x} + \frac{mg}{k}) = g \Rightarrow \ddot{\hat{x}} + \frac{k}{m}\hat{x} = 0$

g) We solve $\ddot{\hat{x}} + \frac{k}{m}\hat{x} = 0$ to get $\hat{x} = c_1 \sin(t\sqrt{\frac{k}{m}}) + c_2\cos(t\sqrt{\frac{k}{m}})$

$\Rightarrow$ Initial conditions: $\hat{x}(0) = D$, $\dot{\hat{x}}(0) = 0$

$\dot{\hat{x}} = c_1\sqrt{\frac{k}{m}}\cos(t\sqrt{\frac{k}{m}}) - c_2\sqrt{\frac{k}{m}}\sin(t\sqrt{\frac{k}{m}})$

$\dot{\hat{x}}(0) = 0 = c_1\sqrt{\frac{k}{m}} \therefore c_1 = 0$

$\hat{x}(0) = c_2 = D \Rightarrow \boxed{\hat{x}(t) = D\cos(t\sqrt{k/m})}$

h) Period $= \frac{2\pi}{\omega}$, where $\omega = \sqrt{k/m}$ $\therefore$ $\boxed{T = 2\pi\sqrt{m/k}}$

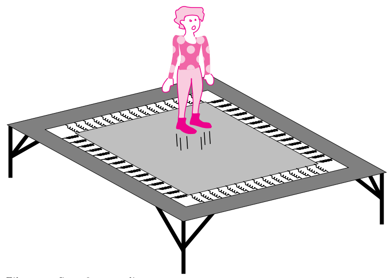
i) This would not make sense because the spring would want to oscillate upwards past the top of the spring (at its support).

9.3.12 A person jumps on a trampoline. The trampoline is modeled as having an effective vertical undamped linear spring with stiffness $k = 200\,\mathrm{lbf/\,ft}$. The person is modeled as a rigid mass $m = 150\,\mathrm{lbm}$. $g = 32.2\,\mathrm{ft/s^2}$.

a) What is the period of motion if the person's motion is so small that her feet never leave the trampoline?

b) What is the maximum amplitude of motion (amplitude of the sine wave) for which her feet never leave the trampoline?

c) (harder) If she repeatedly jumps so that her feet clear the trampoline by a height $h = 5\,\mathrm{ft}$, what is the period of this motion (note, the contact time is *not* exactly half of a vibration period)? [Hint, a neat graph of height vs time will help.]



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Problem 9.12: A person jumps on a trampoline.

9.55

Given: $k = 200\ \frac{\mathrm{lbf}}{\mathrm{ft}}$, $m = 150$ lbm, $g = 32.2\ \mathrm{ft/s^2}$

a) If contact with trampoline never breaks,

$m\ddot{x} + kx = -mg$ OR $\ddot{x} + \omega^2 x = -g$, where $\omega = \sqrt{k/m}$

Solution: $x = C_1 \sin(\omega t) + C_2 \cos(\omega t) - g/\omega^2$

$x(0) = 0$ and $\dot{x}(0) = 0$ (INITIAL COND.)

$\dot{x} = C_1 \omega \cos(\omega t) - C_2 \omega \sin(\omega t)$

$\Rightarrow \dot{x}(0) = C_1\omega = 0 \Rightarrow C_1 = 0$

$x(0) = C_2 - g/\omega^2 = 0 \quad \therefore \quad C_2 = mg/k$

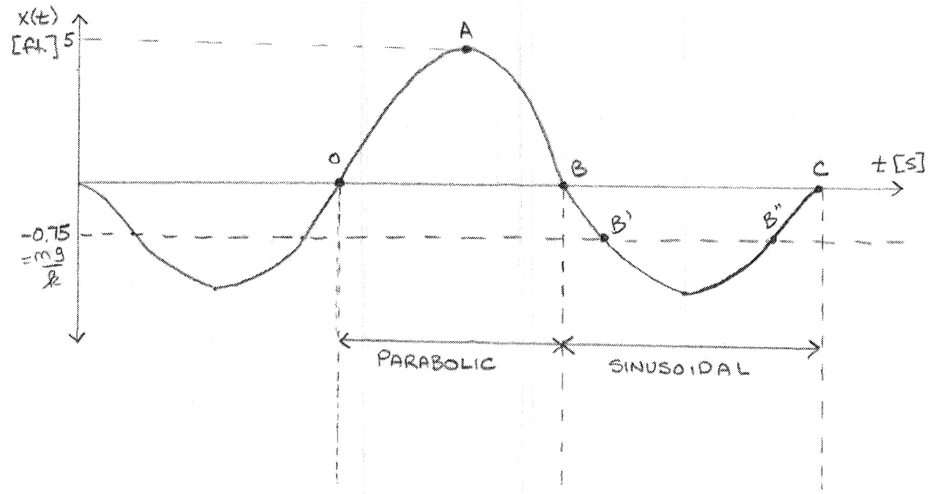
$\therefore \quad x(t) = \frac{mg}{k}\left[\cos(\omega t) - 1\right]$

Period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{150\ \mathrm{lbm}}{200\ \mathrm{lbf/ft}} \times \frac{1\ \mathrm{lbf}}{1\ \mathrm{lbm}\cdot g}}$

$= 2\pi\sqrt{\frac{150}{200\ 1/\mathrm{ft} \times 32.2\ \mathrm{ft/s^2}}} = \boxed{0.959\ \text{seconds}}$

b) Max amplitude $= \frac{mg}{k} = \frac{150\ \mathrm{lbm}\ g}{200\ \mathrm{lbf/ft}} \times \frac{1\ \mathrm{lbf}}{1\ \mathrm{lbm}\cdot g} = \boxed{0.75\ \text{feet}}$

c) If the jumper loses contact with the trampoline, to jump to a height of 5 feet, the motion is sinusoidal while in contact and parabolic (from projectile motion theory) once the jumper is in the air. See next page for a plot of what this looks like.



To analyze, assume we begin at point A, a height of 5 feet above the trampoline. ($x=5$, $\dot{x}=0$)

From $A \to B$, $x(t) = -\frac{1}{2}gt^2 + h_o = 5 - \frac{1}{2}gt^2$

$\therefore$ $x(t) = 0$ when $5 - \frac{1}{2}gt^2 = 0$, or $t = \sqrt{\frac{2(5\,ft)}{32.2\,ft/s^2}}$

$t = 0.5573$ seconds

$\therefore$ Total time from O to B is $2t = 1.115$ seconds

@ B, $x = 0$ and $\dot{x} = -gt\,/_{t=0.5573} = -17.945\ ft/s = \dot{x}_0$

We use this as an initial condition to define a new sine wave, setting point B as $t=0$.

$$x(t) = c_1 \sin\left(\sqrt{\tfrac{k}{m}}\,t\right) + c_2 \cos\left(\sqrt{\tfrac{k}{m}}\,t\right) - \frac{gm}{k}$$

$$x(0) = 0 = C_1(0) + C_2 - \frac{mg}{k} \quad \therefore \quad C_2 = \frac{mg}{k}$$

$$\dot{x}(0) = \dot{x}_0 = C_1\sqrt{\frac{k}{m}}\cos(0) - C_2\sqrt{\frac{k}{m}}\sin(0)$$

$$\therefore \quad C_1\sqrt{\frac{k}{m}} = \dot{x}_0 \quad \text{OR} \quad C_1 = \dot{x}_0\sqrt{\frac{m}{k}}$$

$$\therefore \quad x(t) = \dot{x}_0\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}t\right) + \frac{mg}{k}\cos\left(\sqrt{\frac{k}{m}}t\right) - \frac{mg}{k}$$

We want to find when this expression $= -\frac{mg}{k}$

$\Rightarrow$ This is point B' on previous graph

Using Matlab or a calculator to solve,

$$t = -\tan^{-1}\left(\frac{g}{\dot{x}_0}\sqrt{\frac{m}{k}}\right) \Big/ \sqrt{\frac{k}{m}}$$

$$= -\tan^{-1}\left(\frac{32.2\ \text{ft/s}^2}{-17.945\ \text{ft/s}}\sqrt{\frac{150\ \text{lbm}}{200\ \text{lbf/ft} \times 32.2\ \text{ft/s}^2}}\right) \Big/ \sqrt{\frac{200 \times 32.2}{150}} = 0.04079\ \text{s}$$

$\therefore$ Distance from B to B' $= 0.04079$ s

We know distance from B' to B'' $= \frac{T}{2} = \frac{0.959\ \text{s}}{2}$

$= 0.4795$ seconds

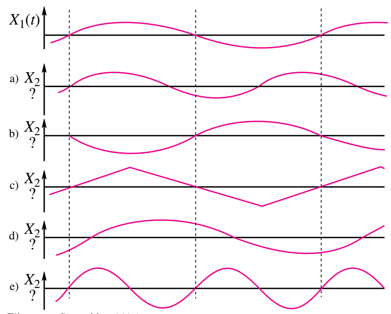
From B'' to C is the same as B to B' $= 0.04079$ s

$\therefore$ Total period $T = 1.115\ \text{s} + 2(0.0408\ \text{s}) + 0.4795\ \text{s}$

$$= \boxed{1.676\ \text{seconds}}$$

The primary emphasis of this section is setting up correct differential equations (without sign errors) and solving these equations on the computer.

9.4.14 $x_1(t)$ and $x_2(t)$ are measured positions on two points of a vibrating structure. $x_1(t)$ is shown. Some candidates for $x_2(t)$ are shown. Which of the $x_2(t)$ could possibly be associated with a normal mode vibration of the structure? Answer "could" or "could not" next to each choice and briefly explain your answer (If a curve looks like it is meant to be a sine/cosine curve, it is.)



Problem 9.14

9.73
* Look at page 466 of text
a) could not because they have different freq.

b) could

c) could not because it is not simple harmonic motion

d) could not because not exactly in (or out) of phase

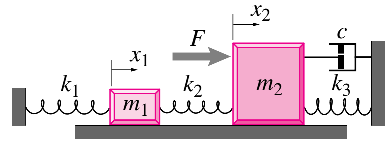
e) could not, reason same as (a)

9.4.17 Two masses are connected to fixed supports and each other with the three springs and dashpot shown. The force F acts on mass 2. The displacements x_1 and x_2 are defined so that $x_1 = x_2 = 0$ when the springs are unstretched. The ground is frictionless. The governing equations for the system shown can be written in first order form if we define $v_1 \equiv \dot{x}_1$ and $v_2 \equiv \dot{x}_2$.

a) Write the governing equations in a neat first order form. Your equations should be in terms of any or all of the constants m_1, m_2, k_1, k_2, k_3, C, the constant force F, and t. Getting the signs right is important.

b) Write computer commands to find and plot $v_1(t)$ for 10 units of time. Make up appropriate initial conditions.

c) For constants and initial conditions of your choosing, plot x_1 vs t for enough time so that decaying erratic oscillations can be observed.



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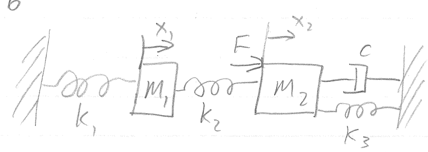
Problem 9.17

9.76

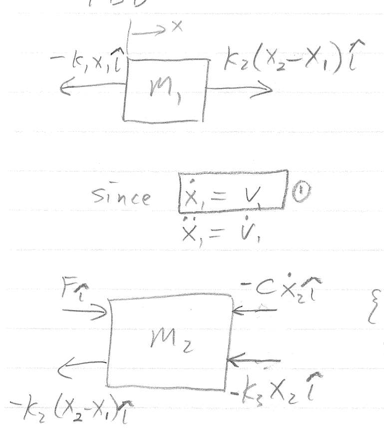


$X_1 = X_2 = 0$ when springs unstretched

ground is frictionless

a) Write governing equation in first order form.

FBD



$\{m_1\ddot{x}_1\hat{\imath} = k_2(x_2-x_1)\hat{\imath} - k_1x_1\hat{\imath}\}$

$\{\}\cdot\hat{\imath} \Rightarrow$

$\ddot{x}_1 = \frac{k_2}{m}(x_2-x_1) - \frac{k_1}{m}x_1$

$\boxed{\dot{v}_1 = \frac{k_2}{m}(x_2-x_1) - \frac{k_1}{m}x_1}$ (3)

since $\boxed{\dot{x}_1 = v_1}$ (1)

$\ddot{x}_1 = \dot{v}_1$

$\{m_2\ddot{x}_2\hat{\imath} = F\hat{\imath} - c\dot{x}_2\hat{\imath} - k_3x_2\hat{\imath} - k_2(x_2-x_1)\hat{\imath}\}$

$\{\}\cdot\hat{\imath} \Rightarrow$

$m_2\ddot{x}_2 = F - c\dot{x}_2 - k_3x_2 - k_2(x_2-x_1)$

since $\boxed{\dot{x}_2 = v_2}$ (2)

$\ddot{x}_2 = \dot{v}_2$

$\boxed{\dot{v}_2 = \frac{F}{m} - \frac{c}{m}\dot{x}_2 - \frac{k_3}{m}x_2 - \frac{k_2}{m}(x_2-x_1)}$ (4)

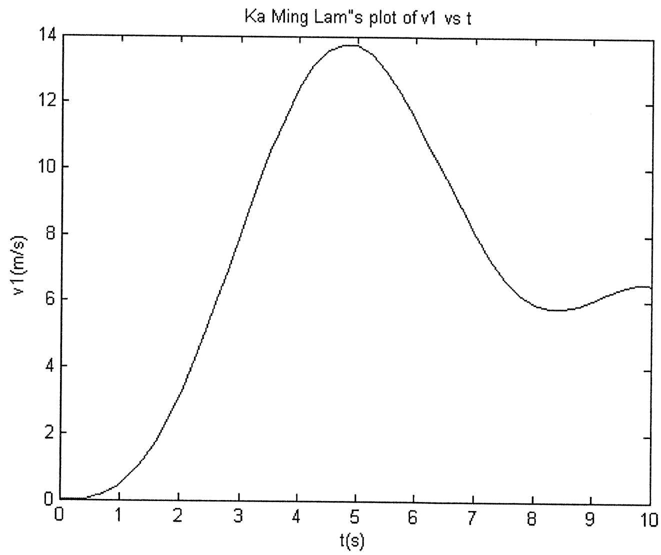
(1)-(4) are gov. eqs in 1st order form

b). See attached files

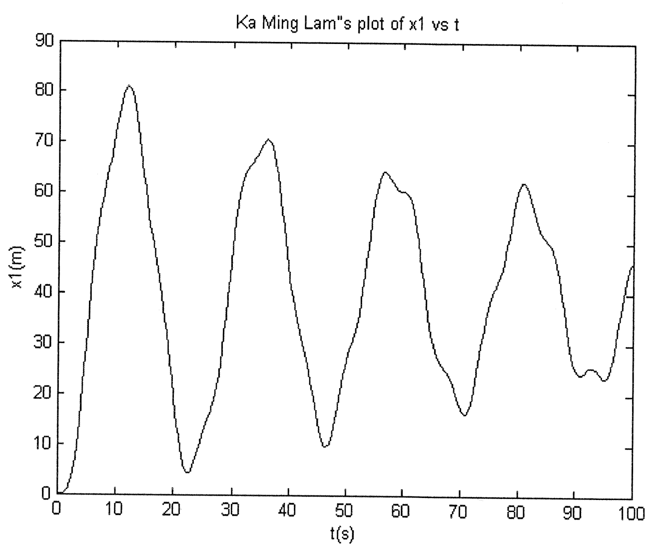
c). Modify the code in b) a little bit to plot $x_1 \sim t$., increase time to see the decaying oscillations.

```
% problem 9.76

function question976
%time span
tspan = [0,10];          %integrate for 10 sec
z0 = [0, 0, 0, 0]';      %initial position and velocity
                         %[x0, vx0, y0, vy0]
%solves the ODEs
[t z] = ode45(@rhs,tspan,z0);

%Unpack the variables
x1 = z(:,1);
v1 = z(:,2);
x2 = z(:,3);
v2 = z(:,4);


%plot the results
plot(t,v1)
title('Ka Ming Lam''s plot of v1 vs t')
xlabel('t(s)')
ylabel('v1(m/s)')
%set grid, xmin, xmax, ymin, ymax


end

%-------------------------------------------------------\
function zdot = rhs(t,z)
x1 = z(1);  v1 = z(2);  x2 = z(3);  v2 = z(4);

%put in vlaues for mass, C and g below
m1 = 2;
m2 = 20;  % masses in kg
C = 0.4;  % in kg/s
F = 120;
k1 = 1;
k2 = 1;
k3 = 1;   % k in N/m

% the linear momentum balance eqns:
x1dot = v1;
v1dot = (-k2-k1)/m1*x1+k2/m1*x2;
x2dot = v2;
v2dot = F/m2-C/m2*v2-(k3+k2)/m2*x2+k2/m2*x1;

zdot = [x1dot;v1dot ; x2dot;v2dot];   %this is what the function returns (column vector)


end
```

b). Here is the plot



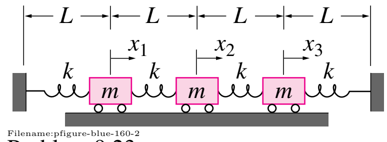
c). In order to have a decaying erratic oscillation we need to increase tspan to [0 100] for this case



9.4.23 For the three-mass system shown, assume $x_1 = x_2 = x_3 = 0$ when all the springs are fully relaxed. One of the normal modes is described with the initial condition $(x_{10}, x_2, x_3) = (1, 0, -1)$.

a) What is the angular frequency ω for this mode? Answer in terms of L, m, k, and g. (Hint: Note that in this mode of vibration the middle mass does not move.)

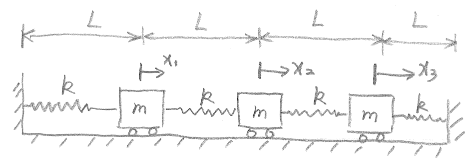
b) Make a neat plot of x_2 versus x_1 for one cycle of vibration with this mode.



Problem 9.23

①

9.82 Normal modes



3-Mass system, connected by springs with stiffness k. All the mass is m.

Assume $x_1 = x_2 = x_3$ where all springs are at rest.

A normal mode of this system can be described as eigenvector (1, 0, -1).

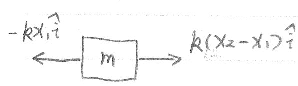
Q. 1). What's the frequency corresponding to this normal mode?

2). Plot x_2 versus x_1 for one cycle of vibration in this mode.

Solution: 1). First, we want to derive the equations of motion for this system, starting from FBD.

FBD: $\longrightarrow \hat{\imath}$ Eq. of motions

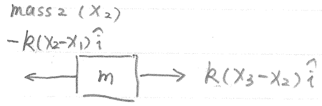
mass 1 (x_1)



$\{ m\ddot{x}_1\hat{\imath} = k(x_2-x_1)\hat{\imath} - kx_1\hat{\imath} \}$

$\{\}\cdot\hat{\imath} \Rightarrow \boxed{m\ddot{x}_1 = -2kx_1 + kx_2}$

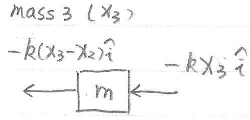
mass 2 (x_2)



$\{ m\ddot{x}_2\hat{\imath} = k(x_3-x_2)\hat{\imath} - k(x_2-x_1)\hat{\imath} \}$

$\{\}\cdot\hat{\imath} \Rightarrow \boxed{m\ddot{x}_2 = kx_1 - 2kx_2 + kx_3}$

mass 3 (x_3)



$\{ m\ddot{x}_3\hat{\imath} = -kx_3\hat{\imath} - k(x_3-x_2)\hat{\imath} \}$

$\{\}\cdot\hat{\imath} \Rightarrow \boxed{m\ddot{x}_3 = kx_2 - 2kx_3}$

(9.82. cont'd) ②

Second, by definition of normal mode, for this normal mode with (1, 0, −1), we can write

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \underbrace{A}_{\text{amplitude}} \underbrace{\cos}_{\text{simple harmonic function}} (\underbrace{\omega}_{\text{same frequency, we want to solve it}} t + \underbrace{\phi}_{\text{exactly in or out of phase.}})$$

$$\Rightarrow \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} A \frac{d^2 \cos(\omega t + \phi)}{dt^2} = - \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} A \omega^2 \cos(\omega t + \phi)$$

Third, substitute $[x]$, $[\ddot{x}]$ for this normal mode into equations of motions:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow - \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} A \omega^2 \cos(\omega t + \phi) + \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} A \cos(\omega t + \phi) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow - \begin{bmatrix} m \\ 0 \\ -m \end{bmatrix} \omega^2 + \begin{bmatrix} 2k \\ 0 \\ -2k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -m\omega^2 + 2k = 0 \qquad \Rightarrow \boxed{\omega = \sqrt{\frac{2k}{m}}}$$

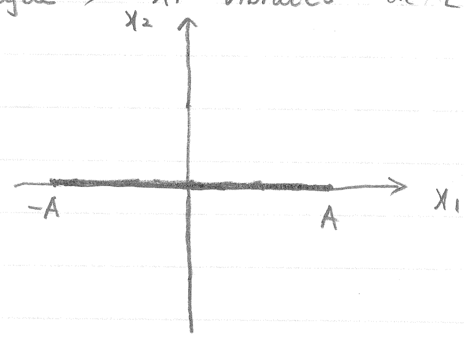
(9.82, cont'd) 3

(2-) From (1-), we know in this mode

$$x_1 = A\cos\left(\sqrt{\frac{2k}{m}}\,t + \phi\right)$$

$$x_2 = 0$$

During one cycle, x_1 vibrates in $[-A, A]$ and x_2 remains 0.



9.5.6 Before a collision two particles, $m_A = 7\,\text{kg}$ and $m_B = 9\,\text{kg}$, have velocities of $v_A^- = 6\,\text{m/s}$ and $v_B^- = 2\,\text{m/s}$. The coefficient of restitution is $e = .5$. Find the impulse of mass A on mass B and the velocities of the two masses after the collision.

9.84

Before a collision two particles,

$m_A = 1\,kg$ $m_B = 2\,kg$

$v_A^- = 10\,m/s$ $v_B^- = 5\,m/s$

After the collision

$v_A^+ = 8\,m/s$

a) momentum of A before collision?

momentum $= m_A v_A^-$

$\boxed{= 10\,kg\,m/s}$

b) momentum of B before collision?

momentum $= m_B v_B^-$

$\boxed{= 10\,kg\,m/s}$

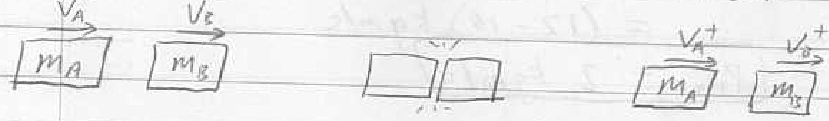
c) System momentum before collision

System momentum $= m_A v_A^- + m_B v_B^-$

$\boxed{= 20\,kg\,m/s}$

FBD

before   collision   After



9.84 continued

d) momentum of A after collision

momentum $= m_A v_A^+$

$\boxed{= 8\ kgm/s}$

e) System momentum after collision

System momentum after = System momentum before

$\boxed{= 20\ kgm/s}$

f) momentum of B after collision

System momentum after = $\text{momentum}_A + \text{momentum}_B$

$\text{momentum}_B = \boxed{12\ kgm/s}$

g) impulse A applies to B?

$P_{A\to B} = m_A(v_A^+ - v_A^-)$

$= (8-10)\ kgm/s$

$\boxed{P_{A\to B} = -2\ kgm/s}$

h) impulse B applies to A?

$P_{B\to A} = m_B(v_B^+ - v_B^-)$

$= (12-10)\ kgm/s$

$\boxed{P_{B\to A} = 2\ kgm/s}$

9.84 continued

i) E_K before collision?

$$E_K = \frac{1}{2} m_A (v_A^-)^2 + \frac{1}{2} m_B (v_B^-)^2$$

$$\boxed{E_K = 75\ \text{J}}$$

j) E_K after collision?

$$E_K = \frac{1}{2} m_A (v_A^+)^2 + \frac{1}{2} m_B (v_B^+)^2$$

$$= \frac{1}{2}(1\,\text{kg})(8\,\text{m/s})^2 + \frac{1}{2}(2\,\text{kg})(6\,\text{m/s})^2$$

$$\boxed{E_K = 68\ \text{J}}$$

k) Coefficient of restitution?

$$(v_B' - v_A') = e\,(v_A - v_B)$$

$$(6 - 8)\,\text{m/s} = e\,(10 - 5)\,\text{m/s}$$

$$-2 = 5e$$

$$\boxed{e = -2/5 = -0.4}$$

Problem 9.84

If you assumed $v_A^+ = 6\ m/s$, than the following answers will change

d) $6\ kg\ m/s$

f) $14\ kg\ m/s$

g) $-4\ kg\ m/s$. You get this by solving $v_B^+ = 7\ m/s$

h) $4\ kg\ m/s$

j) $67\ J$

k) 0.2

9.5.10 A basketball with mass m_b is dropped from height h onto the hard solid ground on which it has coefficient of restitution e_b. Just on top of the basketball, falling with it and then bouncing against it after the basketball hits the ground, is a small rubber ball with mass m_r that has a coefficient of restitution e_r with the basketball.

a) In terms of some or all of m_b, m_r, h, g, e_b and e_r how high does the rubber ball bounce (measure height relative to the collision point)?

b) Assuming the coefficients of restitution are less than or equal to one, for given h, what mass and restitution parameters maximize the height of the bounce of the rubber ball and what is that height?

9.92

Basketball with mass m_b dropped from height h, $e = e_b$

Small rubber ball with mass m_r, $e = e_r$

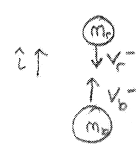
a) Treat this as two collisions:

i) basketball hits ground

$V_0 = 0$, cons. of energy $V_f = -\sqrt{2gh}$ (before hit)

After collison $V = e_b V_f = e_b\sqrt{2gh}$

ii) basketball and rubber ball collide



$V_r^- = -\sqrt{2gh}$, $V_b^- = e_b\sqrt{2gh}$

$$m_b V_b^- + m_r V_r^- = m_b V_b^+ + m_r V_r^+ \quad (1)$$

$$V_r^+ - V_b^+ = e_r(V_b^- - V_r^-) \quad (2)$$

From (2), $V_b^+ = V_r^+ - e_r V_b^- + e_r V_r^-$

$$= V_r^+ - e_r e_b\sqrt{2gh} - e_r\sqrt{2gh}$$

$$= V_r^+ - e_r\sqrt{2gh}\,(1+e_b)$$

Plug into (1), $m_b e_b\sqrt{2gh} - m_r\sqrt{2gh} = m_r V_r^+ + m_b(V_r^+ - e_r\sqrt{2gh}(1+e_b))$

OR $\sqrt{2gh}\,(m_b e_b - m_r) = m_r V_r^+ + m_b V_r^+ - m_b e_r\sqrt{2gh}\,(1+e_b)$

$$= V_r^+(m_r + m_b) - m_b e_r\sqrt{2gh}\,(1+e_b)$$

$\therefore$ $V_r^+(m_r+m_b) = \sqrt{2gh}\,[m_b e_b - m_r + m_b e_r(1+e_b)]$

OR $$V_r^+ = \frac{\sqrt{2gh}\,[m_b e_b - m_r + m_b e_r(1+e_b)]}{m_r + m_b}$$

next $\rightarrow$

Conservation of energy:

$$\cancel{m_r} g h_r = \frac{1}{2}\cancel{m_r}(v_r^+)^2 \quad \therefore \; h_r = \frac{1}{2g}(v_r^+)^2$$

$$h_r = \frac{1}{2\cancel{g}}(2\cancel{g}h)\left[\frac{m_b e_b - m_r + m_b e_r(1+e_b)}{m_r + m_b}\right]^2$$

$$\therefore \; \boxed{h_r = h\left[\frac{m_b e_b - m_r + m_b e_r(1+e_b)}{m_b + m_r}\right]^2}$$

b) To maximize h_r, we can begin by recognizing that letting $e_b = e_r = 1$ maximizes the numerator of the bracketed expression.

$$\therefore \; h_r = h\left(\frac{m_b - m_r + 2m_b}{m_b + m_r}\right)^2 = h\left(\frac{3m_b - m_r}{m_b + m_r}\right)^2$$

This is maximized by increasing m_b and decreasing m_r.

$\therefore$ We want $e_b = e_r = 1$ and the largest ratio of m_b to m_r possible.

$$h_{r,max} = h\left(\frac{3m_b - 0}{m_b + 0}\right)^2 = 9h$$

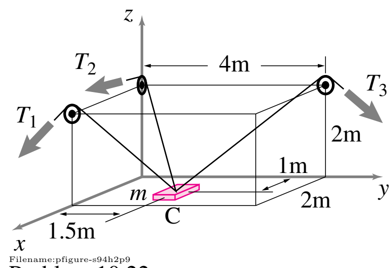
$$\boxed{\text{Theoretical max } h_r = 9h}$$

10.1.22 An object C of mass 2 kg is pulled by three strings as shown. The acceleration of the object at the position shown is $\boldsymbol{a} = \left(-0.6\hat{\imath} - 0.2\hat{\jmath} + 2.0\hat{\boldsymbol{k}}\right)\ \mathrm{m/s^2}$.

a) Draw a free body diagram of the mass.

b) Write the equation of linear momentum balance for the mass. Use $\boldsymbol{\lambda}$'s as unit vectors along the strings.

c) Find the three tensions T_1, T_2, and T_3 at the instant shown. You may find these tensions by using hand algebra with the scalar equations, using a computer with the matrix equation, or by using a cross product on the vector equation.

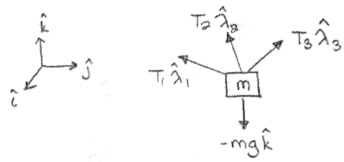


Problem 10.22

10.22

$\vec{a} = -0.6\hat{\imath} - 0.2\hat{\jmath} + 2.0\hat{k}\ [\mathrm{m/s^2}]$, $m = 2\ \mathrm{kg}$

a) Free body diagram:



$\vec{r}_m = 1.0\hat{\imath} + 1.5\hat{\jmath}\quad [\mathrm{m}]$

$\vec{r}_1 = 2.0\hat{\imath} + 2.0\hat{k}\quad [\mathrm{m}]$

$\vec{r}_2 = 2.0\hat{k}\quad [\mathrm{m}]$

$\vec{r}_3 = 4.0\hat{\jmath} + 2.0\hat{k}\quad [\mathrm{m}]$

b) $\vec{r}_{m1} = \vec{r}_1 - \vec{r}_m = 1.0\hat{\imath} - 1.5\hat{\jmath} + 2.0\hat{k}\qquad |\vec{r}_{m1}| = \frac{1}{2}\sqrt{29}$

$\vec{r}_{m2} = \vec{r}_2 - \vec{r}_m = -1.0\hat{\imath} - 1.5\hat{\jmath} + 2.0\hat{k}\qquad |\vec{r}_{m2}| = \frac{1}{2}\sqrt{29}$

$\vec{r}_{m3} = \vec{r}_3 - \vec{r}_m = -1.0\hat{\imath} + 2.5\hat{\jmath} + 2.0\hat{k}\qquad |\vec{r}_{m3}| = \frac{3}{2}\sqrt{5}$

$$\sum \vec{F} = m\vec{a} = T_1\hat{\lambda}_1 + T_2\hat{\lambda}_2 + T_3\hat{\lambda}_3 - mg\hat{k}$$

OR

$$2\,\mathrm{kg}(-0.6\hat{\imath} - 0.2\hat{\jmath} + 2.0\hat{k}) = T_1\left(\tfrac{2\sqrt{29}}{29}\right)(\hat{\imath} - 1.5\hat{\jmath} + 2.0\hat{k})$$
$$+ T_2\left(\tfrac{2\sqrt{29}}{29}\right)(-\hat{\imath} - 1.5\hat{\jmath} + 2.0\hat{k}) + T_3\left(\tfrac{2\sqrt{5}}{15}\right)(-\hat{\imath} + 2.5\hat{\jmath} + 2.0\hat{k}) - 19.62\hat{k}$$

In component form:

$$\hat{\imath}\,(-1.2) = \hat{\imath}\,(0.3714T_1 - 0.3714T_2 - 0.2981T_3)$$
$$\hat{\jmath}\,(-0.4) = \hat{\jmath}\,(-0.5571T_1 - 0.5571T_2 + 0.7454T_3)$$
$$\hat{k}\,(4 + 19.62) = \hat{k}\,(0.7428T_1 + 0.7428T_2 + 0.5963T_3)$$

CONTINUED ON PAGE 4

10.22 continued.

In matrix form, we have:

$$\begin{bmatrix} -1.2 \\ -0.4 \\ 23.62 \end{bmatrix} = \begin{bmatrix} 0.3714 & -0.3714 & -0.2981 \\ -0.5571 & -0.5571 & 0.7454 \\ 0.7428 & 0.7428 & 0.5963 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

Solving in Matlab yields:

$$\boxed{\begin{aligned} T_1 &= 14.28\text{ N} \\ T_2 &= 5.86\text{ N} \\ T_3 &= 14.52\text{ N} \end{aligned}}$$

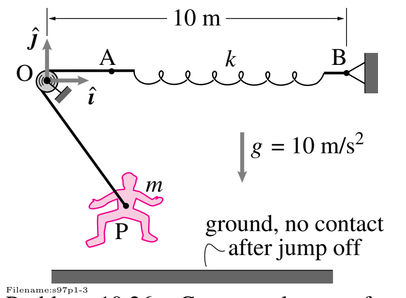
Alternately, see Matlab code (modified from problem 10.17) on previous page.

10.1.26 Bungy Jumping. In a relatively safe bungy jumping system, people jump up from the ground while being pulled up by a rope that runs over a pulley at O and is connected to a stretched spring anchored at B. The ideal pulley has negligible size, mass, and friction. For the situation shown the spring AB has rest length $\ell_0 = 2$ m and a stiffness of $k = 200$ N/m. The inextensible massless rope from A to P has length $\ell_r = 8$ m, the person has a mass of 100 kg. Take O to be the origin of an xy coordinate system aligned with the unit vectors $\hat{\imath}$ and $\hat{\jmath}$

a) Assume you are given the position of the person $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ and the velocity of the person $\boldsymbol{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$. Find her acceleration in terms of some or all of her position, her velocity, and the other parameters given. Then use the numbers given, where supplied, in your final answer.

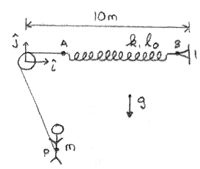
b) Given that bungy jumper's initial position and velocity are $\vec{r}_0 = 1\,\text{m}\hat{\imath} - 5\,\text{m}\hat{\jmath}$ and $\boldsymbol{v}_0 = \boldsymbol{0}$ write computer commands to find her position at $t = \pi/\sqrt{2}$ s.

c) Find the answer to part (b) with pencil and paper (that is, find an analytic solution to the differential equations, a final numerical answer is desired).



Problem 10.26: Conceptual setup for a bungy jumping system.

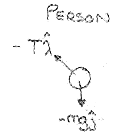
10.26

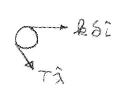


Given: $k = 200$ N/m
$\ell_0 = 2$ m
$\ell_{AP} = 8\text{ m} = \ell_r$
$m = 100$ kg
Assume no air drag.

a) $\vec{r} = x\hat{\imath} + y\hat{\jmath}$, $\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$, Find $\vec{a}$

PERSON: (1) PULLEY: (2)





$$\hat{\lambda} = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2+y^2}}$$

δ = extension of spring = $\ell - \ell_0$, where $\ell = 10 - (\ell_r - \sqrt{x^2+y^2})$

$\therefore \ell = 10 - 8 + \sqrt{x^2+y^2} = 2 + \sqrt{x^2+y^2}$

$\therefore \delta = \ell - \ell_0 = \sqrt{x^2+y^2}$

From FBD 2: $T = k\delta$ (tension on both sides of pulley must be equal) $\therefore T = k\sqrt{x^2+y^2}$

From FBD 1: $\sum \vec{F} = m\vec{a} = -T\hat{\lambda} - mg\hat{\jmath}$

$$\therefore -k\sqrt{x^2+y^2}\,\frac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2+y^2}} - mg\hat{\jmath} = m\vec{a}$$

OR $-kx\hat{\imath} - (ky + mg)\hat{\jmath} = m\vec{a}$ - plug in values

$$\vec{a} = \frac{-200\,\text{N/m}\,x}{100\,\text{kg}}\hat{\imath} - \frac{200\,\text{N/m}\,y + 100\,\text{kg}\,(10\,\text{m/s}^2)}{100\,\text{kg}}\hat{\jmath} = \boxed{-2x\,[\text{s}^{-2}]\hat{\imath} - (2y + 10\,\text{m})[\text{s}^{-2}]\hat{\jmath}}$$

Problem 10.26 (b).

```
function Prob1026()
% Problem 10.26 Solution
% March 11, 2008

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%  VARIABLES (Assume consistent units)
%    r  = displacement (vector with x and y components)
%    v  = dr/dt

% INTIAL CONDITIONS
r0= [1 -5]'; % initial position
v0= [0 0]'; % initial velocity
z0= [r0;v0]; % pack variables

tspan =[0 pi/sqrt(2)]; %time interval of integration

[t zarray] = ode45(@rhs,tspan, z0);

% Unpack Variables
r= zarray(:,1:2);

disp(r(end,:));

% ANSWER:
% ans =
%
%    -1.0000    -5.0000   (meters)
%

end

% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function zdot = rhs(t,z)

%Unpack variables
r= z(1:2);
v= z(3:4);

%The equations
rdot= v;
vdot= [-2*r(1) -2*r(2)-10]';

% Pack the rate of change of r and v
zdot= [rdot; vdot];

end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

b) See Matlab code on previous page

c) From part (a),

$$\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} = -2x\hat{\imath} - (2y+10)\hat{\jmath}$$

$$\therefore \quad (\ddot{x} = -2x)\hat{\imath}\cdot\hat{\imath} \Rightarrow \ddot{x} = -2x \quad (1)$$

$$(\ddot{y} = -2y-10)\hat{\jmath}\cdot\hat{\jmath} \Rightarrow \ddot{y} = -2y-10 \quad (2)$$

Solve (1) and (2) with $\vec{r}(0) = \hat{\imath} - 5\hat{\jmath}$, $\vec{v}(0) = \vec{0}$

(1) $\ddot{x} = -2x$, so $x(t) = A\sin(\sqrt{2}t) + B\cos(\sqrt{2}t)$

$\dot{x}(t) = \sqrt{2}A\cos(\sqrt{2}t) - \sqrt{2}B\sin(\sqrt{2}t)$

$\dot{x}(0) = 0 = \sqrt{2}A \quad \therefore A = 0$

$x(0) = 1 = B\cos(0) \quad \therefore B = 1$

$\therefore \quad x(t) = \cos(\sqrt{2}t)$

(2) $\ddot{y} = -2y - 10$, so $y(t) = C\sin(\sqrt{2}t) + D\cos(\sqrt{2}t) - \underbrace{5}_{y_p}$

$\dot{y}(0) = 0 = \sqrt{2}C \quad \therefore C = 0$

$y(0) = -5 = D\cos(0) - 5 \quad \therefore D = 0$

$\therefore \quad y(t) = -5$

$$\vec{r}(t) = \cos\sqrt{2}t\,\hat{\imath} - 5\hat{\jmath} = x(t)\hat{\imath} + y(t)\hat{\jmath}$$

$$\therefore \quad \vec{r}\left(\tfrac{\pi}{\sqrt{2}}\right) = \cos\left(\sqrt{2}\tfrac{\pi}{\sqrt{2}}\right)\hat{\imath} - 5\hat{\jmath} = \boxed{-\hat{\imath} - 5\hat{\jmath}\ [\mathrm{m}]}$$

10.1.30 The equations of motion from problem **??** are nonlinear and cannot be solved in closed form for the position of the baseball. Instead, solve the equations numerically. Make a computer simulation of the flight of the baseball, as follows.

a) Convert the equation of motion into a system of first order differential equations.

b) Pick values for the gravitational constant g, the coefficient of resistance b, and initial speed v_0, solve for the x and y coordinates of the ball and make a plots its trajectory for various initial angles θ_0.

c) Use Euler's, Runge-Kutta, or other suitable method to numerically integrate the system of equations.

d) Use your simulation to find the initial angle that maximizes the distance of travel for ball, with and without air resistance.

e) If the air resistance is very high, what is a qualitative description for the curve described by the path of the ball? Show this with an accurate plot of the trajectory. (Make sure to integrate long enough for the ball to get back to the ground.)

10.30

a) $v^2 = \dot{x}^2 + \dot{y}^2$ ← speed of ball

$\vec{F}_d = -bv^2\hat{e}_t$ ← taking into account air resistance

$$m\vec{a} = \underbrace{-bv^2\hat{e}_t}_{\text{air resistance}} \underbrace{-mg\hat{\jmath}}_{\text{gravity}}$$

$$m\ddot{x} = -b(\dot{x}^2+\dot{y}^2)\cdot(\hat{e}_t\cdot\hat{\imath})$$
$$m\ddot{y} = -b(\dot{x}^2+\dot{y}^2)\cdot(\hat{e}_t\cdot\hat{\jmath}) - mg$$

$$\boxed{\begin{aligned}\ddot{x} &= -\tfrac{b}{m}(\dot{x}^2+\dot{y}^2)\cos\theta \\ \ddot{y} &= -g - \tfrac{b}{m}(\dot{x}^2+\dot{y}^2)\sin\theta\end{aligned}}$$

$$\theta = \tan^{-1}\frac{dy}{dx} = \tan^{-1}\frac{dy/dt}{dx/dt} = \tan^{-1}\frac{\dot{x}}{\dot{y}}$$

$$\cos\theta = \frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}} \qquad \sin\theta = \frac{\dot{y}}{\sqrt{\dot{x}^2+\dot{y}^2}}$$

$$\ddot{x} = -\frac{b}{m}\dot{x}\sqrt{\dot{x}^2+\dot{y}^2}$$
$$\ddot{y} = -\frac{b}{m}\dot{y}\sqrt{\dot{x}^2+\dot{y}^2} - g$$

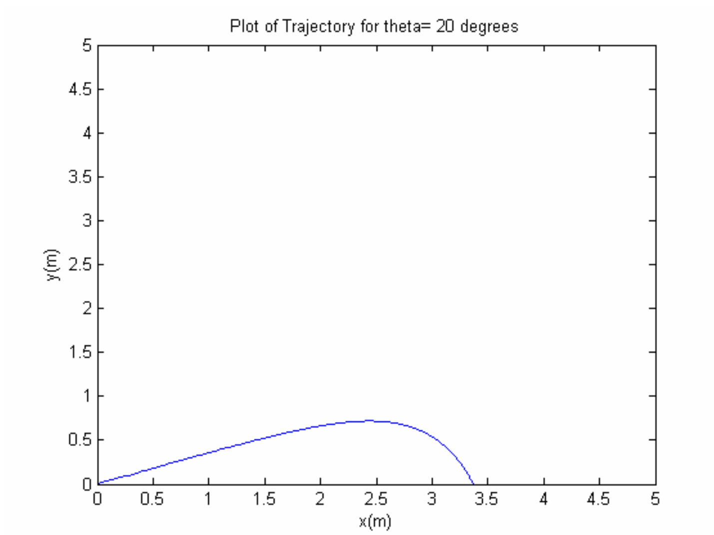
$$\boxed{\begin{aligned}&\text{for } v_x = \dot{x},\quad v_y = \dot{y} \\ &\dot{v}_x = -\tfrac{b}{m}v_x\sqrt{v_x^2+v_y^2} \\ &\dot{v}_y = -\tfrac{b}{m}v_y\sqrt{v_x^2+v_y^2} - g\end{aligned}}$$

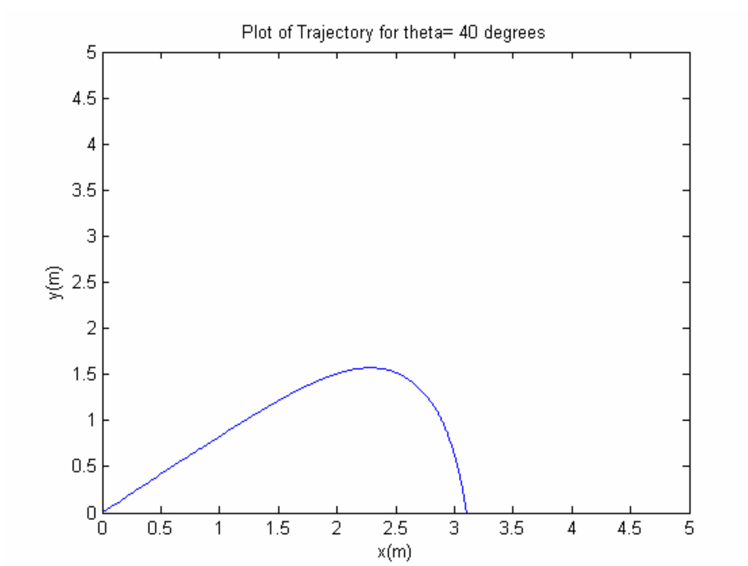
10.30 (continued)

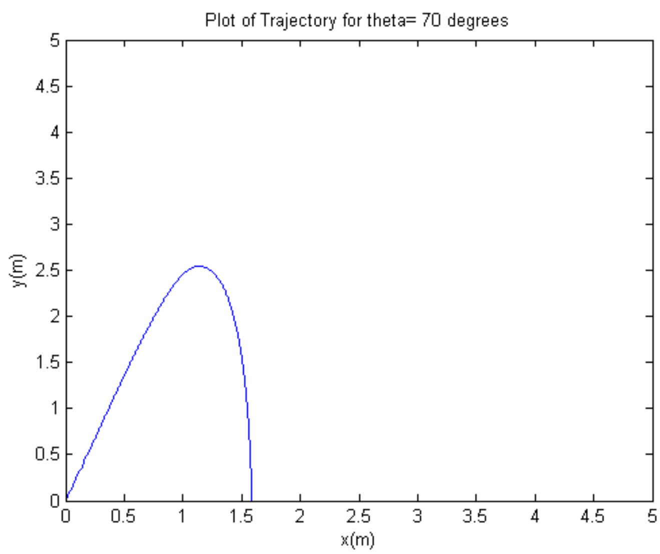
b). See attached codes and results

```
%problem 10.30(a)

function solution1030a
%solution to 10.30
%September 23,2008
b=1; m=1; g=10; % give values for b,m and g here
%Initial conditions and time span
tspan=[0:0.001:5]; %integrate for 50 seconds
x0=0;
y0=0;           %initial position
v0=50;        %magnitude of initial velocity (m/s)
theta0=20;    %angle of initial velocity (in degrees)

z0=[x0,y0,v0*cos(theta0*pi/180),v0*sin(theta0*pi/180)]';

%solves the ODEs
[t,z] = ode45(@rhs,tspan,z0,[],b,m,g);

%Unpack the variables
x= z(:,1);
y =z(:,2);
v_x = z(:,3);
v_y=z(:,4);

%plot the results
plot(x,y);
xlabel('x(m)');
ylabel('y(m)');
%set grid,xmin,xmax,ymin,ymax
axis([0,5,0,5]);
title(['Plot of Trajectory for theta= ',num2str(theta0),' degrees']);

end

%-----------------------------------------------------------------------%
function zdot = rhs(t,z,b,m,g)                    %function to define ODE
x=z(1); y=z(2); v_x=z(3); v_y=z(4);

%the linear momentum balance eqns
xdot=v_x;
v_xdot=-(b/m)*v_x*(v_x^2+v_y^2)^0.5;
ydot=v_y;
```

```
v_ydot=-g-(b/m)*v_y*(v_x^2+v_y^2)^0.5;

zdot=[xdot; ydot; v_xdot; v_ydot]; %this is what the function returns (column vector)

end
%----------------------------------------------------------------------%
```







c). Disregard this question. This question intends to ask you develop your own ode solver similar to ode45, using Euler's method or more sophisticated method (Ruger-Kutta method).

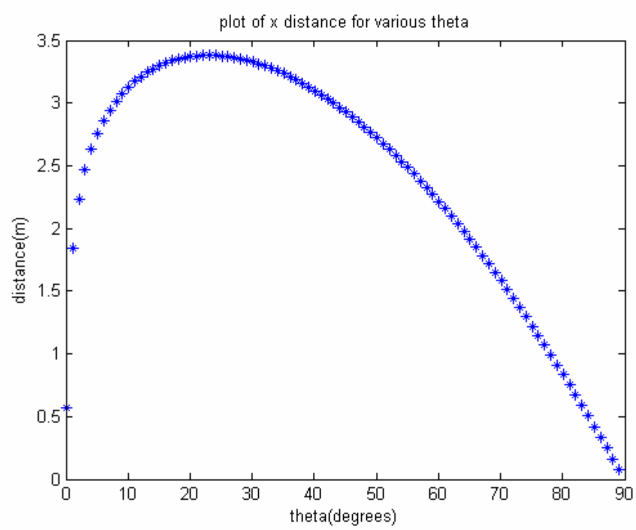
d). To find out x distance, we use 'stopevent' to terminate the integration at y=0. Then loop over for theta from 0.1 to 89.1 degree with an increment of 1 degree.

```
%problem 10.30(d)

function solution1030d
%solution to 10.30
%September 23,2008

b=1; m=1; g=10; % give values for b,m and g here

%Initial conditions and time span
tspan=[0 50]; %integrate for 50 seconds
x0=0;
y0=0;           %initial position
v0=50;       %magnitude of initial velocity (m/s)

theta0=[0.1:1:89.1]';    %angle of initial velocity (in degrees)
distance=zeros(size(theta0));    %arrays to record x distance at y=0 for each angle

for i=1:length(theta0)
```

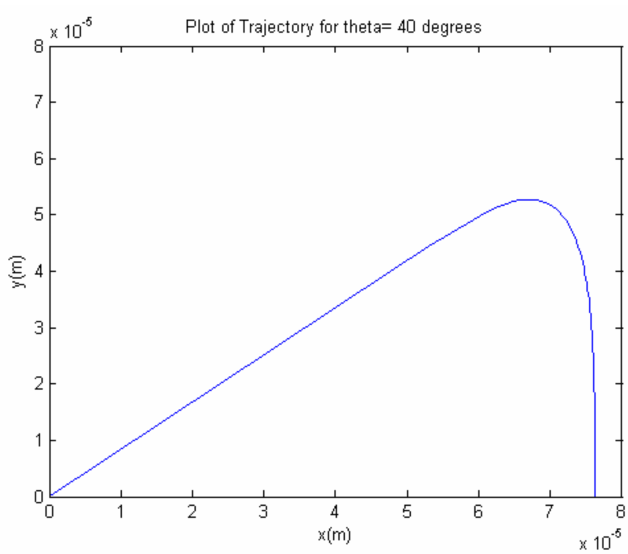
```
z0=[x0,y0,v0*cos(theta0(i)*pi/180),v0*sin(theta0(i)*pi/180)]';

options=odeset('events', @stopevent);
%solves the ODEs
[t,z] = ode45(@rhs,tspan,z0,options,b,m,g);

%Unpack the variables
x= z(:,1);
distance(i)=x(end);% the last component of x is the distance we want
end
plot(theta0,distance,'*')
xlabel('theta(degrees)');
ylabel('distance(m)');
%set grid,xmin,xmax,ymin,ymax
title(['plot of x distance for various theta']);

[maxd,j]=max(distance);
fprintf(1,'\nThe maximum distance is %6.4f m when theta=%2.0f degrees\n', maxd,theta0(j));
%print the results
end

%-----------------------------------------------------------------------%
function zdot = rhs(t,z,b,m,g)                    %function to define ODE
x=z(1); y=z(2); v_x=z(3); v_y=z(4);

%the linear momentum balance eqns
xdot=v_x;
v_xdot=-(b/m)*v_x*(v_x^2+v_y^2)^0.5;
ydot=v_y;
v_ydot=-g-(b/m)*v_y*(v_x^2+v_y^2)^0.5;

zdot=[xdot; ydot; v_xdot; v_ydot]; %this is what the function returns (column vector)

end
%-----------------------------------------------------------------------%
function [value, isterminal, dir]= stopevent(t,z,b,m,g,v0,theta)
% terminate the integration at y=0
x=z(1);
y=z(2);
value= y;
isterminal=1;
dir=-1;
end
```

Matlab out put: The maximum distance is 3.3806 m when theta=23 degrees

10.30 (Continued)
The x distance at y=0 for various theta is plotted below



e). Use the code for (a) and change *b* to a very large number, 100000. The trajectory looks like



,

which is approximately a triangle.

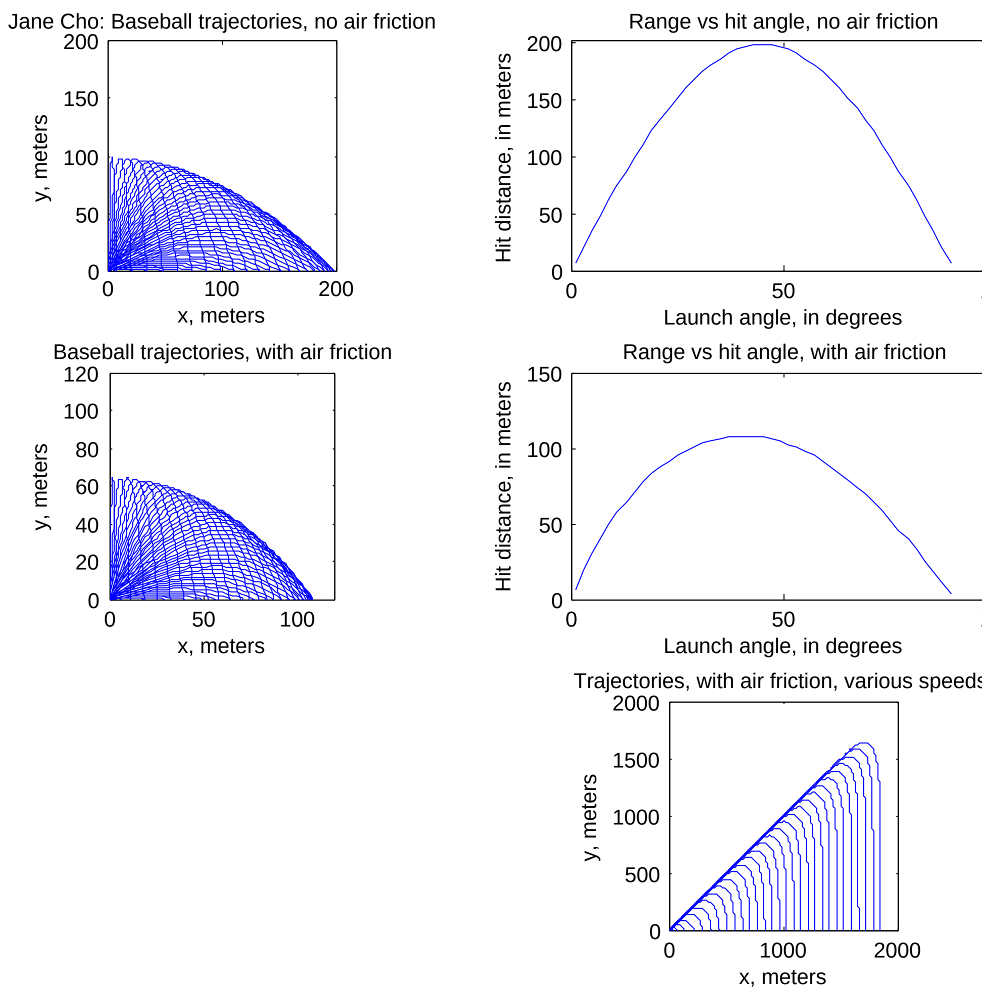
```
10.30 Another solution (more detailed)

The m file attached does the following.

a) uses events and  x(end) to calculate range.
b) has that embedded in a loop so that there is an angle(i) and
   a range(i)
c) Makes a nice plot of range vs angle
d) uses MAX to find the maximum range and corresponding angle
e) has good numerics to show that the trajectory shape converges to
   a triangle as the speed -> infinity.
```

```
function baseball_trajectory
% Calculates the trajectory of a baseball.
% Calculates maximum range for given speed,
% with and without air friction.
% Shows shape of path at high speed.
disp(['Start time:  ' datestr(now)])
cla

% (a) ODEs are in the function rhs far below.
%     The 'event' fn that stops the integration
%     when the ball hits the ground is in 'eventfn'
%     even further below.
% (b) Coefficients for a real baseball taken
% from a google search, which finds a paper
% Sawicki et al, Am. J. Phys. 71(11), Nov 2003.
% Greg Sawicki, by the way, learned some dynamics
% in TAM 203 from Ruina at Cornell.

% All parameters in MKS.
m   = 0.145;    % mass of baseball, 5.1 oz
rho = 1.23;     % density of air in kg/m^3
r   = 0.0366;   % baseball radius (1.44 in)
A   = pi*r^2;   % cross sectional area of ball
C_d = 0.35;     % varies, this is typical
g   = 9.81;     % typical g on earth
b   = C_d*rho*A/2; % net coeff of v^2 in drag force

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% (b-d)  Use typical homerun hit speed and look
% at various  angles of hit.

tspan=linspace(0,100,1001); % give plenty of time
n = 45;  % number of simulations
angle = linspace(1,89,n);  %  launch from 1 to 89 degrees
r0=[0 0]';   % Launch x and y position.

% First case:  No air friction.
b = 0;
subplot(3,2,1)
hold off

% Try lots of launch angles, one simulation for
% each launch angle.
for i = 1:n
inspeed = 44;  % typical homerun hit (m/s), 98 mph.

theta0 = angle(i)*pi/180; % initial angle this simulation
v0=inspeed*[cos(theta0) sin(theta0)]'; %launch velocity
z0=[r0; v0]; % initial position and velocity
```

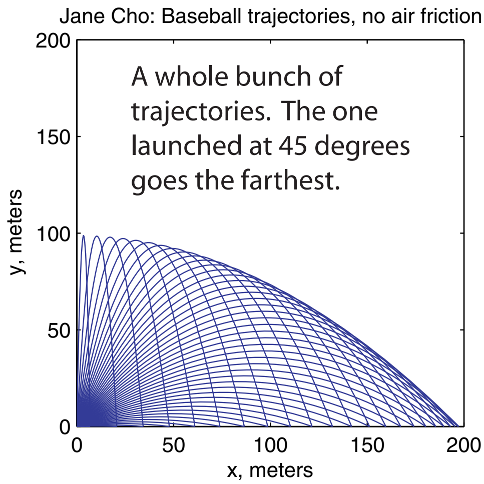
```
options=odeset('events',@eventfn);
[t zarray]=ode45(@rhs,tspan,z0,options,g,b,m); %Solve ODE

x=zarray(:,1); y=zarray(:,2); %Unpack positions
range(i)= x(end); % x value at end,  when ball hits ground

plot(x,y); title('Jane Cho: Baseball trajectories, no air friction')
xlabel('x, meters'); ylabel('y, meters'); axis('equal')
axis([0 200 0 200])
hold on  % save plot for over-writing
end % end of for loop for no-friction trajectories

%Plot range vs angle, no friction case
subplot(3,2,2); hold off;
plot(angle,range);
title('Range vs hit angle, no air friction')
xlabel('Launch angle, in degrees')
ylabel('Hit distance, in meters')

% Pick out best angle and distance
[bestx besti] = max(range);
disp(['No friction case:'])
best_theta_deg = angle(besti)
bestx


% Second case:  WITH air friction
% Identical to code above but now b is NOT zero.
b   = C_d*rho*A/2; % net coeff of v^2 in drag force

subplot(3,2,3)
hold off % clear plot overwrites

% Try lots of launch angles
for i = 1:n  %
inspeed = 44;  % typical homerun hit (m/s), 98 mph.

theta0 = angle(i)*pi/180; % initial angle this simulation
v0=inspeed*[cos(theta0) sin(theta0)]'; %launch velocity
z0=[r0; v0]; % initial position and velocity

options=odeset('events',@eventfn);
[t zarray]=ode45(@rhs,tspan,z0,options,g,b,m); %Solve ODE

x=zarray(:,1); y=zarray(:,2); %Unpack positions
range(i)= x(end); % x value at end,  when ball hits ground

plot(x,y); title('Baseball trajectories, with air friction')
xlabel('x, meters'); ylabel('y, meters'); axis('equal')
axis([0 120 0 120])
```

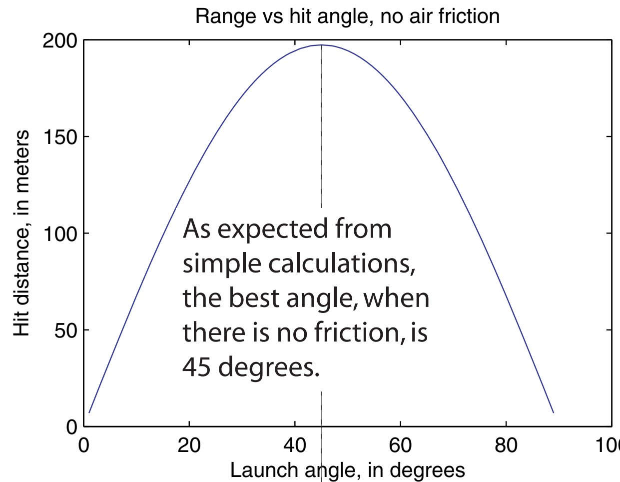
```
hold on  % save plot for over-writing
end % end of for loop for with-friction trajectories

%Plot range vs angle, no friction case
subplot(3,2,4);
plot(angle,range);
title('Range vs hit angle, with air friction')
xlabel('Launch angle, in degrees')
ylabel('Hit distance, in meters')

%Find Max range and corresponding launch angle
[bestx besti] = max(range);
disp(['With Friction:'])
best_theta_deg = angle(besti)
bestx


%%%%%%%%%%%%%%%%%%%%%%%%%%
% Now look at trajectories at a variety of speeds
% Try lots of launch angles
subplot(3,2,6)
hold off
speeds = 10.^linspace(1,8,30); % speeds from 1 to 100 million m/s
for i = 1:30  %
inspeed = speeds(i);  % typical homerun hit (m/s), 98 mph.

theta0 = pi/4; % initial angle is 45 degrees at all speeds
v0=inspeed*[cos(theta0) sin(theta0)]'; %launch velocity
z0=[r0; v0]; % initial position and velocity

options=odeset('events',@eventfn);
[t zarray]=ode45(@rhs,tspan,z0,options,g,b,m); %Solve ODE

x=zarray(:,1); y=zarray(:,2); %Unpack positions
range(i)= x(end); % x value at end,  when ball hits ground

plot(x,y); title('Trajectories, with air friction, various speeds ')
xlabel('x, meters'); ylabel('y, meters'); axis('equal')
axis([0 2000 0 2000])
hold on  % save plot for over-writing
end % end of for loop for range at various speeds

disp(['End time:  ' datestr(now)])
end % end of Baseball_trajectory.m




%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Governing Ord Diff Eqs.
```

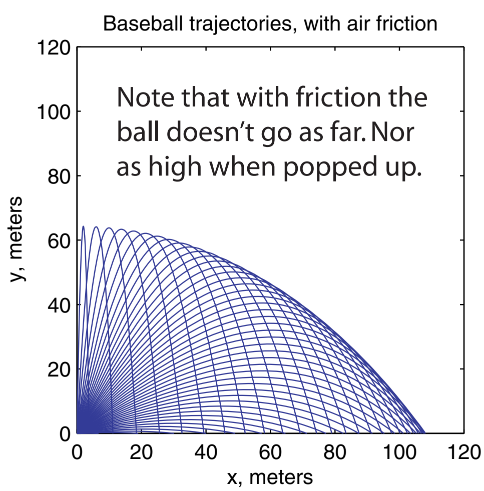
```
function zdot=rhs(t,z,g,b,m)
% Unpack the variables
x=z(1); y=z(2);
vx=z(3); vy=z(4);

%The ODEs
xdot=vx; ydot=vy; v = sqrt(vx^2+vy^2);
vxdot=-b*vx*v/m;
vydot=-b*vy*v/m - g;

zdot= [xdot;ydot;vxdot;vydot]; % Packed up again.
end



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 'Event' that ball hits the ground
function [value isterminal dir] = eventfn(t,z,g,b,m)
y=z(2);
value = y;      % When this is zero, integration stops
isterminal = 1; % 1 means stop.
dir= -1;        % -1 means ball is falling when it hits
end
```

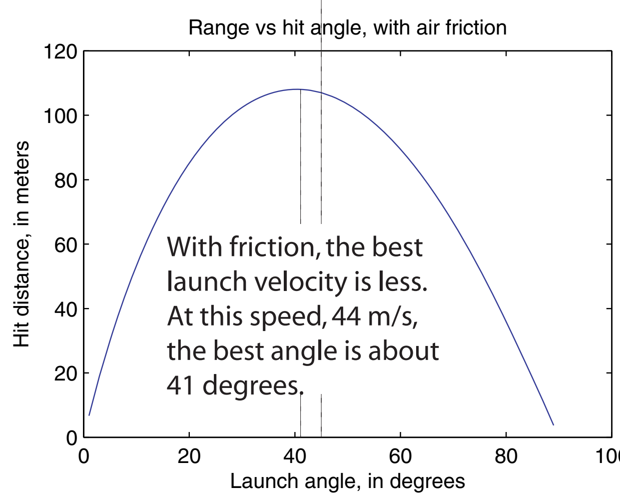

Jane Cho: Baseball trajectories, no air friction
y, meters
x, meters
Range vs hit angle, no air friction
Hit distance, in meters
Launch angle, in degrees
Baseball trajectories, with air friction
y, meters
x, meters
Range vs hit angle, with air friction
Hit distance, in meters
Launch angle, in degrees
Trajectories, with air friction, various speeds
y, meters
x, meters

Baseball. For the first 4 plots realistic ball properties are used and the launch speed is always 44 m/s (typical home run hit). Spin is ignored.



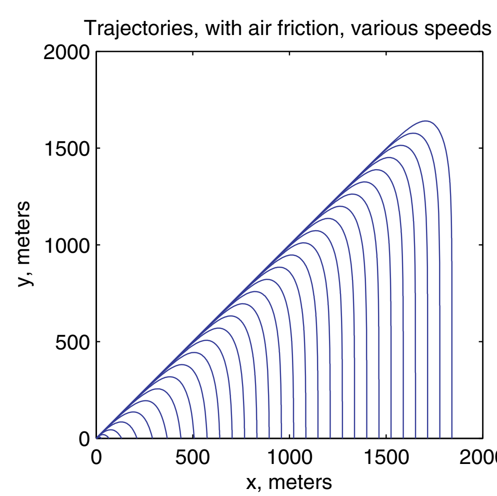






At right are a bunch of trajectories. The slowest launch is 10 m/s, the fastest is 100,000,000 m/s. Such a ball would burn up, tear apart etc... but ignore that.

Note that as the speed gets large the trajectory gets closer and closer to, its a strange and beautiful shape, to a triangle. The same would happen if the speed were fixed and the drag progressively increased.



With no friction the range increases with the square of the speed. With quadratic drag, at high speeds the range goes up with the log of the launch speed. Like the penetration distance of a bullet.

10.2.22 At a time of interest, a particle with mass $m_1 = 5\,\text{kg}$ has position, velocity, and acceleration $\vec{r}_1 = 3\,\text{m}\hat{\imath}$, $\vec{v}_1 = -4\,\text{m/s}\hat{\jmath}$, and $\vec{a}_1 = 6\,\text{m/s}^2\hat{\jmath}$, respectively. Another particle with mass $m_2 = 5\,\text{kg}$ has position, velocity, and acceleration $\vec{r}_2 = -6\,\text{m}\hat{\imath}$, $\vec{v}_2 = 5\,\text{m/s}\hat{\jmath}$, and $\vec{a}_2 = -4\,\text{m/s}^2\hat{\jmath}$, respectively. For this system of two particles, and at this time, find its

a) linear momentum $\vec{L}$,

b) rate of change of linear momentum $\dot{\vec{L}}$

c) angular momentum about the origin $\vec{H}_{/O}$,

d) rate of change of angular momentum about the origin $\dot{\vec{H}}_{/O}$,

e) kinetic energy E_K, and

f) rate of change of kinetic energy $\dot{E}_K$.

10.55

At a particular instant, two particles of interest has the mass, position, velocity and accelerative below

$m_1 = 5\,kg$ $\qquad m_2 = 5\,kg$

$\vec{r}_1 = 3m\hat{\imath}$ $\qquad \vec{r}_2 = -6m\hat{\imath}$

$\vec{v}_1 = -4m/s\,\hat{\jmath}$ $\qquad \vec{v}_2 = 5\,m/s\,\hat{\jmath}$

$\vec{a}_1 = 6\,m/s^2\,\hat{\jmath}$ $\qquad \vec{a}_2 = -4\,m/s^2\,\hat{\jmath}$

a) Find linear momentum $\vec{L}$

$\vec{L} = m\vec{v}$

$\vec{L}_1 = m_1\vec{v}_1 = \boxed{-20\,kgm/s\,\hat{\jmath} = \vec{L}_1}$

$\vec{L}_2 = m_2\vec{v}_2 = \boxed{25\,kg\,m/s\,\hat{\jmath} = \vec{L}_2}$

$\vec{L}_{system} = (-20+25)\,kgm/s\,\hat{\jmath} = \boxed{5\,kgm/s\,\hat{\jmath}}$

b) Find $\dot{\vec{L}}$

$\dot{\vec{L}} = \vec{F} = m\vec{a}$

$\dot{\vec{L}}_1 = m_1\vec{a}_1 = \boxed{30\,N\hat{\jmath} = \dot{\vec{L}}_1}$

$\dot{\vec{L}}_2 = m_2\vec{a}_2 = \boxed{-20N\hat{\jmath} = \dot{\vec{L}}_2}$

$\dot{\vec{L}}_{system} = (30-20)\,N\hat{\jmath} = \boxed{10\,N\hat{\jmath}}$

c) Find $H_{/o}$

$\vec{H}_{/o} = \vec{r}_{/o} \times (m\vec{v})$

$\vec{H}_1 = \vec{r}_1 \times (m_1\vec{v}_1) = \boxed{-60\,kgm^2/s\,\hat{k}}$

$\vec{H}_2 = \vec{r}_2 \times (m_2\vec{v}_2) = \boxed{-150\,kgm^2/s\,\hat{k}}$

$\vec{H}_{system} = (-60-150)\,kgm^2/s\,\hat{k} = \boxed{-210\,kgm^2/s\,\hat{k}}$

10.55 continued

d) Find $\dot{\vec{H}}_{/0}$

$$\dot{\vec{H}} = \vec{r}_{/0} \times (m\vec{a})$$

$$\dot{\vec{H}}_1 = \vec{r}_1 \times (m_1 \vec{a}_1) = \boxed{90 N_m \hat{k}}$$

$$\dot{\vec{H}}_2 = \vec{r}_2 \times (m_2 \vec{a}_2) = \boxed{120 \, Nm \, \hat{k}}$$

$$\dot{\vec{H}}_{system} = (90 + 120) Nm \hat{k} = \boxed{210 Nm \, \hat{k}}$$

e) $E_K = \frac{1}{2} m v^2$

$$E_{K1} = \frac{1}{2} m_1 |\vec{v}_1|^2$$
$$= \frac{1}{2}(5\,kg)(4\,m/s)^2$$
$$\boxed{E_{K1} = 40\,J}$$

$$E_{K2} = \frac{1}{2} m_2 |\vec{v}_2|^2$$
$$= \frac{1}{2}(5\,kg)(5\,m/s)^2$$
$$\boxed{E_{K2} = 62.5\,J}$$

$$\boxed{E_{K\,system} = 102.5\,J}$$

f) Find $\dot{E}_K$

$$\dot{E}_K = \vec{F} \cdot \vec{v}$$

$$\dot{E}_{K1} = (m_1 \vec{a}_1) \cdot \vec{v}_1$$
$$= (30 N \hat{j}) \cdot (-4\,m/s\,\hat{j}) = \boxed{-120\,W}$$

$$\dot{E}_{K2} = (m_2 \vec{a}_2) \cdot \vec{v}_2$$
$$= (-20 N \hat{j}) \cdot (5\,m/s\,\hat{j}) = \boxed{-100\,W}$$

$$\dot{E}_{K\,system} = -220\,W$$

Experts note that these problems do not use polar coordinates or any other fancy coordinate systems. Such descriptions come later in the text. At this point we want to lay out the basic equations and the qualitative features that can be found by numerical integration of the equations using Cartesian (xyz) coordinates.

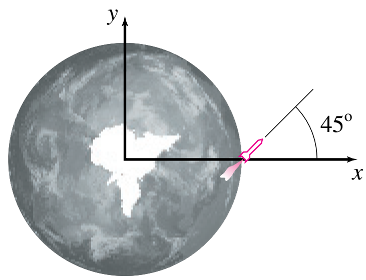
10.3.5 An intercontinental misile, modelled as a particle, is launched on a ballistic trajectory from the surface of the earth. The force on the missile from the earth's gravity is $F = mgR^2/r^2$ and is directed towards the center of the earth. When it is launched from the equator it has speed v_0 and in the direction shown, 45° from horizontal (both measured relative to a Newtonian reference frame). For the purposes of this calculation ignore the earth's rotation. You can think of this problem as two-dimensional in the plane shown. If you need numbers, use the following values:

$m = 1000\,\text{kg}$ = missile mass
$g = 10\,\text{m/s}^2$ at the earth's surface,
$R = 6,400,000\,\text{m}$ = earth's radius, and
$v_0 = 9000\,\text{m/s}$.

The distance of the missile from the center of the earth is $r(t)$.

a) Draw a free body diagram of the missile. Write the linear momentum balance equation. Break this equation into x and y components. Rewrite these equations as a system of-4 first order ODE's suitable for computer solution. Write appropriate initial conditions for the ODE's.

b) Using the computer (or any other means) plot the trajectory of the rocket after it is launched for a time of 6670 seconds. [Hint: use a much shorter time when debugging your program.] On the same plot draw a (round) circle for the earth.



Filename:pfigure-s94q12p1

Problem 10.5: In intercontinental ballistic missile launch.

TAM 203
Homework Solutions

Due 3/27/08

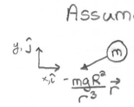
10.61

Given: $F = mgR^2/r^2$, V_0, 45° from horizontal (θ)

$m = 1000\,\text{kg}$, $R = 6.4\times10^6\,\text{m}$, $V_0 = 9000\,\text{m/s}$

a) Free body diagram:

Assume $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$



$\sum \vec{F} = m\vec{a}$, $\quad -\frac{mgR^2}{r^3}\vec{r} = m\vec{a}$

$$\frac{-\cancel{m}gR^2}{(x^2+y^2)^{3/2}}(x\hat{\imath}+y\hat{\jmath}) = \cancel{m}\ddot{x}\hat{\imath} + \cancel{m}\ddot{y}\hat{\jmath}$$

In component form,

$$\ddot{x} + \frac{gR^2}{(x^2+y^2)^{3/2}}x = 0 \quad \text{AND} \quad \ddot{y} + \frac{gR^2}{(x^2+y^2)^{3/2}}y = 0$$

Differential equations:

(1) $\dot{x} = V_x$ $\qquad$ (3) $\dot{y} = V_y$

(2) $\dot{V}_x = \frac{-gR^2}{(x^2+y^2)^{3/2}}x$ $\qquad$ (4) $\dot{V}_y = \frac{-gR^2}{(x^2+y^2)^{3/2}}y$

Initial conditions:

(1) $x(0) = R$ $\qquad$ (3) $y(0) = 0$

(2) $V_x(0) = V_0\cos\theta$ $\qquad$ (4) $V_y(0) = V_0\sin\theta$

b) See Matlab code and plot on pages 2-3.

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10.61b – Matlab code

```
function Prob1061()
% Problem 10.61 Solution
% March 27, 2008

% VARIABLES (Assume consistent units)
% r = displacement vector [x,y]
% v = velocity vector = dr/dt [vx,vy]

m= 1000;      % Mass of satellite (kg)
R= 6400000;   % Radius of Earth (m)
g= 9.81;      % Gravity acceleration (m/s^2)
v0= 9000;     % Initial velocity (m/s)
theta= 45;    % Launch angle (degrees)

% INITIAL CONDITIONS
x0= R;
y0= 0;
vx0= v0*cosd(theta);
vy0= v0*sind(theta);
z0= [x0 y0 vx0 vy0]'; % pack variables

tspan= [0 6670]; % seconds

[t zarray]= ode45(@rhs,tspan,z0,[],m,R,g);

% Unpack Variables
x= zarray(:,1);
y= zarray(:,2);

plot(x,y,'r--');
  title('Plot of Earth and Satellite Orbit')
  xlabel('x [m]')
  ylabel('y [m]')
  axis(1000000*[-8 15 -8 15])
hold on;

% Draw the Earth
t= 0:pi/100:2*pi;
ex= R*cos(t);
ey= R*sin(t);
plot(ex,ey,'b');

end

% THE DIFFERENTIAL EQUATION 'The Right Hand Side'
function zdot = rhs(t,z,m,R,g)

% Unpack variables
x= z(1);
y= z(2);
vx= z(3);
vy= z(4);
```

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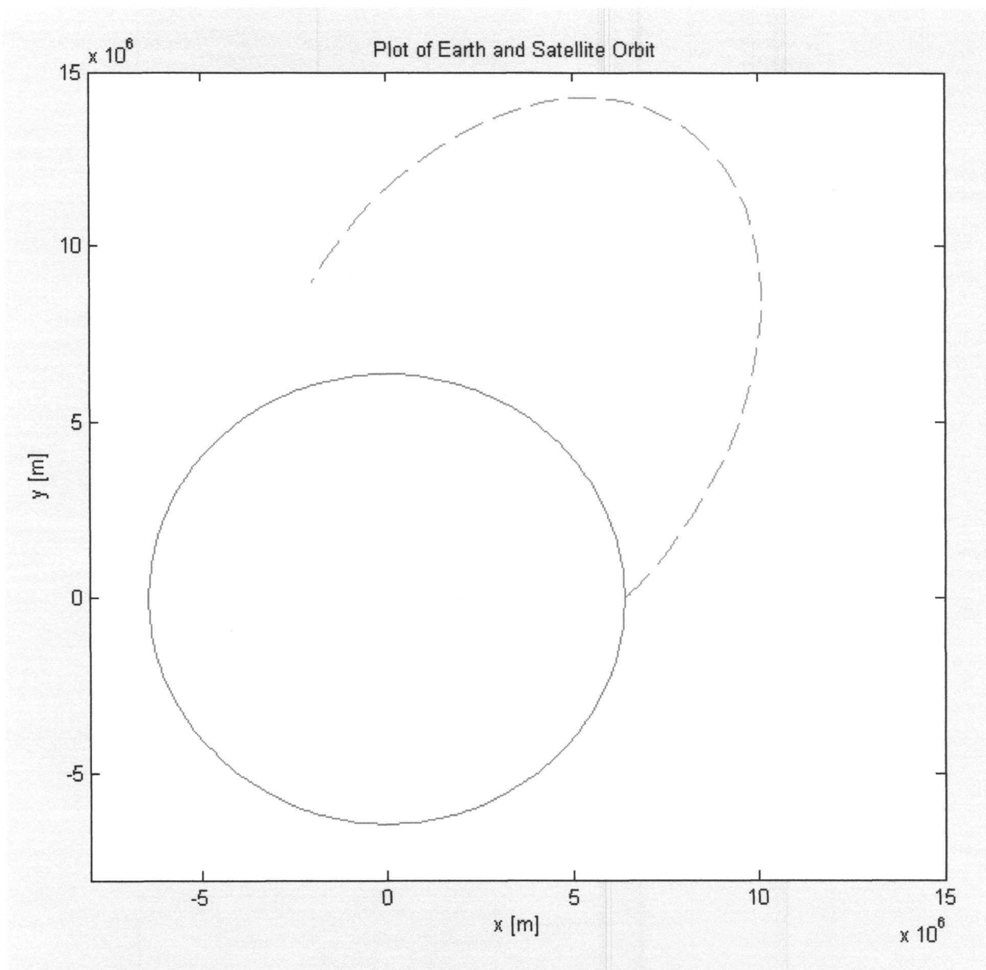
```
% The equations
xdot= vx;
vxdot= -g*R^2/(x^2+y^2)^(3/2)*x;
ydot= vy;
vydot= -g*R^2/(x^2+y^2)^(3/2)*y;

% Pack the rate of change of x,y,vx and vy
zdot= [xdot ydot vxdot vydot]';

end
```

10.61b – Satellite Orbit Plot



11.1.10 Montgomery's eight. Three equal masses, say $m = 1$, are attracted by an inverse-square gravity law with $G = 1$. That is, each mass is attracted to the other by $F = Gm_1m_2/r^2$ where r is the distance between them. Use these unusual and special initial positions:

$$\begin{aligned}(x1, y1) &= (-0.97000436, 0.24308753)\\ (x2, y2) &= (-x1, -y1)\\ (x3, y3) &= (0, 0)\end{aligned}$$

and initial velocities

$$\begin{aligned}(vx3, vy3) &= (0.93240737, 0.86473146)\\ (vx1, vy1) &= -(vx3, vy3)/2\\ (vx2, vy2) &= -(vx3, vy3)/2.\end{aligned}$$

For each of the problems below show accurate computer plots and explain any curiosities.

a) Use computer integration to find and plot the motions of the particles. Plot each with a different color. Run the program for 2.1 time units.

b) Same as above, but run for 10 time units.

c) Same as above, but change the initial conditions slightly.

d) Same as above, but change the initial conditions more and run for a much longer time.

11.10

a) See attached Matlab code and plots for (a) - (d), recognizing that:

$$m\ddot{a}_1 = \frac{Gm^2}{|\vec r_2-\vec r_1|^3}(\vec r_2-\vec r_1) + \frac{Gm^2}{|\vec r_3-\vec r_1|^3}(\vec r_3-\vec r_1)$$

$$\therefore\quad \ddot{\vec a}_1 = Gm\left(\frac{\vec r_2-\vec r_1}{|\vec r_2-\vec r_1|^3} + \frac{\vec r_3-\vec r_1}{|\vec r_3-\vec r_1|^3}\right)$$

We can turn this, and perform similar linear momentum balance on (2) and (3), into six first-order vector differential equations:

$$\begin{aligned}\dot{\vec r}_1 &= \vec v_1 & \dot{\vec v}_1 &= Gm\left(\frac{\vec r_2-\vec r_1}{|\vec r_2-\vec r_1|^3} + \frac{\vec r_3-\vec r_1}{|\vec r_3-\vec r_1|^3}\right)\\ \dot{\vec r}_2 &= \vec v_2 & \dot{\vec v}_2 &= Gm\left(\frac{\vec r_1-\vec r_2}{|\vec r_1-\vec r_2|^3} + \frac{\vec r_3-\vec r_2}{|\vec r_3-\vec r_2|^3}\right)\\ \dot{\vec r}_3 &= \vec v_3 & \dot{\vec v}_3 &= Gm\left(\frac{\vec r_1-\vec r_3}{|\vec r_1-\vec r_3|^3} + \frac{\vec r_2-\vec r_3}{|\vec r_2-\vec r_3|^3}\right)\end{aligned}$$

From plots on subsequent pages, we can see that these initial conditions provide for a very specific displacement function for each mass. If these conditions are modified slightly, as in (c) and (d), the displacement plot is very different.

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```
function Prob1110()
% Problem 11.10 Solution
% April 1, 2008

% VARIABLES
G= 1;
m= 1;

% Initial Conditions
r01= [-0.97000436 0.24308753]'; r02= -r01; r03= [0 0]';
v03= [0.93240737 0.86473146]'; v01= -1/2*v03; v02= -1/2*v03;

z0= [r01; r02; r03; v01; v02; v03]; % pack variables

tspan= [0 10];

[t zarray]= ode45(@rhs,tspan,z0,[],G,m);

% Unpack variables
r1= zarray(:,1:2);
r2= zarray(:,3:4);
r3= zarray(:,5:6);

plot(r1(:,1), r1(:,2), 'r');
hold on;
plot(r2(:,1), r2(:,2),'b--');
plot(r3(:,1), r3(:,2),'g-.');

end

% THE DIFFERENTIAL EQUATIONS (RIGHT HAND SIDE)
function zdot = rhs(t,z,G,m)

% Unpack variables
r1= z(1:2);
r2= z(3:4);
r3= z(5:6);
v1= z(7:8);
v2= z(9:10);
v3= z(11:12);

% The equations
r1dot= v1; r2dot= v2; r3dot= v3;
v1dot= G*m*((r3-r1)/(sqrt(sum((r3-r1).^2)))^3+...
    (r2-r1)/(sqrt(sum((r2-r1).^2)))^3);
v2dot= G*m*((r1-r2)/(sqrt(sum((r1-r2).^2)))^3+...
    (r3-r2)/(sqrt(sum((r3-r2).^2)))^3);
v3dot= G*m*((r1-r3)/(sqrt(sum((r1-r3).^2)))^3+...
    (r2-r3)/(sqrt(sum((r2-r3).^2)))^3);

% Pack the rate of change variables
zdot= [r1dot; r2dot; r3dot; v1dot; v2dot; v3dot];

end
```

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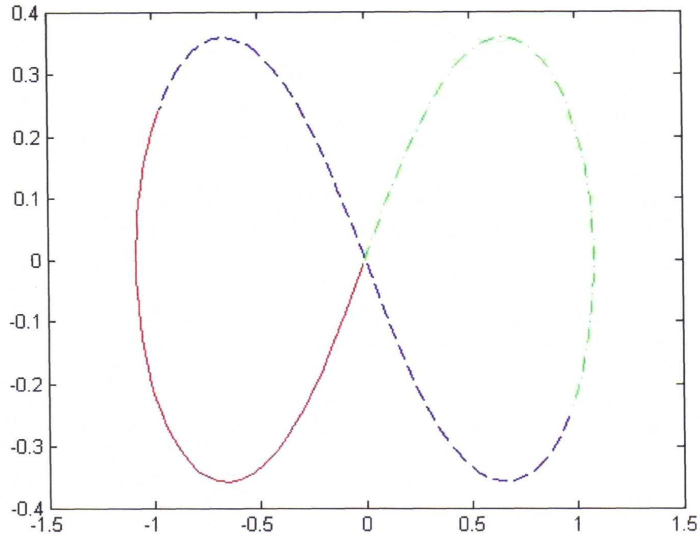
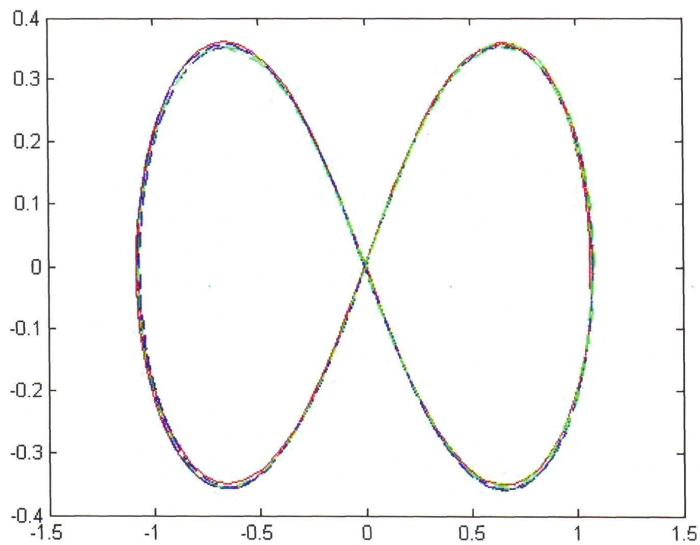


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-1.5
-1
-0.5
0
0.5
1
1.5

(a)



0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-1.5
-1
-0.5
0
0.5
1
1.5

(b)

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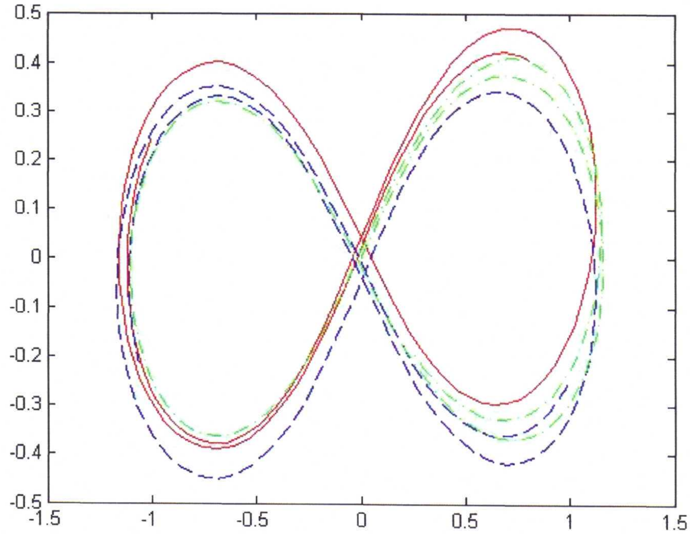
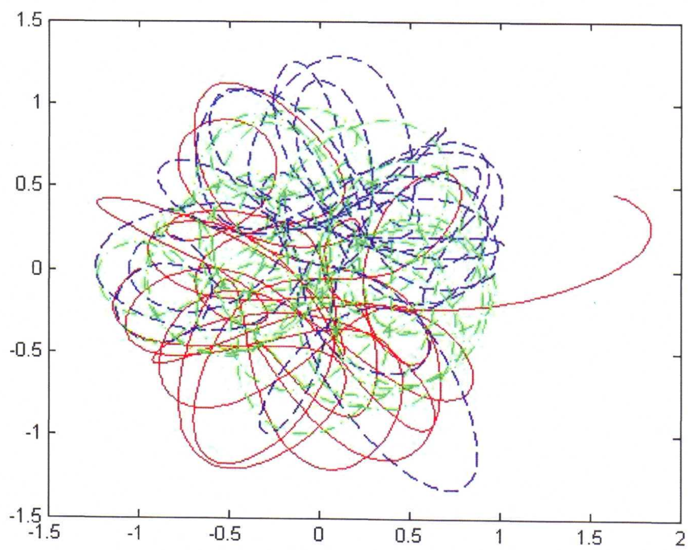
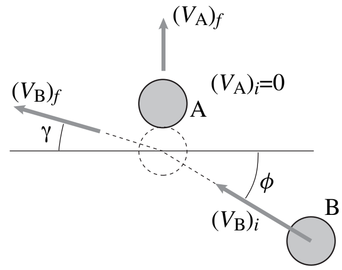

0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-1.5
-1
-0.5
0
0.5
1
1.5

(c)


1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2

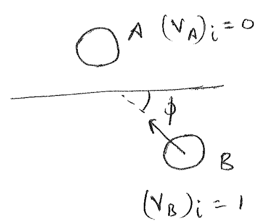
(d)

11.2.7 Two frictionless equal-mass pucks sliding on a plane collide as shown below. Puck A is initially at rest. Given that $(V_B)_i = 1.0\,\text{m/s}$, $(V_A)_i = 0$, and $(V_A)_f = 0.5\,\text{m/s}$, find the approach angle ϕ and rebound angle γ. The coefficient of restitution is $e = 0.9$.

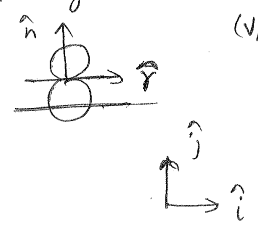


Problem 11.7

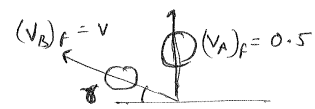
Before Collision



During collision



After Collision



Given: $(V_A)_i = 0$; $(V_B)_i = 1$; $(V_A)_f = 0.5$; $e = 0.9$

Find ϕ, γ

Let $(V_B)_f = v$

$$(\vec{V}_A)_i = 0$$

$$(\vec{V}_B)_i = -(V_B)_i \cos\phi\,\hat{i} + (V_B)_i \sin\phi\,\hat{j}$$

$$= -\cos\phi\,\hat{i} + \sin\phi\,\hat{j}$$

$$(\vec{V}_A)_f = 0.5\,\hat{j}$$

$$(\vec{V}_B)_f = -(V_B)_f \cos\gamma\,\hat{i} + (V_B)_f \sin\gamma\,\hat{j}$$

$$= -v\cos\gamma\,\hat{i} + v\sin\gamma\,\hat{j}$$

Conservation of linear momentum gives

$$m_A(\vec{V}_A)_i + m_B(\vec{V}_B)_i = m_A(\vec{V}_A)_f + m_B(\vec{V}_B)_f$$

Assuming masses are equal $m_A = m_B = m$

$$\Rightarrow m(0) + m(-\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) = m\,0.5\,\hat{j} + m(-v\cos\gamma\,\hat{i} + v\sin\gamma\,\hat{j})$$

$$\Rightarrow -\cos\phi\,\hat{i} + \sin\phi\,\hat{j} = (0.5 + v\sin\gamma)\,\hat{j} - v\cos\gamma\,\hat{i}$$

$$\{\}\cdot\hat{i} \qquad -\cos\phi = -v\cos\gamma \qquad \text{— (I)}$$

$$\{\}\cdot\hat{j} \qquad \sin\phi = 0.5 + v\sin\gamma \qquad \text{— (II)}$$

We have 2 equation (I), (II) & 3 unknowns, namely, v, ϕ, γ.

Let's use the coefficient of restitution to generate the third equation

$$-e(\vec{v}_A - \vec{v}_B)_i\cdot\hat{n} = (\vec{v}_A - \vec{v}_B)_f\cdot\hat{n}$$

where $\hat{n} = \hat{j}$ {see figure: during collision}

$$\Rightarrow -0.9\,(0 - (-\cos\phi\,\hat{i} + \sin\phi\,\hat{j}))\cdot\hat{j} = (0.5\,\hat{j} - (-v\cos\gamma\,\hat{i} + v\sin\gamma\,\hat{j}))\cdot\hat{j}$$

$$\Rightarrow 0.9\sin\phi = 0.5 - v\sin\gamma \qquad \text{— (III)}$$

~~Subtracting~~ Adding (II) and (III)

$$1.9\sin\phi = 1$$

$\therefore \sin\phi = \frac{1}{1.9} = 0.53 \Rightarrow \phi = 32^\circ$

Putting $\phi = 32^\circ$ in (I) & (II) gives

$$v\cos\gamma = 0.85 \quad \text{— (IV)}$$

$$v\sin\gamma = 0.03 \quad \text{— (V)}$$

Dividing (V)/(IV) $\tan\gamma = \frac{0.03}{0.85} \Rightarrow \gamma = 2.02^\circ$ or $\gamma = 182.02^\circ$

square and add (V), (IV)

$$v^2 = 0.7235 \Rightarrow v = \pm 0.85$$

Again from IV, V we observe

$v = 0.85$ $\gamma = 2.02$ is one pair

$v = -0.85$ $\gamma = 182.02$ is the other

But really both solutions above are equivalent.

Thus final answer

$$\boxed{\begin{aligned}\gamma &= 2.02^\circ = 0.036 \text{ rad} \\ \phi &= 32^\circ = 0.56 \text{ rad}\end{aligned}}$$

11.2.10 Solve the general two-particle frictionless collision problem. For example, write computer code that has lines like this near the start :

`m1=3; m2=19`	Set values of masses
`v1zero=[10 20]`	Initial velocity of mass 1
`v2zero=[-5 3]`	Initial velocity of mass 2
`e=.5`	Set coefficient of restitution
`theta=pi/4`	Angle that the normal to contact plane makes, measured CCW from +x axis, in radians

Your program (function, code, script) should calculate the impulse of mass 1 on mass 2, and the velocities of the two masses after the collision. Your program should assume consistent units for all quantities.

a) You should demonstrate that your program works by solving at least 4 different problems for which you can check your answer by simple pencil-and-paper calculations. These problems should have as much variety as possible. Sketch these problems clearly, show their analytic solution, and show that the computer agrees.

b) Solve the problem given in the sample text given in the initial problem statement.

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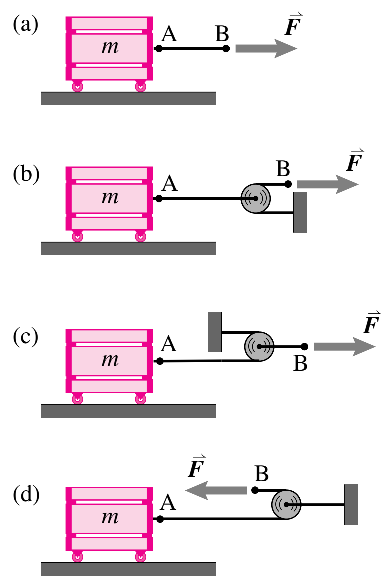
```
% Two-Particle Collisions
% Problem 11.20 Solution
% April 1, 2008

theta = 45;          % angle (degrees) between n and plus x axis
nx = cosd(theta);
ny = sind(theta);
n = [nx ny]';        % impulse direction
v1bef = [10 20]';    % vel of m1 before collision
v2bef = [-5  3]';    % vel of m2 before collision
m1 = 3;  m2 = 19;    % values of two masses
e = .5;              % coefficient of restitution

% Write governing equations in form of Az=b
% where z is a list of unknowns representing
% the particle velocities after the collision
% and the magnitude of the impulse.

A = [ m1    0   m2   0     0        % x comp of lin mom bal
       0   m1    0   m2    0        % y comp of lin mom bal
     -nx  -ny   nx   ny    0        % restitution equation
       0    0   m2   0   -nx        % impulse-momentum for m2, x comp
       0    0    0   m2  -ny];      % impulse-momentum for m2, y comp

b = [m1*v1bef + m2*v2bef;           % x & y comps of lin mom bal for syst
     -e*sum((v2bef-v1bef).*n);      % restitution equation
     m2*v2bef];                     % impulse-momentum for m2, x & y comps

z= A\b;

% Type out the solution (crudely).
disp('   v1xaft     v1yaft     v2xaft     v2yaft      P');
disp(z');

%   ANSWER:
%    v1xaft     v1yaft     v2xaft     v2yaft      P
%  -10.7273   -0.7273   -1.7273     6.2727   87.9384
```

A ball m is thrown horizontally at height h and speed v_0. It then has a sequence of bounces on the horizontal ground. Treating each collision as frictionless with restitution coefficient e how far has the ball travelled horizontally when it just finishes bouncing? Answer in terms of some or all of m, g, h, v_0 and e. A ball m is thrown horizontally at height h and speed v_0. It then has a sequence of bounces on the horizontal ground. Treating each collision as frictionless with restitution coefficient e how far has the ball travelled horizontally when it just finishes bouncing? Answer in terms of some or all of m, g, h, v_0 and e.

For all problems, unless stated otherwise, treat all strings as inextensible, flexible and massless. Treat all pulleys and wheels as round, frictionless and massless. Assume all massive objects are prevented from rotating (e.g., wheels stay on the ground, *etc.*). When numbers are called for use $g = 10\,\mathrm{m/s^2}$ or $g = 32\,\mathrm{ft/s^2}$.

12.1.6 For the various situations pictured, find the acceleration of mass A and point B. Clearly define any variables, coordinates or sign conventions that you use.



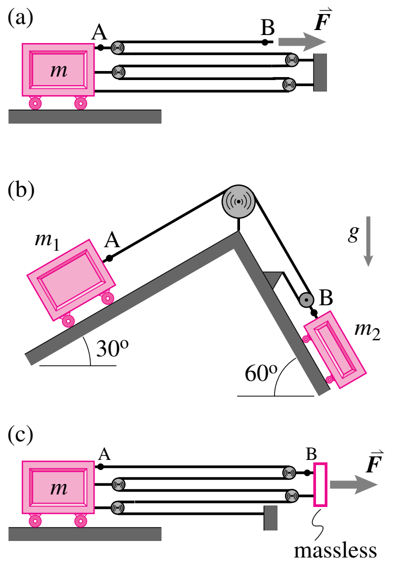
Problem 12.6: Four different ways to pull a mass.

12.1.14 For the situations pictured, find the accelerations of mass A and of point B. Clearly define any variables, coordinates or sign conventions that you use.

a) A single mass and four pulleys.

b) Two masses and two pulleys.

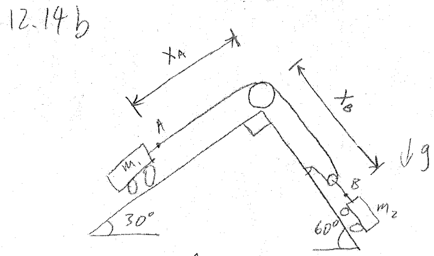
c) A single mass and four pulleys.

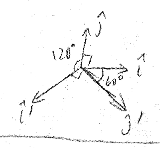


Filename:pulley4

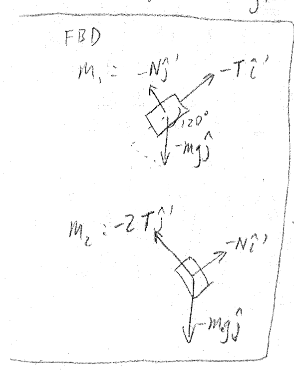
Problem 12.14: Various pulley arrangements.

12.14 b





T is the tension force in the cable.



$m_1 \ddot{x}_A = -m_1 g \hat{j} - T\hat{\imath}' - N\hat{j}'$

$\{\}\cdot\hat{\imath}'$:

$m_1 \ddot{x}_A = -m_1 g(\hat{j}\cdot\hat{\imath}') - T$

$\hat{j}\cdot\hat{\imath}' = \cos(120) = -\sin(30)$

$\boxed{m_1 \ddot{x}_A = m_1 g \sin 30 - T}$ (1)

$m_2 \ddot{x}_B = -2T\hat{j}' - N\hat{\imath}' - m_2 g\hat{j}$

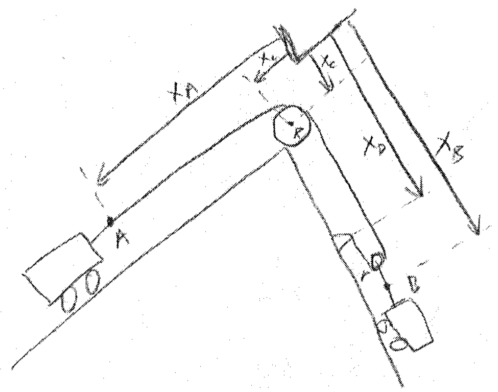
$\{\}\cdot\hat{j}'$:

$m_2 \ddot{x}_B = -2T - m_2 g(\hat{j}\cdot\hat{j}')$

$\hat{j}\cdot\hat{j}' = \cos(150) = -\sin 60$

$\boxed{m_2 \ddot{x}_B = -2T + m_2 g \sin 60}$ (2)

12.14 b continued



$$\ell_{tot} = (x_A - x_c) + \frac{\pi R}{2} + (x_B - x_c) + (x_B - x_D) + \pi r$$

$$\ddot{\{\}}: \quad \ddot{x}_A + 2\ddot{x}_B = 0$$

$$\boxed{\ddot{x}_A = -2\ddot{x}_B} \quad (3)$$

We have 3 unknowns : $\ddot{x}_A$, $\ddot{x}_B$, T

(1) $\Rightarrow$ $T = m_1 g \sin 30 - m_1 \ddot{x}_A$

substitute into (2)

$$m_2 \ddot{x}_B = -2(m_1 g \sin 30 - m_1 \ddot{x}_A) + m_2 g \sin 60$$

(3) $\Rightarrow$ $\ddot{x}_A = -2\ddot{x}_B$ substitute into above

$$m_2 \ddot{x}_B = -2m_1 g \sin 30 - 4m_1 \ddot{x}_B + m_2 g \sin 60$$

$$(m_2 + 4m_1)\ddot{x}_B = -2m_1 g \sin 30 + m_2 g \sin 60$$

$$\boxed{\begin{aligned} \ddot{x}_B &= \frac{-2m_1 g \sin 30^\circ + m_2 g \sin 60^\circ}{m_2 + 4m_1} \\ &= \frac{-2m_1 + \sqrt{3}\, m_2}{2m_2 + 8m_1}\, g \\ \ddot{x}_A &= -2\ddot{x}_B = \frac{2m_1 - \sqrt{3}\, m_2}{m_2 + 4m_1}\, g \end{aligned}}$$

$\therefore$ Acceleration of point A is

$$\vec{a}_A = \ddot{x}_A \hat{\imath}' = \frac{2m_1 - \sqrt{3}\, m_2}{m_2 + 4m_1}\, g\, \hat{\imath}'$$

$$= \frac{2m_1 - \sqrt{3}\, m_2}{m_2 + 4m_1}\, g \left(-\frac{\sqrt{3}}{2}\hat{\imath} - \frac{1}{2}\hat{\jmath}\right)$$

Acceleration of point B is

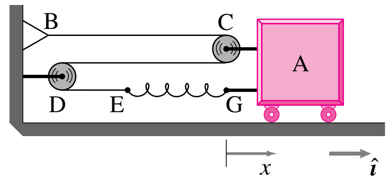
$$\vec{a}_B = \ddot{x}_B \hat{\jmath}' = \frac{\sqrt{3}\, m_2 - 2m_1}{2m_2 + 8m_1}\, g\, \hat{\jmath}'$$

$$= \frac{\sqrt{3}\, m_2 - 2m_1}{2m_2 + 8m_1}\, g \left(\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right)$$

12.1.26 Block A, with mass m_A, is pulled to the right a distance d from the position it would have if the spring were relaxed. It is then released from rest. Assume ideal string, pulleys and wheels. The spring has constant k.

a) What is the acceleration of block A just after it is released (in terms of k, m_A, and d)?

b) What is the speed of the mass when the mass passes through the position where the spring is relaxed? .



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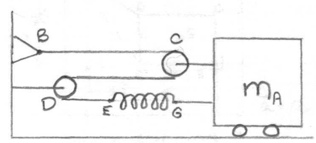
Problem 12.26

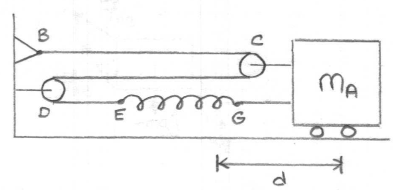
TAM 203 Homework Solution | Due 4/8/08

12.26

ORIGINAL:



PULLED:

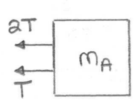


a) Find the stretching of the spring:

We know BC has increased by d, as well as CD and DG. Since the string is inextensible, the spring must have stretched by 3d.

$\therefore F_{spring} = 3kd$, which must equal T_{cable}

FBD:



$\Sigma F = ma$; so $-3T = m_A a$ $\quad T = 3kd$ $\quad \therefore \boxed{a = \frac{-9kd}{m_A}}$

b) We know $\ddot{x} = -\frac{9kx}{m_A}$, so $x(t) = c_1 \cos\left(3\sqrt{\frac{k}{m_A}}\,t\right) + c_2 \sin\left(3\sqrt{\frac{k}{m_A}}\,t\right)$

$\dot{x}(0) = 0 = -c_1\left(3\sqrt{\frac{k}{m_A}}\right)\sin(0) + c_2\left(3\sqrt{\frac{k}{m_A}}\right)\cos(0) \quad \therefore c_2 = 0$

$x(0) = d = c_1\cos(0) \quad \therefore c_1 = d$

$x(t) = d\cos\left(3\sqrt{\frac{k}{m_A}}\,t\right)$ AND $\dot{x}(t) = -3d\sqrt{\frac{k}{m_A}}\sin\left(3\sqrt{\frac{k}{m_A}}\,t\right)$

$x(t) = 0 = d\cos\left(3\sqrt{\frac{k}{m_A}}\,t\right)$

$\therefore 3t\sqrt{\frac{k}{m_A}} = \frac{\pi}{2}$

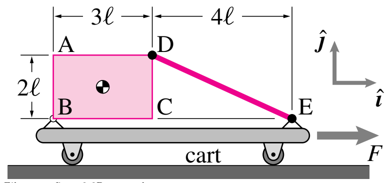
OR $t = \frac{\pi}{6}\sqrt{\frac{m_A}{k}}$

$\dot{x}\left(\frac{\pi}{6}\sqrt{\frac{m_A}{k}}\right) = -3d\sqrt{\frac{k}{m_A}}\sin\left(\frac{\pi}{2}\right)$

$\therefore$ when spring is relaxed,

$$\boxed{\dot{x} = -3d\sqrt{k/m_A}}$$

12.2.11 Guyed plate on a cart A uniform rectangular plate $ABCD$ of mass m is supported by a rod DE and a hinge joint at point B. The dimensions are as shown. There is gravity. What must the acceleration of the cart be in order for massless rod DE to be in tension?



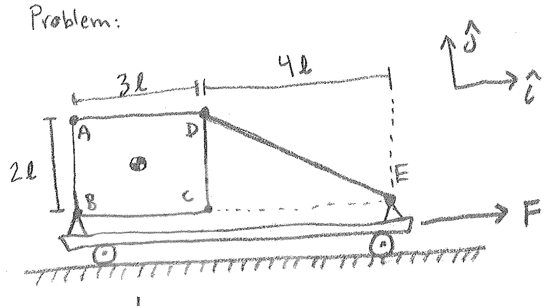
Problem 12.11: Uniform plate supported by a hinge and a cable on an accelerating cart.

10:10 AM Section 205
TA: Pranav Bhounsule

Alan Argondizza
TAM 2030
HW 10, Due February 24, 2009

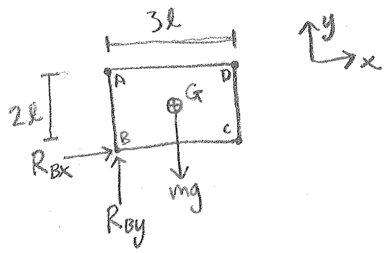
$\boxed{12.40}$ SOLUTION

Problem:



What must the acceleration of the cart be for massless rod DE to be in tension?

Free Body Diagram:



* Consider that at some threshold acceleration (as a increases), the rod DE will go from compression to zero-load to tension. Solve the problem where DE carries no load to find the minimum acceleration, past which DE will be in tension. This approach is reflected in the FBD by the absence of forces at point D, and the assumption that the mass is not rotating about the z axis.

LMB:

$\Sigma F_x = ma = R_{Bx}$

$\Sigma F_y = 0 = R_{By} - mg$

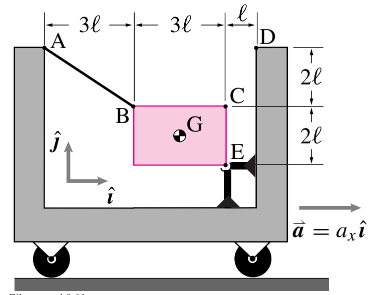
$\Sigma M_G = 0 = \ell \cdot R_{Bx} - 1.5\ell \cdot R_{By}$
(no rotation)

$\rightarrow R_{Bx} = \frac{3}{2} R_{By}$

$ma = \frac{3}{2} mg$

$\rightarrow a = \frac{3}{2} g \rightarrow$ so a must be greater than $\frac{3}{2}g$ for DE to be in tension.

12.2.14 A uniform rectangular plate of mass m is supported by an inextensible cable AB and a hinge joint at point E on the cart as shown. The hinge joint is attached to a rigid column welded to the floor of the cart. The cart has acceleration $a_x\hat{\imath}$. There is gravity. Find the tension in cable AB. (What's 'wrong' with this problem? What if instead point B were at the bottom left hand corner of the plate?)



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Problem 12.14

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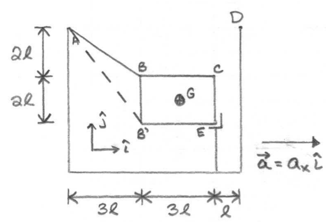
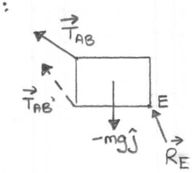
12.43



FBD:



$$\sum \vec{M}_E = \dot{\vec{H}}_E = \vec{r}_{GE} \times (ma_x\hat{\imath})$$

$$\Rightarrow \vec{r}_{GE} \times (-mg\hat{\jmath}) = \vec{r}_{GE} \times (ma_x\hat{\imath}) \qquad (1)$$

The problem with this problem is that the tension acts in a direction through the support E. We cannot determine $\vec{T}_{AB}$ just by summing moments, and unidirectional motion only exists for a particular a_x:

$$\vec{r}_{GE} = -1.5\ell\hat{\imath} + \ell\hat{\jmath}$$

$$\therefore \vec{r}_{GE} \times (-mg\hat{\jmath}) = 1.5mg\ell\hat{k}$$

$$\vec{r}_{GE} \times (ma_x\hat{\imath}) = -ma_x\ell\hat{k}$$

From equation (1), $a_x = -1.5g$ for unidirectional motion. If the cable went from A to B' (dashed above), this problem is avoided:

$$\sum \vec{M}_E = \vec{r}_{B'E} \times (T_{AB'}\hat{\lambda}_{AB'}) + \vec{r}_{GE} \times (-mg\hat{\jmath}) = \vec{r}_{GE} \times (ma_x\hat{\imath})$$

$$\Rightarrow \hat{\lambda}_{AB'} = \frac{3\ell\hat{\imath} - 4\ell\hat{\jmath}}{\sqrt{9\ell^2 + 16\ell^2}} = \frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath} \quad \text{AND} \quad \vec{r}_{B'E} = 3\ell\hat{\imath}$$

$$\therefore \vec{r}_{B'E} \times (T_{AB'}\hat{\lambda}_{AB'}) = T_{AB'}\left(-\frac{12}{5}\ell\hat{k}\right)$$

$$\text{So, } \left\{-\frac{12}{5}\ell T_{AB'}\hat{k} + 1.5mg\ell\hat{k} = -ma_x\ell\hat{k}\right\} \cdot \hat{k}$$

$$\boxed{T_{AB'} = \frac{5m}{12}\left(a_x + \frac{3}{2}g\right)}$$

12.2.25 Car braking: front brakes versus rear brakes versus all four brakes. What is the peak deceleration of a car when you apply: the front brakes till they skid, the rear brakes till they skid, and all four brakes till they skid? Assume that the coefficient of friction between rubber and road is $\mu = 1$ (about right, the coefficient of friction between rubber and road varies between about .7 and 1.3) and that $g = 10\,\mathrm{m/s^2}$ (2% error). Pick the dimensions and mass of the car, but assume the center of mass height h is greater than zero but is less than half the wheel base w, the distance between the front and rear wheel. Also assume that the CM is halfway between the front and back wheels (*i.e.,* $l_f = l_r = w/2$). The car has a stiff suspension so the car does not move up or down or tip appreciably during braking. Neglect the mass of the rotating wheels in the linear and angular momentum balance equations. Treat this problem as two-dimensional problem; i.e., the car is symmetric left to right, does not turn left or right, and that the left and right wheels carry the same loads. To organize your work, here are some steps to follow.

a) Draw a FBD of the car assuming rear wheel is skidding. The FBD should show the dimensions, the gravity force, what you know *a priori* about the forces on the wheels from the ground (i.e., that the friction force $F_r = \mu N_r$, and that there is no friction at the front wheels), and the coordinate directions. Label points of interest that you will use in your momentum balance equations. (Hint: also draw a free body diagram of the rear wheel.)

b) Write the equation of linear momentum balance.

c) Write the equation of angular momentum balance relative to a point of your choosing. Some particularly useful points to use are:

   - the point above the front wheel and at the height of the center of mass;
   - the point at the height of the center of mass, behind the rear wheel that makes a 45 degree angle line down to the rear wheel ground contact point; and
   - the point on the ground straight under the front wheel that is as far below ground as the wheel base is long.

d) Solve the momentum balance equations for the wheel contact forces and the deceleration of the car. If you have used any or all of the recommendations from part (c) you will have the pleasure of only solving one equation in one unknown at a time.

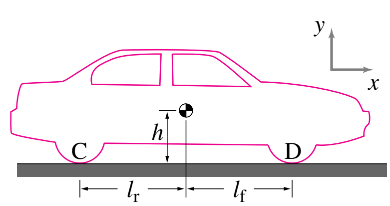
e) Repeat steps (a) to (d) for front-wheel skidding. Note that the advantageous points to use for angular momentum balance are now different. Does a car stop faster or slower or the same by skidding the front instead of the rear wheels? Would your solution to (e) be different if the center of mass of the car were at ground level(h=0)?

f) Repeat steps (a) to (d) for all-wheel skidding. There are some shortcuts here. You determine the car deceleration without ever knowing the wheel reactions (or using angular momentum balance) if you look at the linear momentum balance equations carefully.

g) Does the deceleration in (f) equal the sum of the decelerations in (d) and (e)? Why or why not?

h) What peculiarity occurs in the solution for front-wheel skidding if the wheel base is twice the height of the CM above ground and $\mu = 1$?

i) What impossibility does the solution predict if the wheel base is shorter than twice the CM height? What wrong assumption gives rise to this impossibility? What would really happen if one tried to skid a car this way?



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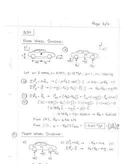
Problem 12.25



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d) From (3), $-\hat{\imath}\times(-mg\hat{\jmath}) + (-2\hat{\imath}+0.75\hat{\jmath})\times(-R_A\hat{\imath}+R_A\hat{\jmath}) = \vec{0}$

$$\{mg\hat{k} - 1.25R_A\hat{k} = 0\hat{k}\}\cdot\hat{k}$$

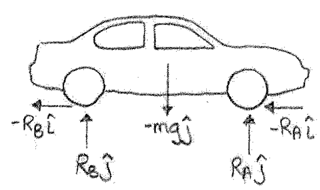
$$\therefore\ R_A = \frac{mg}{1.25} = \frac{10}{1.25} = 8.0\text{ kN}$$

From (2), $R_B = 2.0$ kN

From (1), $a = -R_A/m = \boxed{-8.0\ \text{m/s}^2} = \left(\frac{-g}{w-h}\right)\cdot\frac{w}{2}$

(f) ALL WHEEL SKIDDING:

a)



b) $\Sigma\vec{F}_x = m\vec{a} \rightarrow -R_B - R_A = ma$ (1)

$\Sigma\vec{F}_y = \vec{0} \rightarrow R_A + R_B = mg$ (2)

Plug (2) into (1): $-mg = ma \quad \therefore\ a = -g = \boxed{-10\ \text{m/s}^2}$

(g) No, the acceleration in (f) is not equal to the sum of those found in (d) and (e). The normal forces and friction forces are distributed differently, so there is no reason to believe they would be the same.

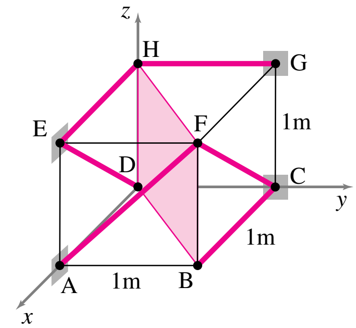
(h) If $w = 2h$, with front-wheel skidding, $\vec{r}_{AC} = (-2\hat{\imath}+\hat{\jmath})h$, $\vec{r}_{AC}\times R_A(-\hat{\imath}+\hat{\jmath}) = -R_A\hat{k}$, so $a = -g$

(i) If $w < 2h$, with front-wheel skidding, $\vec{r}_{AC} = (-2\hat{\imath}+{>}1)h$, $\vec{r}_{AC}\times R_A(-\hat{\imath}+\hat{\jmath}) < -R_A\hat{k}$, so $a < -g$ or $|a| > g$. This is only because we assumed a non-rotating rigid body, which would no longer hold.

12.2.43 The uniform 2 kg plate DBFH is held by six massless rods (AF, CB, CF, GH, ED, and EH) which are hinged at their ends. The support points A, C, G, and E are all accelerating in the x-direction with acceleration $\boldsymbol{a} = 3\,\mathrm{m/s^2}\,\hat{\imath}$. There is no gravity.

a) What is $\{\sum \vec{F}\}\cdot\hat{\imath}$ for the forces acting on the plate?

b) What is the tension in bar CB?



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Problem 12.43

TAM 203
Homework Solution

Due 4/10/08

12.72

let P be the centroid of plate

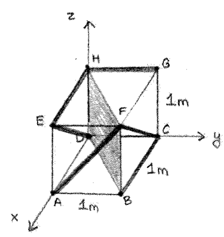


Plate DBFH has mass $m = 2$ kg, held by six massless rods:
AF, CB, CF, GH, ED, EH

Points A, C, E + G accelerate with
$\vec{a} = 3\,\mathrm{m/s^2}\,\hat{\imath}$

a) $\{\Sigma \vec{F}\}\cdot\hat{\imath} = m\vec{a}\cdot\hat{\imath} = (2\,\mathrm{kg})(3\,\mathrm{m/s^2}) = \boxed{6\,\mathrm{N}}$

b) To find $\vec{F}_{CB}$, take angular momentum balance about F:

$$\Sigma \vec{M}_{/F} = \dot{\vec{H}}_F = \vec{r}_{P/F} \times m\vec{a} = \tfrac{1}{2}(\hat{\imath}+\hat{\jmath}+\hat{k}) \times (6\hat{\imath})$$
$$= 3\left[(\hat{\imath}\times\hat{\imath})+(\hat{\jmath}\times\hat{\imath})+(\hat{k}\times\hat{\imath})\right] = 3(\hat{\jmath} - \hat{k})$$

$\therefore\ \Sigma \vec{M}_{/F} = 3\,\mathrm{N\text{-}m}\,(\hat{\jmath}-\hat{k})$

$$\Sigma \vec{M}_F = \vec{r}_{H/F} \times \vec{T}_{HE} + \vec{r}_{H/F} \times \vec{T}_{HG} + \vec{r}_{D/F} \times \vec{T}_{DE} + \vec{r}_{B/F} \times \vec{T}_{BC}$$

$$= T_{HE}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} + T_{HG}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} + \frac{T_{DE}}{\sqrt{3}}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} + T_{BC}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{vmatrix}$$

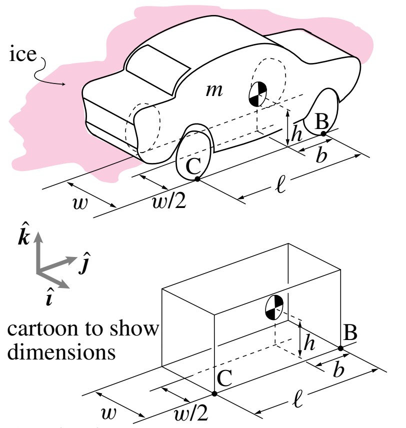
$$= T_{HE}(-\hat{k}) + T_{HG}(\hat{k}) + \frac{T_{DE}}{\sqrt{3}}(\hat{\imath}-\hat{k}) + T_{BC}(-\hat{\jmath})$$

$$\therefore\ \frac{T_{DE}}{\sqrt{3}}\hat{\imath} - T_{BC}\hat{\jmath} + \left(T_{HG} - T_{HE} - \frac{T_{DE}}{\sqrt{3}}\right)\hat{k} = 3\hat{\jmath} - 3\hat{k}$$

$\{\}\cdot\hat{\jmath} \longrightarrow -T_{BC} = 3$ $\qquad \therefore\ T_{BC} = -3\,\mathrm{N}$

OR $\boxed{\vec{T}_{BC} = (-3\,\mathrm{N})\,\hat{\imath}}$

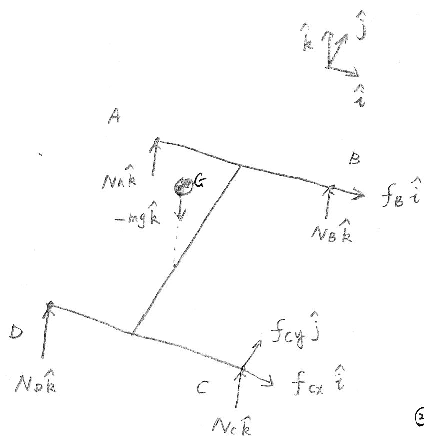
12.2.47 A rear-wheel drive car on level ground. The two left wheels are on perfectly slippery ice. The right wheels are on dry pavement. The negligible-mass front right wheel at B is steered straight ahead and rolls without slip. The right rear wheel at C also rolls without slip and drives the car forward with velocity $\vec{v} = v\hat{j}$ and acceleration $\vec{a} = a\hat{j}$. Dimensions are as shown and the car has mass m. What is the sideways force from the ground on the right front wheel at B? Answer in terms of any or all of m, g, a, b, ℓ, w, and $\hat{i}$.



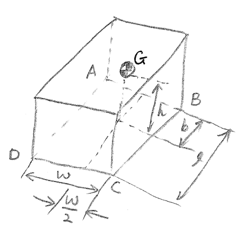
Problem 12.47: The left wheels of this car are on ice.

12.76

FBD



Dimensions.



Note:

① Supporting forces $N_i\hat{k}$ on four wheels, but they don't equal.

② No friction force on A & D since they are on ice.

③ Side way friction force on B and C.

④ Since C is the driving wheel, there is a driving force. (essentially friction force) $f_{cy}\hat{j}$ on C.

$\vec{f} = f_{cx}\hat{i} + f_{cy}\hat{j}$ is the friction force acting on C.

Known: $\vec{V} = v\hat{j}$, $\vec{a} = a\hat{j}$ (Velocity and acceleration)

Want to solve for $f_B\hat{i}$.

7 unknowns : f_B, N_B, N_A, N_D, N_c, f_{cx}, f_{cy}

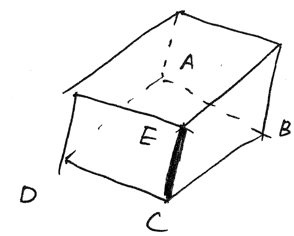
6 equations : LMB (3 equations), AMB (3 equations)

Can't solve for all forces.

But you can still write down AMB, LMB and manipulate the algebra to get f_B.

However, we can find a short cut for f_B.

Take AMB about vertical line at C: CE



All forces have no moments about CE except $f_B \hat{\imath}$

$\therefore$ AMB about CE $\Rightarrow$

$$(\vec{r}_{B/C} \times f_B \hat{\imath}) \cdot \hat{k} = (\vec{r}_{G/C} \times ma\hat{\jmath}) \cdot \hat{k}$$

$$\Rightarrow (\ell \hat{\jmath} \times f_B \hat{\imath}) \cdot \hat{k} = \left[\left(-\frac{w}{2}\hat{\imath} + (\ell - b)\hat{\jmath} + h\hat{k} \right) \times ma\hat{\jmath} \right] \cdot \hat{k}$$

$$\Rightarrow -f_B \ell = -\frac{maw}{2}$$

$$\therefore \boxed{f_B = \frac{maw}{2\ell}}$$

Sideway force from the ground on B is $\boxed{\frac{maw}{2\ell}\hat{\jmath}}$

Note: if you are famaliar withe moment about a line, you can directly write down

$$\boxed{f_B \ell = \frac{maw}{2}}$$

13.1.1 A particle goes on a circular path with radius R making the angle $\theta = ct$ measured counter clockwise from the positive x axis. Assume $R = 5\,\mathrm{cm}$ and $c = 2\pi\ \mathrm{s}^{-1}$.

a) Plot the path.

b) What is the angular rate in revolutions per second?

c) Put a dot on the path for the location of the particle at $t = t^* = 1/6$ s.

d) What are the x and y coordinates of the particle position at $t = t^*$? Mark them on your plot.

e) Draw the vectors $\hat{\boldsymbol{e}}_\theta$ and $\hat{\boldsymbol{e}}_R$ at $t = t^*$.

f) What are the x and y components of $\hat{\boldsymbol{e}}_R$ and $\hat{\boldsymbol{e}}_\theta$ at $t = t^*$?

g) What are the R and θ components of $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ at $t = t^*$?

h) Draw an arrow representing both the velocity and the acceleration at $t = t^*$.

i) Find the $\hat{\boldsymbol{e}}_R$ and $\hat{\boldsymbol{e}}_\theta$ components of position $\vec{\boldsymbol{r}}$, velocity $\vec{\boldsymbol{v}}$ and acceleration $\vec{\boldsymbol{a}}$ at $t = t^*$.

j) Find the x and y components of position $\vec{\boldsymbol{r}}$, velocity $\vec{\boldsymbol{v}}$ and acceleration $\vec{\boldsymbol{a}}$ at $t = t^*$. Find the velocity and acceleration two ways:

   1. Differentiate the position given as $\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$.
   2. Differentiate the position give as $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{e}}_r$ and then convert the results to Cartesian coordinates.

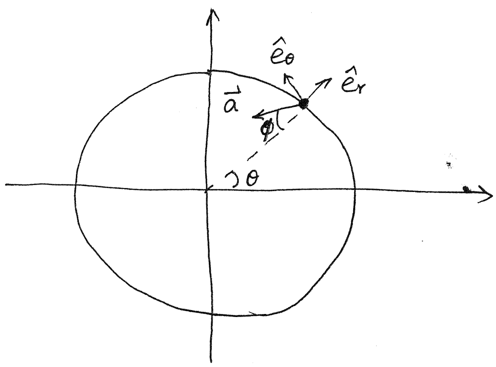
13.1.15 A particle moves in circles so that its acceleration $\vec{a}$ always makes a fixed angle ϕ with the position vector $-\vec{r}$, with $0 \le \phi \le \pi/2$. For example, $\phi = 0$ would be constant rate circular motion. Assume $\phi = \pi/4$, $R = 1\,\text{m}$ and $\dot\theta_0 = 1\,\text{rad/s}$. How long does it take the particle to reach

a) the speed of sound ($\approx 300\,\text{m/s}$)?

b) the speed of light ($\approx 3 \cdot 10^8\,\text{m/s}$)?

c) ∞?

13.15



Acceleration $\vec{a}$ always makes a fixed angle $\phi = \frac{\pi}{4}$ with $-\hat{e}_r$.

Let $a = |\vec{a}|$

$$\begin{aligned}\vec{a} &= -a\cos\phi\,\hat{e}_r + a\sin\phi\,\hat{e}_\theta \\ &= -a\cos\tfrac{\pi}{4}\,\hat{e}_r + a\sin\tfrac{\pi}{4}\,\hat{e}_r \\ &= -\tfrac{\sqrt{2}}{2}a\,\hat{e}_r + \tfrac{\sqrt{2}}{2}a\,\hat{e}_\theta \\ &= -R\dot\theta^2\,\hat{e}_r + R\ddot\theta\,\hat{e}_\theta\end{aligned}$$

$$\Rightarrow \left.\begin{aligned} -R\dot\theta^2 &= -a\tfrac{\sqrt{2}}{2} \\ R\ddot\theta &= a\tfrac{\sqrt{2}}{2}\end{aligned}\right\} \Rightarrow \cancel{R}\ddot\theta = \cancel{R}\dot\theta^2$$

$$\boxed{\ddot\theta = \dot\theta^2} \quad \#$$

Let $\omega = \dot\theta$, we have $\boxed{\dot\omega = \omega^2}$ (ω: Angular velocity (magnitude))

$$\Rightarrow \frac{d\omega}{dt} = \omega^2 \Rightarrow -\frac{1}{\omega} = t + \underbrace{c}_{\text{constant}}$$

$$\Rightarrow \boxed{\omega = -\frac{1}{t+c}}$$

At $t = 0$, $\dot\theta_0 = 1\,\text{rad/s}$, $\Rightarrow c = -1$ (s)

(Note: in the book, it says $\dot\phi_0 = 1\,\text{rad/s}$, this should be $\dot\theta_0 = 1\,\text{rad/s}$)

$$\therefore \omega = -\frac{1}{t-1} \quad (\text{rad/s})$$

i) Magnitude of velocity $|\vec{v}| = \omega R = 300\,\text{m/s}$

$$\Rightarrow -\frac{R}{t-1} = 300\,\text{m/s}$$, since $R = 1\,\text{m}$, $\boxed{t = 0.99667\,\text{s}}$

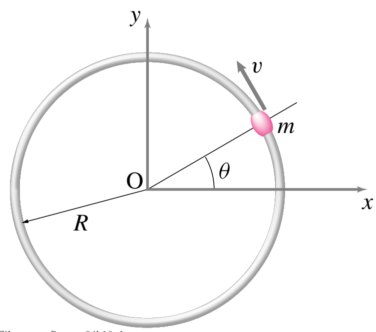
ii) $|\vec{v}| = 3\times10^8\,\text{m/s}$, similarly to i), we have $-\frac{R}{t-1} = 3\times10^8\,\text{m/s}$

$$\Rightarrow t = (1 - 0.3333\times10^{-8})\ \text{s}$$

13.2.30 Bead on a hoop with friction. A bead slides on a rigid, stationary, circular wire. The coefficient of friction between the bead and the wire is μ. The bead is loose on the wire (not a tight fit but not so loose that you have to worry about rattling). Assume gravity is negligible.

a) Given v, m, R, & μ; what is $\dot{v}$?

b) If $v(\theta = 0) = v_0$, how does v depend on θ, μ, v_0 and m?



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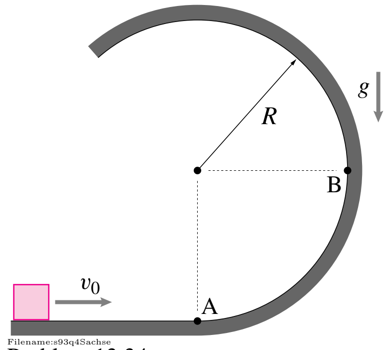
Problem 13.30

13.2.34 A block with mass m is moving to the right at speed v_0 when it reaches a circular frictionless portion of the ramp.

a) What is the speed of the block when it reaches point B? Solve in terms of R, v_0, m and g.

b) What is the force on the block from the ramp just after it gets onto the ramp at point A? Solve in terms of R, v_0, m and g. Remember, force is a vector.



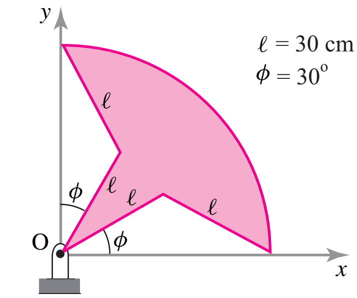
Problem 13.34

13.3.8 Write a computer program to animate the rotation of an object. Your input should be a set of x and y coordinates defining the object (such that `plot y vs x` draws the object on the screen) and the rotation angle θ. The output should be the rotated coordinates of the object.

a) From the geometric information given in the figure, generate coordinates of enough points to define the given object.

b) Using your program, plot the object at $\theta = 20°, 60°, 100°, 160°$, and $270°$.

c) Assume that the object rotates with constant angular speed $\omega = 2\,\text{rad/s}$. Find and plot the position of the object at $t = 1\,\text{s},\ 2\,\text{s},\ \text{and} 3\,\text{s}$.

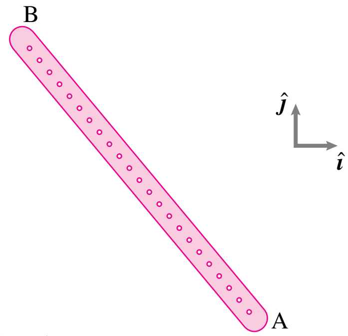


Problem 13.8

13.4.14 A $0.4\,\mathrm{m}$ long rod AB has many holes along its length such that it can be pegged at any of the various locations. It rotates counter-clockwise at a constant angular speed about a peg whose location is not known. At some instant t, the velocity of end B is $\vec{v}_B = -3\,\mathrm{m/s}\hat{\jmath}$. After $\frac{\pi}{20}$ s, the velocity of end B is $\vec{v}_B = -3\,\mathrm{m/s}\hat{\imath}$. If the rod has not completed one revolution during this period,

a) find the angular velocity of the rod, and

b) find the location of the peg along the length of the rod.



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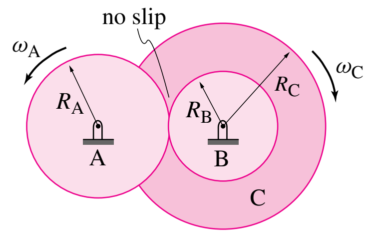
Problem 13.14

13.4.22 2-D constant rate gear train. The angular velocity of the input shaft (driven by a motor not shown) is a constant, $\omega_{\text{input}} = \omega_A$. What is the angular velocity $\omega_{\text{output}} = \omega_C$ of the output shaft and the speed of a point on the outer edge of disc C, in terms of R_A, R_B, R_C, and ω_A?



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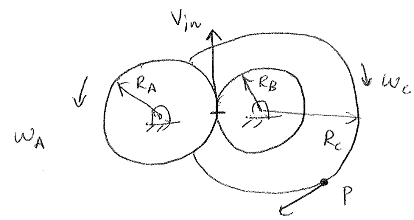
Problem 13.22: Gear B is welded to C and engages with A.

13.81



Given: R_A, R_B, R_C, ω_A

Find ω_c, V_p ?

$\Rightarrow$ For ~~non~~ slip between the gear R_A, R_B

$$V_{in} = \omega_A R_A = \omega_B R_B$$

$$\Rightarrow \omega_B = \omega_A \frac{R_A}{R_B}$$

Also $\omega_c = \omega_B$

Thus $\boxed{\omega_c = \omega_A \cdot \frac{R_A}{R_B}}$

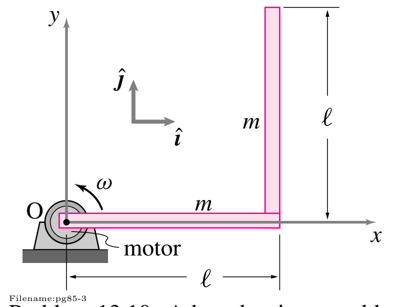
$$\Rightarrow V_p = \omega_c R_c$$

$$\boxed{V_p = \omega_A \frac{R_A R_c}{R_B}}$$

13.6.10 Motor turns a bent bar. Two uniform bars of length ℓ and uniform mass m are welded at right angles. One end is attached to a hinge at O where a motor keeps the structure rotating at a constant rate ω (counterclockwise). What is the net force and moment that the motor and hinge cause on the structure at the instant shown.

a) neglecting gravity

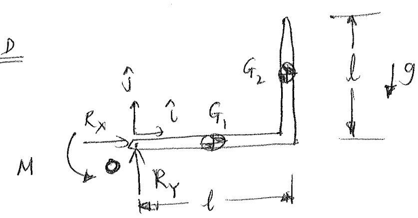
b) including gravity.



Problem 13.10: A bent bar is rotated by a motor.

13.112

FBD



Given : $m, \ell, g, \omega, \alpha = 0$

Find : R_x, R_Y, M

We will solve the general case involving gravity and than reduce to no gravity case by putting gravity $=0$.

AMB/o $\quad \vec{M}_{/O} = \dot{\vec{H}}_{/O}$

$$\vec{r}_{G_1/O} \times mg(-\hat{j}) + \vec{r}_{G_2/O} \times mg(-\hat{j}) + M\hat{k} = \vec{r}_{G_1/O} \times m\vec{a}_{G_1} + \vec{r}_{G_2/O} \times m\vec{a}_{G_2}$$

$$\therefore \frac{\ell}{2}\hat{\imath} \times mg(-\hat{j}) + (\ell\hat{\imath} + \frac{\ell}{2}\hat{j}) \times mg(-\hat{j}) + M\hat{k} = \vec{0}$$

why?

{Because $a_{G_1} = -\omega^2 r_{G_1/O}$
$a_{G_2} = -\omega^2 r_{G_2/O}$ }

{AMB/o} $\cdot k$

$$-\frac{3}{2}mg\ell + M = 0$$

$$\Rightarrow \quad M = \frac{3}{2}mg\ell \quad \text{—(I)}$$

LMB

$$R_x \hat{\imath} + R_y \hat{\jmath} - mg\hat{\jmath} - mg\hat{\jmath} = m\vec{a}_{G_1} + m\vec{a}_{G_2}$$

But $\vec{a}_{G_1} = \vec{a}_{G_1/0} = -\omega^2 r_{G_1/0} = -\omega^2 \{ \frac{\ell}{2} \hat{\imath} \}$

$$\vec{a}_{G_2} = \vec{a}_{G_2/0} = -\omega^2 r_{G_2/0} = -\omega^2 \{ \ell \hat{\imath} + \frac{\ell}{2} \hat{\jmath} \}$$

Thus

$$R_x \hat{\imath} + \{R_y - 2mg\} \hat{\jmath} = m \{ -\frac{3\omega^2 \ell}{2} \hat{\imath} - \frac{\omega^2 \ell}{2} \hat{\jmath} \}$$

$\{LMB\} \cdot \hat{\imath}$

$$R_x = -\frac{3m\omega^2 \ell}{2} \quad \text{— II}$$

$\{LMB\} \cdot \hat{\jmath}$

$$R_y = 2mg - \frac{m\omega^2 \ell}{2} \quad \text{— III}$$

a) Neglect gravity

Put $g = 0$ in I, II, III

$$\boxed{\begin{aligned} M &= 0 \\ R_x &= -\frac{3m\omega^2 \ell}{2} \\ R_y &= -\frac{m\omega^2 \ell}{2} \end{aligned}}$$

b) Including gravity

From I, II, III

$$\boxed{\begin{aligned} M &= \frac{3}{2} mg\ell \\ R_x &= -3m\omega^2 \ell/2 \\ R_y &= 2mg - \frac{3m\omega^2 \ell}{2} \end{aligned}}$$

13.6.20 At the input to a gear box a 100 lbf force is applied to gear A. At the output, the machinery (not shown) applies a force of F_B to the output gear. Gear A rotates at constant angular rate $\omega = 2\,\text{rad/s}$, clockwise.

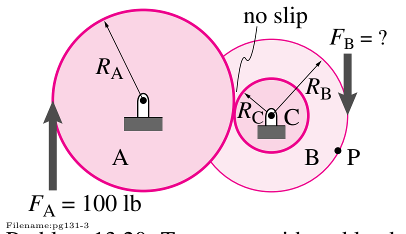
a) What is the angular speed of the right gear?

b) What is the velocity of point P?

c) What is F_B?

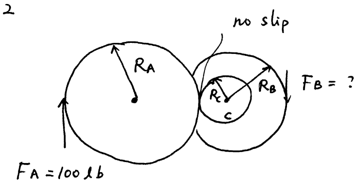
d) If the gear bearings had friction, would F_B have to be larger or smaller in order to achieve the same constant velocity?

e) If instead of applying a 100 lbf to the left gear it is driven by a motor (not shown) at constant angular speed ω, what is the angular speed of the right gear?

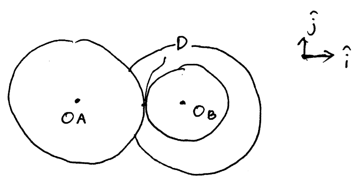


Problem 13.20: Two gears with end loads.

13. 122



(a)



$\hat{i}$ is along the line connecting O_A and O_B. D is the point of contact

$\therefore\ \vec{r}_{D/O_A} = R_A\,\hat{i}\ ,\qquad \vec{r}_{D/O_B} = -R_C\,\hat{i}$

No slip condition:

$$\vec{V}_D\big|_{\text{on gear }A} = \vec{V}_D\big|_{\text{on gear }B}$$

$$\cancel{\vec{V}_{O_A}}^{\,0} + \vec{\omega}_A \times \vec{r}_{D/O_A} = \cancel{\vec{V}_{O_B}}^{\,0} + \vec{\omega}_B \times \vec{r}_{D/O_B}$$

Gear A rotates clockwise with $\omega = 2\,\text{rad/s}$.

$\therefore\ \vec{\omega}_A = -\omega\,\hat{k}$

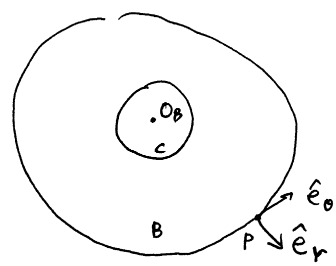
Assume $\vec{\omega}_B = \omega_B\,\hat{k}$

$\Rightarrow\ -\omega\hat{k} \times R_A\hat{i} = \omega_B\hat{k} \times (-R_C\hat{i})$

$\Rightarrow\ \boxed{\omega_B = \omega\dfrac{R_A}{R_C}}\ ,\quad \boxed{\omega_B > 0}$ means the right gear rotates counterclockwise

$\therefore$ gear B,C rotates counterclockwise with ~~spee~~ angular velocity $\omega_B = \omega\dfrac{R_A}{R_C}$

b.).



From (a), we know the angular velocity of B, $\vec{\omega}_B = \omega \frac{R_A}{R_C} \hat{k}$

$\therefore \quad \vec{V}_P = \vec{V}_{OB} + \vec{\omega}_B \times \vec{r}_{P/OB} = \omega \frac{R_A}{R_C} R_B \, \hat{e}_\theta$

$\therefore$ The velocity of P is $\boxed{\frac{\omega R_A R_B}{R_C}}$ and is in $\hat{e}_\theta$ direction.

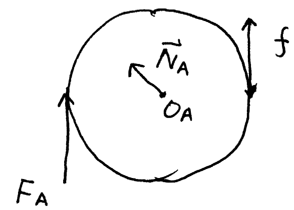
c). Assume friction less bearing.

FBD $\hat{j}$ $\hat{i}$

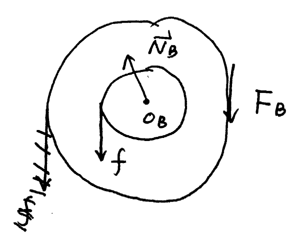
f: friction frorce at contact point

$\vec{N}_A$: reaction force at gear bearing

Left gear:



Right gear:



AMB of left gear about OA

$$\Rightarrow \quad \sum \vec{M}_{/OA} = \dot{\vec{H}}_{/OA} = \frac{d}{dt}(I_{OA} \underbrace{\vec{\omega}_A}_{\text{constant angular velocity}}) = 0$$

$$\Rightarrow \ -F_A R_A + f R_A = 0$$

$$\Rightarrow \ F_A = f$$

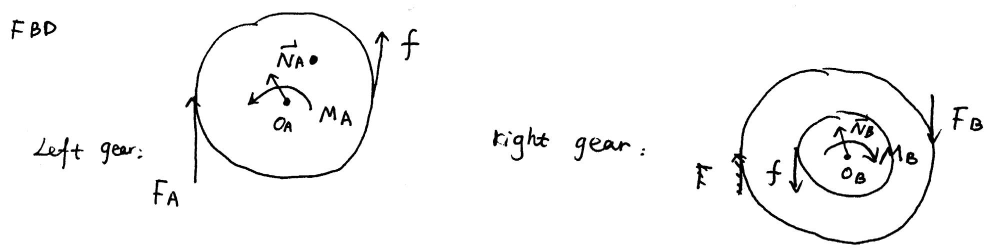
AMB of right gear about O_B

$$\Rightarrow \sum \vec{M}_{/O_B} = \dot{\vec{H}}_{/O_B} = \frac{d}{dt}(I_{O_B}\dot{\vec{\omega}}_B) = 0$$

$$\Rightarrow -F_B R_B + f R_C = 0 \qquad \Rightarrow F_B = \frac{f R_C}{R_B}$$

$$\therefore \boxed{F_B = \frac{F_A R_C}{R_B}}$$

d). If gear bearing had friction,



then there will be moment from the bearing resisting rotation of the gears.

$\because$ A is rotating clockwise, so $\vec{M}_A = M_A\hat{k}$, $M_A > 0$.
i.e., the moment is counterclockwise

B, C is rotating counterclockwise, so $\vec{M}_B = -M_B\hat{k}$, $M_B > 0$
i.e., the moment on B, C. is clockwise.

Use the same argument as in (c).,

Left gear: $\sum \vec{M}_{/O_A} = 0$

$$\Rightarrow -F_A R_A + f R_A + M_A = 0$$

$$\Rightarrow f = F_A - \frac{M_A}{R_A}$$

right gear: $\sum \vec{M}_{/O_B} = 0$

$$\Rightarrow -F_B R_B + f R_C - M_B = 0 \Rightarrow F_B = f\frac{R_C}{R_B} - \frac{M_B}{R_B}$$

$$\therefore \boxed{F_B = \frac{F_A R_C}{R_B} - \frac{M_A R_C}{R_A R_B} - \frac{M_B}{R_B}} < \frac{F_A R_C}{R_B} \quad \text{since } M_A, M_B > 0$$

$\therefore$ F_B is smaller because of friction

e). If the left gear is driven by a motor, angular speed of the right gear is $\boxed{\text{still} \quad \omega_B = \omega \frac{R_A}{R_C}}$, counter clockwise.

Because this result comes from the kinematic constraint that there is no slip between left gear and right gear. It doesn't depend on how the left gear is driven.

13.6.34 A pegged compound pendulum. A uniform bar of mass m and length ℓ hangs from a peg at point C and swings in the vertical plane about an axis passing through the peg. The distance d from the center of mass of the rod to the peg can be changed by putting the peg at some other point along the length of the rod.

a) Find the angular momentum of the rod about point C.

b) Find the rate of change of angular momentum of the rod about C.

c) How does the period of the pendulum vary with d? Show the variation by plotting the period against $\frac{d}{\ell}$. [Hint, you must first find the equations of motion, linearize for small θ, and then solve.]

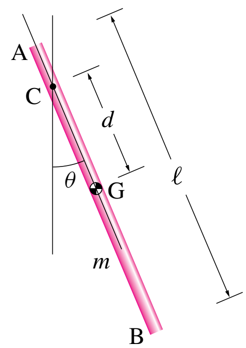
d) Find the total energy of the rod (using point C as a datum for potential energy).

e) Find $\ddot{\theta}$ when $\theta = \pi/6$.

f) Find the reaction force on the rod at C, as a function of m, d, ℓ, θ, and $\dot{\theta}$.

g) For the given rod, what should be the value of d (in terms of ℓ) in order to have the fastest pendulum?

h) **Test of Schuler's pendulum.** The pendulum with the value of d obtained in (g) is called the Schuler's pendulum. It is not only the fastest pendulum but also the "most accurate pendulum". The claim is that even if d changes slightly over time due to wear at the support point, the period of the pendulum does not change much. Verify this claim by calculating the percent error in the time period of a pendulum of length $\ell = 1\,\text{m}$ under the following three conditions: (i) initial $d = 0.15\,\text{m}$ and after some wear $d = 0.16\,\text{m}$, (ii) initial $d = 0.29\,\text{m}$ and after some wear $d = 0.30\,\text{m}$, and (iii) initial $d = 0.45\,\text{m}$ and after some wear $d = 0.46\,\text{m}$. Which pendulum shows the least error in its time period? What is the connection between this result and the plot obtained in (c)?



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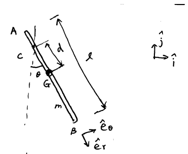
Problem 13.34

13.136.



a). $\vec{H}_{/C} = I_C\,\vec{\omega} = (I_G + md^2)(\dot{\theta}\hat{k}) = \boxed{\left(\frac{m\ell^2}{12} + md^2\right)\dot{\theta}\hat{k}}$

or $\vec{H}_{/C} = \vec{r}_{G/C} \times m\vec{V}_G + I_G\vec{\omega} = d\hat{e}_r \times md\dot{\theta}\hat{e}_\theta + \frac{m\ell^2}{12}\dot{\theta}\hat{k}$

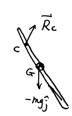
$= \left(\frac{m\ell^2}{12} + md^2\right)\dot{\theta}\hat{k}$

b). $\dot{\vec{H}}_{/C} = \frac{d}{dt}(\vec{H}_{/C}) = \frac{d}{dt}\left[\underbrace{\left(\frac{m\ell^2}{12} + md^2\right)}_{\text{constant}}\dot{\theta}\underbrace{\hat{k}}_{\text{constant}}\right]$

$= \boxed{\left(\frac{m\ell^2}{12} + md^2\right)\ddot{\theta}\hat{k}}$

c). AMB about C.

FBD



$\vec{R}_C$: reaction force at the hinge.

$\sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}$

$\vec{r}_{G/C} \times (-mg\hat{j}) = \left(\frac{m\ell^2}{12} + md^2\right)\ddot{\theta}\hat{k}$

$\Rightarrow \left(\frac{m\ell^2}{12} + md^2\right)\ddot{\theta} + mgd\sin\theta = 0$

$\Rightarrow \ddot{\theta} + \frac{12gd}{12d^2 + \ell^2}\sin\theta = 0$

For small θ, $\sin\theta \approx \theta$, $\Rightarrow$ $\boxed{\ddot{\theta} + \frac{12gd}{12d^2+\ell^2}\theta = 0}$

The period $\boxed{T = \frac{2\pi}{\sqrt{\frac{12gd}{12d^2+\ell^2}}}} = 2\pi\sqrt{\frac{12d^2+\ell^2}{12gd}} = \frac{2\pi\sqrt{\ell}}{\sqrt{g}}\sqrt{D + \frac{1}{12D}}$

where $D = \frac{d}{\ell}$.

Can plot $\underbrace{\frac{T}{2\pi}\sqrt{\frac{g}{\ell}}}_{} \sim D$ to ~~th~~ show the variation

normalized T : $\frac{T}{2\pi}\sqrt{\frac{g}{\ell}} = \sqrt{D + \frac{1}{12D}}$

d). Use the height of point c as a datum for potential energy.
At a position with angle θ.

$$E_p = -mgd\cos\theta$$

$$E_k = \frac{1}{2}mV_G^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}I_c\omega^2 = \frac{1}{2}(md^2 + \frac{ml^2}{12})\dot\theta^2$$

$$\therefore E_T = E_k + E_p = \boxed{\frac{1}{2}(md^2 + \frac{ml^2}{12})\dot\theta^2 - mgd\cos\theta}$$

e). $\theta = \frac{\pi}{6} \Rightarrow \sin\theta = \frac{1}{2}$

$\because \ddot\theta + \frac{12gd}{12d^2+l^2}\sin\theta = 0$ is satisfied all the time

$$\therefore \ddot\theta = -\frac{12gd}{12d^2+l^2}\sin\theta = \boxed{-\frac{6gd}{12d^2+l^2}} \quad \text{if } \theta = \frac{\pi}{6}$$

f). Use LMB

$$\Sigma\vec F = m\vec a_G \Rightarrow \vec R_c - mg\hat j = m\vec a_G$$

where $\vec a_G = \ddot\theta d\hat e_\theta - \dot\theta^2 d\hat e_r = (-\frac{12gd}{12d^2+l^2}\sin\theta\,\hat e_\theta) - \dot\theta^2 d\hat e_r$

$\because \hat e_\theta = \cos\theta\hat i + \sin\theta\hat j$, $\hat e_r = \sin\theta\hat i - \cos\theta\hat j$

$$\Rightarrow \vec a_G = -(\frac{12gd}{12d^2+l^2}\cos\theta\sin\theta + \dot\theta^2 d\sin\theta)\hat i - (\frac{12gd}{12d^2+l^2}\sin^2\theta - \dot\theta^2 d\cos\theta)\hat j$$

$\therefore$ The reaction force

$$\boxed{\vec R_c = -m(\frac{12gd}{12d^2+l^2}\cos\theta\sin\theta + \dot\theta^2 d\sin\theta)\hat i + (mg - \frac{12mgd}{12d^2+l^2}\sin^2\theta + m\dot\theta^2 d\cos\theta)\hat j}$$

g). $\because T = 2\pi\sqrt{\frac{12d^2+l^2}{12gd}}$

To find the minimum T, set $\frac{\partial T}{\partial d} = 0$

$$\because \frac{\partial T}{\partial d} = 2\pi\sqrt{\frac{12gd}{12d^2+l^2}}\left(\frac{1}{g} - \frac{l^2}{12gd^2}\right) = 0$$

$$\Rightarrow \frac{1}{g} - \frac{l^2}{12gd^2} = 0 \Rightarrow \boxed{d_m = \sqrt{\frac{1}{12}}\,l} \simeq 0.2887l$$

when $d < d_m$, $\frac{\partial T}{\partial d} < 0$; when $d > d_m$, $\frac{\partial T}{\partial d} > 0$

$\therefore$ $d_m = \sqrt{\frac{1}{12}}\, \ell$ is the minimum point for T.

h). $\ell = 1\ m$, $g = 9.8\ m/s^2$, $T = 2\pi\sqrt{\frac{12d^2+\ell^2}{12gd}} = \frac{2\pi\sqrt{\ell}}{\sqrt{g}}\sqrt{D+\frac{1}{12D}}$ $\boxed{D = \frac{d}{\ell}}$

i). ~~initid~~ initial $d_o = 0.15m$

$T_o = 1.6859\ s$

after some wear, $d = 0.16\ m$, $T = 1.6561\ s$

error $\left|\frac{T-T_o}{T_o}\right| = 1.767\ \%$

ii) initial $d_o = 0.29\ m$, $T_o = 1.5251\ s$

after some wear $d = 0.30\ m$, $T = 1.5256\ s$

error $\left|\frac{T-T_o}{T_o}\right| = 0.0343\%$

iii) initial $d_o = 0.45\ m$, $T_o = 1.5996\ s$

after some wear $d = 0.46m$, $T = 1.6071s$

error $\left|\frac{T-T_o}{T_o}\right| = 0.470\ \%$

The second case where $d_o = 0.29m$ shows the least error.

Since $d_o = 0.29m$ is close to $d_m = \sqrt{\frac{1}{12}}\,\ell \approx 0.2887\,\ell$,

and

$$\Delta T = |T - T_o| \approx \left|\frac{\partial T}{\partial d}\Big|_{d=d_o}(d-d_o)\right|$$

For all the 3 cases, $d - d_o = 0.1m$. However, $d_o = 0.29\ m$ is close to $d_m \approx 0.2887\,\ell$ where $\frac{\partial T}{\partial d} = 0$. So $\left|\frac{\partial T}{\partial d}\Big|_{d=d_o}\right|$ is the least when $d_o = 0.29\ m$.

From the graph given in c). one can see the slope near $d_m = \sqrt{\frac{1}{12}}\,\ell$ is very small, that is, the curve is flat near d_m.

14.1.1 A disk of radius R is hinged at point O at the edge of the disk, approximately as shown. It rotates counterclockwise with angular velocity $\dot{\theta} = \vec{\omega}$. A bolt is fixed on the disk at point P at a distance r from the center of the disk. A frame $x'y'$ is fixed to the disk with its origin at the center C of the disk. The bolt position P makes an angle ϕ with the x'-axis. At the instant of interest, the disk has rotated by an angle θ.

a) Write the position vector of point P relative to C in the $x'y'$ coordinates in terms of given quantities.

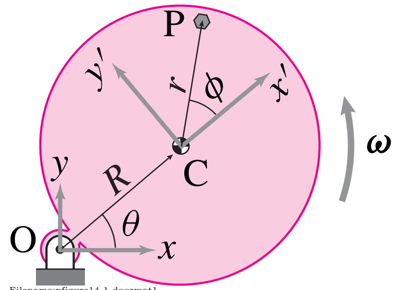
b) Write the position vector of point P relative to O in the xy coordinates in terms of given quantities.

c) Write the expressions for the rotation matrix $R(\theta)$ and the angular velocity matrix $\mathcal{S}(\vec{\omega})$.

d) Find the velocity of point P relative to C using $R(\theta)$ and the angular velocity matrix $\mathcal{S}(\vec{\omega})$.

e) Using $R = 30\,\text{cm}$, $r = 25\,\text{cm}$, $\theta = 60°$, and $\phi = 45°$, find $[\vec{r}_{C/0}]_{xy}$, and $[\vec{r}_{P/0}]_{xy}$ at the instant shown.

f) Assuming that the angular speed is $\omega = 10\,\text{rad/s}$ at the instant shown, find $[\vec{v}_{C/0}]_{xy}$ and $[\vec{v}_{P/0}]_{xy}$ taking other quantities as specified above.



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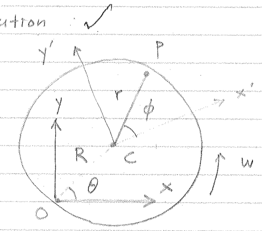
Problem 14.1

14.1 Solution ✓



a) $$\boxed{[\vec{r}_{P/C}]_{x'y'} = \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}}$$

b) $$[\vec{r}_{P/O}]_{xy} = [\vec{r}_{C/O}]_{xy} + [R(\theta)][\vec{r}_{P/C}]_{x'y'}$$

$$= \boxed{\begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}}$$

c) $$\boxed{R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$$ Rotation matrix

Book ⇒ Pg. 664 $$S(\vec{w}) = \dot{R}R^{-1} = \begin{bmatrix} -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta \\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\theta}(-\sin\theta\cos\theta + \sin\theta\cos\theta) & -\dot{\theta}(\sin^2\theta + \cos^2\theta) \\ \dot{\theta}(\cos^2\theta + \sin^2\theta) & \dot{\theta}(\sin\theta\cos\theta - \sin\theta\cos\theta) \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix}}$$

d)
$$[\dot{\vec{r}}_{P/C}]_{x'y'} = S(\vec{\omega})\, R(\theta)\, [\vec{r}_{P/C}]_{x'y'}$$

$$= \begin{bmatrix} 0 & -\dot{\theta} \\ \dot{\theta} & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta \\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -\dot{\theta}\,(r\cos\phi\sin\theta + r\cos\phi\cos\theta) \\ \dot{\theta}\,(r\cos\phi\cos\theta - r\sin\phi\sin\theta) \end{bmatrix}}$$

e) $R = 30\,cm, \quad r = 25\,cm, \quad \theta = 60°, \quad \phi = 45°$

$$[\vec{r}_{C/O}]_{xy} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} = \begin{bmatrix} 30\cos 60° \\ 30\sin 60° \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} [cm]}$$

$$[\vec{r}_{P/O}]_{xy} = \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} + \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 25\cos 45° \\ 25\sin 45° \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} + 25 \begin{bmatrix} \sqrt{2}/4 - \sqrt{6}/4 \\ \sqrt{6}/4 + \sqrt{2}/4 \end{bmatrix} [cm]$$

$$= \boxed{\begin{bmatrix} 8.53 \\ 50.13 \end{bmatrix} [cm]}$$

f) $\omega = 10\ \text{rad/s}$

$$[\vec{v}_{C/O}]_{xy} = [\dot{\vec{r}}_{C/O}]_{xy} = \begin{bmatrix} \dot{R\cos\theta} \\ R\sin\theta \end{bmatrix} = \dot{\theta} \begin{bmatrix} R\cos\theta \\ R\sin\theta \end{bmatrix}$$

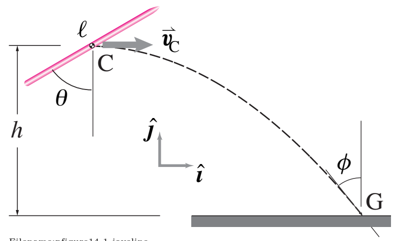
$$= \dot{\theta}\,[\vec{r}_{C/O}]_{xy} = (10)\begin{bmatrix} 15 \\ 15\sqrt{3} \end{bmatrix} = \boxed{\begin{bmatrix} 150 \\ 150\sqrt{3} \end{bmatrix} [\text{cm/s}]}$$

$$[\vec{v}_{P/O}]_{xy} = \dot{\theta}\,[\vec{r}_{P/O}]_{xy} = (10)\begin{bmatrix} 8.53 \\ 50.13 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 85.3 \\ 500.13 \end{bmatrix} [\text{cm/s}]}$$

14.1.12 The center of mass of a javelin travels on a more or less parabolic path while the javelin rotates during its flight. In a particular throw, the velocity of the center of mass of a javelin is measured to be $\vec{v}_C = 10\,\mathrm{m/s}\hat{\imath}$ when the center of mass is at its highest point $h = 6\,\mathrm{m}$. As the javelin lands on the ground, its nose hits the ground at G such that the javelin is almost tangent to the path of the center of mass at G. Neglect the air drag and lift on the javelin.

a) Given that the javelin is at an angle $\theta = 45°$ at the highest point, find the angular velocity of the javelin. Assume the angular velocity isconstant during the flight and that the javelin makes less than a full revolution.



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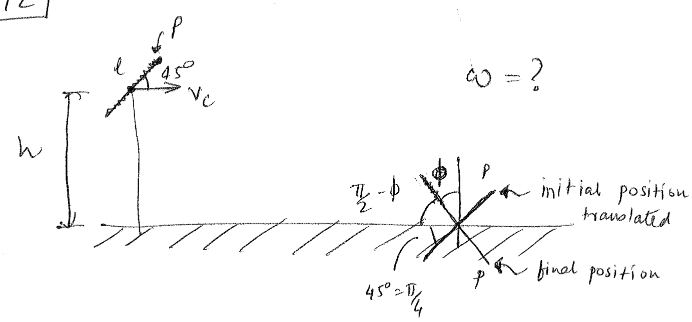
Problem 14.12

14.12)



— Ignore the length of the javelin in calculation

— Consider linear motion, solve for ϕ and time to hit ground (t)

— Solve for angular speed using ϕ and t.

For projectile motion:

$$h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 6}{10}}$$

$$\Rightarrow t = 1.1\ s$$

$$\phi = \tan^{-1}\left(\frac{V_x}{V_y}\right) = \tan^{-1}\left(\frac{V_c}{gt}\right) = \tan^{-1}\left(\frac{10}{10 \times 1.1}\right)$$

$$\Rightarrow \phi = 0.7378$$

Total angle turned from figure above $= \frac{\pi}{4} + \left(\frac{\pi}{2} - \phi\right)$

$$= 1.62$$

Angular velocity, $\omega = \dfrac{\text{Angle turned}}{\text{time taken}} = \dfrac{1.62}{1.1}$

$$\boxed{\omega = 1.47\ \mathrm{rad/s}}$$

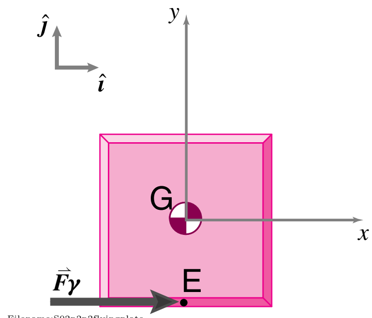
14.2.7 A uniform 1kg plate that is one meter on a side is initially at rest in the position shown. A constant force $\vec{F} = 1\,\mathrm{N}\hat{\imath}$ is applied at $t = 0$ and maintained henceforth. If you need to calculate any quantity that you don't know, but can't do the calculation to find it, assume that the value is given.

a) Find the position of G as a function of time (the answer should have numbers and units).

b) Find a differential equation, and initial conditions, that when solved would give θ as a function of time. θ is the counterclockwise rotation of the plate from the configuration shown.

c) Write computer commands that would generate a drawing of the outline of the plate at $t = 1\,\mathrm{s}$. You can use hand calculations or the computer for as many of the intermediate commands as you like. Hand work and sketches should be provided as needed to justify or explain the computer work.

d) Run your code and show clear output with labeled plots. Mark output by hand to clarify any points.



Problem 14.7

10:10AM Wed section 205
TA: Pranav Bhounsule

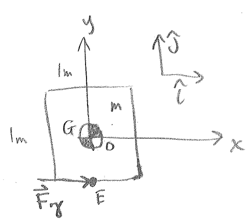
Alan Argondizza
TAM 2030
HW 17 Due 3/26/09

14.12, 14.19, 14.21, 14.31

10/10

$\boxed{14.12}$ See $\boxed{\text{SOLUTION}}$

$\boxed{14.19}$ ✓



$\vec{F} = 1\mathrm{N}\hat{\imath}$

$m = 1\,\mathrm{kg}$

a) LMB: $\Sigma \vec{F} = m\vec{a}_G$

$\vec{a}_G = \frac{\vec{F}}{m}$

$\left\{ \vec{a}_G = \frac{1\mathrm{N}\hat{\imath}}{1\mathrm{kg}} = a_x\hat{\imath} + a_y\hat{\jmath} \right\}$

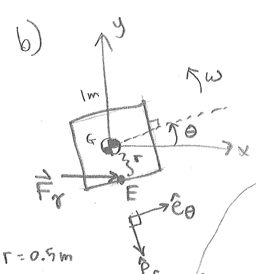
$\{\ldots\}\cdot\hat{\jmath} \Rightarrow 1\,\mathrm{m/s^2} = a_x = \ddot{x}$

Position:

$\vec{r}_{G/O} = x\hat{\imath} = (\cancel{x_o}^{\,0} + \cancel{\dot{x}_o}^{\,0} + \frac{1}{2}\ddot{x}t^2)\hat{\imath}$

$$\boxed{\vec{r}_{G/O} = \frac{t^2}{2}\hat{\imath}\ \mathrm{m}}$$

b)



$r = 0.5\,\mathrm{m}$

Pretend G is fixed:

AMB $\Sigma\vec{M}_{/G} = \dot{\vec{H}}_{/G}$

$I_{cm} = \frac{m}{12}(a^2+b^2) = \frac{m}{12}(1^2+1^2)$

$= \frac{m}{6} = \underline{\frac{1}{6} = I_{cm}}$

$r(\vec{F}_\gamma\cdot\hat{e}_\theta)\,\hat{k} = I^{cm}\dot{\vec{\omega}} + \cancel{\vec{\omega}\times(I^{cm}\vec{\omega})}^{\,0}$

$\{ rF_\gamma\cos\theta\,\hat{k} = I_{cm}\ddot{\theta}\hat{k} \}$

$\{\ldots\}\cdot\hat{k} \Rightarrow rF_\gamma\cos\theta = I_{cm}\ddot{\theta} \Rightarrow (\frac{1}{2})(1)\cos\theta = \frac{1}{6}\ddot{\theta}$

OR

$\vec{r}_{E/G}\times\vec{F}_\gamma$

$= r\hat{e}_r\times\{F_\gamma\cos\theta\,\hat{e}_\theta + F_\gamma\sin\theta\,\hat{e}_r\}$

$= rF_\gamma\cos\theta\,\hat{k}$

$$\boxed{\ddot{\theta} = 3\cos\theta}$$

$$\boxed{\begin{aligned}\theta\big|_{t=0} &= 0\\ \dot{\theta}\big|_{t=0} &= 0\end{aligned}}$$

```
function square_plate
% Alan Argondizza
% Solution to 14.19 part c and d

% Initial conditions and time span
time= 3;
tspan= linspace(0,time,101);  %Integrate for time seconds
z0 = [0,0]';                 % initial [angle,omega] both zero

% solve the ODE:
[t,z] = ode45(@rhs, tspan, z0);

% Unpack the variables
theta = z(:,1);      %first column of z
thetadot = z(:,2);   %second column of z

clf
tag=0

%plot using a loop:
for i= 1:1:length(t)

    %entire square:
    subplot(2,1,1)
    %create initial square:
    square= [.5,-.5,-.5,.5,.5;.5,.5,-.5,-.5,.5];
    %create rotation matrix:
    R= [cos(theta(i)),-sin(theta(i));...
        sin(theta(i)),cos(theta(i))];
    %determine displacement of G:
    xdisp= .5*t(i)^2;
    rotatedsquare= R*square + [xdisp,xdisp,xdisp,xdisp,xdisp;0,0,0,0,0];

    plot(rotatedsquare(1,:),rotatedsquare(2,:));

    %this conditional marks the square at time t= 1 second:
    if floor(t(i)) == 1
        if tag ~= 55
            line(rotatedsquare(1,:),rotatedsquare(2,:),'LineWidth',10,'Color','red');
        end
    end
    title('Trajectory of Square (Alan Argondizza)');
    xlabel('X');
    ylabel('Y');
    axis('equal');
    hold on

    %verticicies of square:
    subplot(2,1,2)
    line(rotatedsquare(1,:),rotatedsquare
(2,:),'LineStyle','none','Color','red','Marker','.');
```

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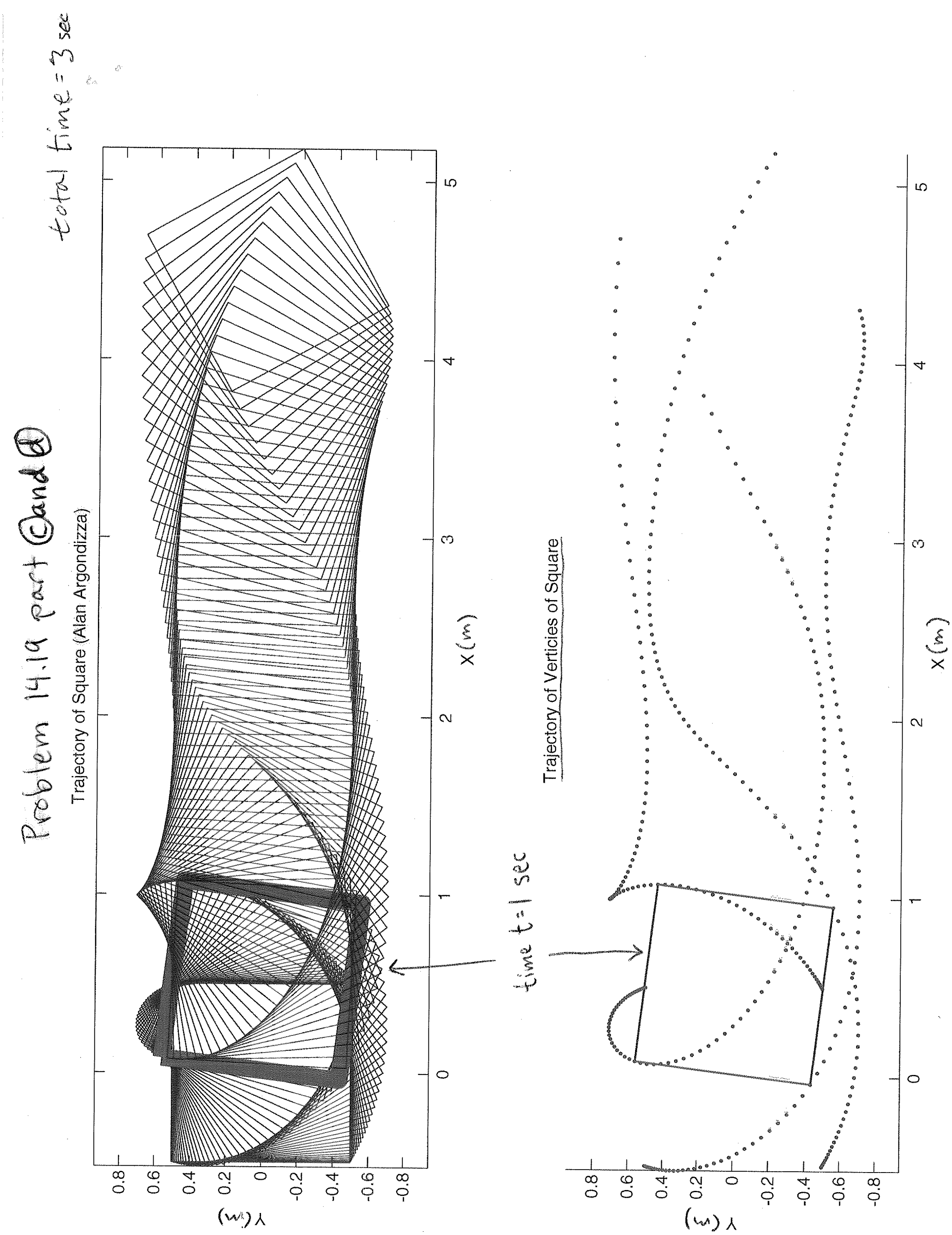
```
    %this conditional marks the square at time t= 1 second:
    if floor(t(i)) == 1
        if tag ~= 55
            line(rotatedsquare(1,:),rotatedsquare(2,:),'LineWidth',1,'Color','red');
        end
        tag=55;
    end
    title('Trajectory of Verticies of Square');
    xlabel('X');
    ylabel('Y');
    axis('equal');
    hold on
end
end

function zdot = rhs(t,z)

theta = z(1);       % unpack z into readable variables
thetadot = z(2);

%RHS:
omega = thetadot;
omegadot = 3*cos(theta);
% pack up the derivatives:
z1dot = omega;
z2dot = omegadot;
%function return:
zdot = [z1dot z2dot]';
end
```

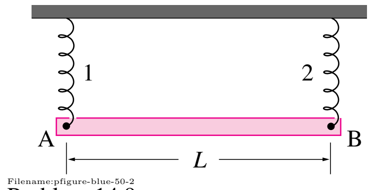

Problem 14.19 part c and d
total time = 3 sec
Trajectory of Square (Alan Argondizza)
X (m)
Y (m)
time t = 1 sec
Trajectory of Verticies of Square
X (m)
Y (m)

14.2.9 A uniform slender bar AB of mass m is suspended from two springs (each of spring constant K) as shown. Immediately after spring 2 breaks, determine

a) the angular acceleration of the bar,

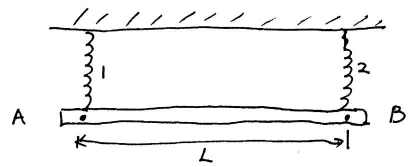
b) the acceleration of point A, and

c) the acceleration of point B.



Problem 14.9

14.21



1, 2 : two springs with spring const k

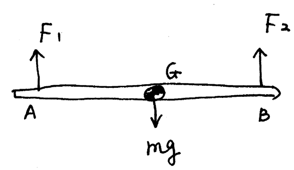
If spring 2 breaks, determine at that instant,

(a) angular acceleration of the bar

(b) the acceleration of point A

(c) the acceleration of point B.

Before "2" breaks, the bar is in equilibrium.



It's easy to get

$F_1 = F_2 = \frac{mg}{2}$ from LMB, AMB $/_G$.

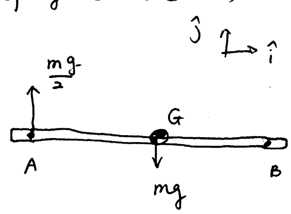
After "2" breaks,

The distance between A ~~bet~~ and the ceiling ~~is~~ at that instant is the same as that before "2" breaks.

$\therefore$ The stretch of spring 1 reamains unchanged. So the tension force on spring is still $\frac{mg}{2}$.

But spring 2 breaks, so $F_2 = 0$

$\therefore$



AMB $/_G$ $\Rightarrow$

$$\sum \vec{M}_{/G} = I_G \dot{\vec{\omega}}$$

$$\Rightarrow -\frac{mgL}{4}\hat{k} = \frac{mL^2}{12}\dot{\vec{\omega}}$$

$$\Rightarrow \boxed{\dot{\vec{\omega}} = -\frac{3g}{L}\hat{k}}$$

$\therefore$ angular acceleration of the bar is $-\frac{3g}{L}\hat{k}$ at that instant.

LMB $\Rightarrow$

$$-mg\hat{j} + \frac{mg}{2}\hat{j} = m\vec{a}_G$$

$$\Rightarrow \quad \vec{a}_G = -\frac{g}{2}\hat{j}$$

$$\therefore \quad \vec{a}_A = \vec{a}_G + \dot{\vec{\omega}} \times \vec{r}_{A/G} - \omega^2 \vec{r}_{A/G}$$

At this instant $\omega = 0$

$$\therefore \quad \vec{a}_A = -\frac{g}{2}\hat{j} + \left(-\frac{3g}{L}\hat{k}\right) \times \left(-\frac{L}{2}\hat{i}\right)$$

$$= -\frac{g}{2}\hat{j} + \frac{3g}{2}\hat{j} = g\hat{j}$$

Similarly

$$\vec{a}_B = \vec{a}_G + \dot{\vec{\omega}} \times \vec{r}_{B/G} - \overset{0}{\cancel{\omega^2}}\,\vec{r}_{B/G}$$

$$= -\frac{g}{2}\hat{j} + \left(-\frac{3g}{L}\hat{k}\right) \times \left(\frac{L}{2}\hat{i}\right)$$

$$= -\frac{g}{2}\hat{j} - \frac{3g}{2L}\hat{j}$$

$$= -2g\hat{j}$$

$\therefore$ accelerations of point A, B at that instant are

$$\boxed{\vec{a}_A = g\hat{j}}$$

$$\boxed{\vec{a}_B = -2g\hat{j}}$$

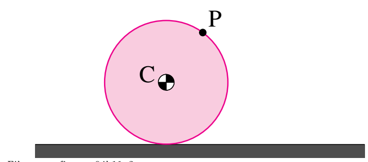
The next several problems concern Work, power and energy

14.3.3 Rolling at constant rate. A round disk rolls on the ground at constant rate. It rolls $1\frac{1}{4}$ revolutions over the time of interest.

a) **Particle paths.** Accurately plot the paths of three points: the center of the disk C, a point on the outer edge that is initially on the ground, and a point that is initially half way between the former two points. [Hint: Write a parametric equation for the position of the points. First find a relation between ω and v_C. Then note that the position of a point is the position of the center plus the position of the point relative to the center.] Draw the paths on the computer, make sure x and y scales are the same.

b) **Velocity of points.** Find the velocity of the points at a few instants in the motion: after $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 revolution. Draw the velocity vector (by hand) on your plot. Draw the direction accurately and draw the lengths of the vectors in proportion to their magnitude. You can find the velocity by differentiating the position vector or by using relative motion formulas appropriately. Draw the disk at its position after one quarter revolution. Note that the velocity of the points is perpendicular to the line connecting the points to the ground contact.

c) **Acceleration of points.** Do the same as above but for acceleration. Note that the acceleration of the points is parallel to the line connecting the points to the center of the disk.



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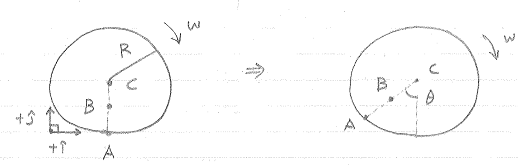
Problem 14.3

For this problem, I looked at the sample derivation done on page 767 in textbook.

14.3.1



a) point C : $\vec{r}_C = \boxed{R\theta\,\hat{\imath} + R\,\hat{\jmath}}$

point A : $\vec{r}_A = \vec{r}_C + \vec{r}_{A/C}$

$= \vec{r}_C + (-R\sin\theta\,\hat{\imath} - R\cos\theta\,\hat{\jmath})$

$= R\theta\,\hat{\imath} + R\hat{\jmath} - R\sin\theta\,\hat{\imath} - R\cos\theta\,\hat{\jmath}$

$= \boxed{R(\theta - \sin\theta)\,\hat{\imath} + R(1 - \cos\theta)\,\hat{\jmath}}$

point B : $\vec{r}_B = \vec{r}_C + \vec{r}_{B/C}$

$= \vec{r}_C + (-\frac{1}{2}R\sin\theta\,\hat{\imath} - \frac{1}{2}R\cos\theta\,\hat{\jmath})$

$= R\theta\,\hat{\imath} + R\hat{\jmath} - \frac{1}{2}R\sin\theta\,\hat{\imath} - \frac{1}{2}R\cos\theta\,\hat{\jmath}$

$= \boxed{R(\theta - \frac{1}{2}\sin\theta)\,\hat{\imath} + R(1 - \frac{1}{2}\cos\theta)\,\hat{\jmath}}$

Plot of path on separate sheets.

b) <u>point C:</u> $\vec{v}_C = \dot{\vec{r}}_C = R\dot{\theta}\hat{\imath}$

$\frac{1}{4}$ rev. $\left(\theta = \frac{\pi}{2}\right) \Rightarrow \vec{v}_C = \boxed{R\dot{\theta}\hat{\imath}}$

$\frac{1}{2}$ rev. $(\theta = \pi) \Rightarrow \vec{v}_C = \boxed{R\dot{\theta}\hat{\imath}}$

$\frac{3}{4}$ rev. $\left(\theta = \frac{3\pi}{2}\right) \Rightarrow \vec{v}_C = \boxed{R\dot{\theta}\hat{\imath}}$

1 rev. $(\theta = 2\pi) \Rightarrow \vec{v}_C = \boxed{R\dot{\theta}\hat{\imath}}$

<u>point A:</u> $\vec{v}_A = \vec{v}_C + \vec{\omega} \times \vec{r}_{A/C}$

$\frac{1}{4}$ rev. $\Rightarrow \vec{v}_A = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times R(-\hat{\imath})$

$= R\dot{\theta}\hat{\imath} + R\dot{\theta}\hat{\jmath} = \boxed{R\dot{\theta}(\hat{\imath}+\hat{\jmath})}$

$\frac{1}{2}$ rev. $\Rightarrow \vec{v}_A = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times R(+\hat{\jmath})$

$= R\dot{\theta}\hat{\imath} + R\dot{\theta}\hat{\imath} = \boxed{2R\dot{\theta}\hat{\imath}}$

$\frac{3}{4}$ rev. $\Rightarrow \vec{v}_A = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times R(+\hat{\imath})$

$= R\dot{\theta}\hat{\imath} - R\dot{\theta}\hat{\jmath} = \boxed{R\dot{\theta}(\hat{\imath}-\hat{\jmath})}$

1 rev. $\Rightarrow \vec{v}_A = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times R(-\hat{\jmath})$

$= R\dot{\theta}\hat{\imath} - R\dot{\theta}\hat{\imath} = \boxed{\vec{0}}$

✓

point B: $\vec{v}_B = \vec{v}_C + \vec{\omega} \times \vec{r}_{B/C}$

$\frac{1}{4}$ rev. $\Rightarrow$ $\vec{v}_B = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times \frac{1}{2}R(-\hat{\imath})$

$= \boxed{R\dot{\theta}\,(\hat{\imath} + \frac{1}{2}\hat{\jmath})}$

$\frac{1}{2}$ rev. $\Rightarrow$ $\vec{v}_B = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times \frac{1}{2}R(+\hat{\jmath})$

$= \boxed{\frac{3}{2}R\dot{\theta}\hat{\imath}}$

$\frac{3}{4}$ rev. $\Rightarrow$ $\vec{v}_B = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times \frac{1}{2}R(+\hat{\imath})$

$= \boxed{R\dot{\theta}(\hat{\imath} - \frac{1}{2}\hat{\jmath})}$

1 rev. $\Rightarrow$ $\vec{v}_B = R\dot{\theta}\hat{\imath} + \dot{\theta}(-\hat{k}) \times \frac{1}{2}R(-\hat{\jmath})$

$= \boxed{\frac{1}{2}R\dot{\theta}\hat{\imath}}$

velocity vectors shown on separate sheets. $\longrightarrow$

c) point C: $\vec{a}_C = \dot{\vec{v}}_C = R\ddot{\theta}\hat{\imath} = \boxed{\vec{0}}$ $(\ddot{\theta} = 0)$

point A: $\vec{a}_A = \vec{a}_C + \vec{a}_{A/C}$

$$= \vec{a}_C + \cancelto{0}{\dot{\vec{\omega}} \times \vec{r}_{A/C}} - \omega^2 \vec{r}_{A/C}$$

$\frac{1}{4}$ rev. $\Rightarrow \vec{a}_A = \vec{0} - \dot{\theta}^2 R(-\hat{\imath}) = \boxed{R\dot{\theta}^2\hat{\imath}}$

$\frac{1}{2}$ rev. $\Rightarrow \vec{a}_A = \vec{0} - \dot{\theta}^2 R(+\hat{\jmath}) = \boxed{-R\dot{\theta}^2\hat{\jmath}}$

$\frac{3}{4}$ rev. $\Rightarrow \vec{a}_A = \vec{0} - \dot{\theta}^2 R(+\hat{\imath}) = \boxed{-R\dot{\theta}^2\hat{\imath}}$

1 rev. $\Rightarrow \vec{a}_A = \vec{0} - \dot{\theta}^2 R(-\hat{\jmath}) = \boxed{R\dot{\theta}^2\hat{\jmath}}$

point B: $\vec{a}_B = \vec{a}_C + \vec{a}_{B/C}$

$$= \vec{a}_C + \cancelto{0}{\dot{\vec{\omega}} \times \vec{r}_{B/C}} - \omega^2 \vec{r}_{B/C}$$

$\frac{1}{4}$ rev. $\Rightarrow \vec{a}_A = \boxed{\frac{1}{2}R\dot{\theta}^2\hat{\imath}}$

$\frac{1}{2}$ rev. $\Rightarrow \vec{a}_A = \boxed{-\frac{1}{2}R\dot{\theta}^2\hat{\jmath}}$

$\frac{3}{4}$ rev. $\Rightarrow \vec{a}_A = \boxed{-\frac{1}{2}R\dot{\theta}^2\hat{\imath}}$

1 rev. $\Rightarrow \vec{a}_A = \boxed{\frac{1}{2}R\dot{\theta}^2\hat{\jmath}}$

$\boxed{\text{acceleration vectors shown on separate sheets.}}$

✓

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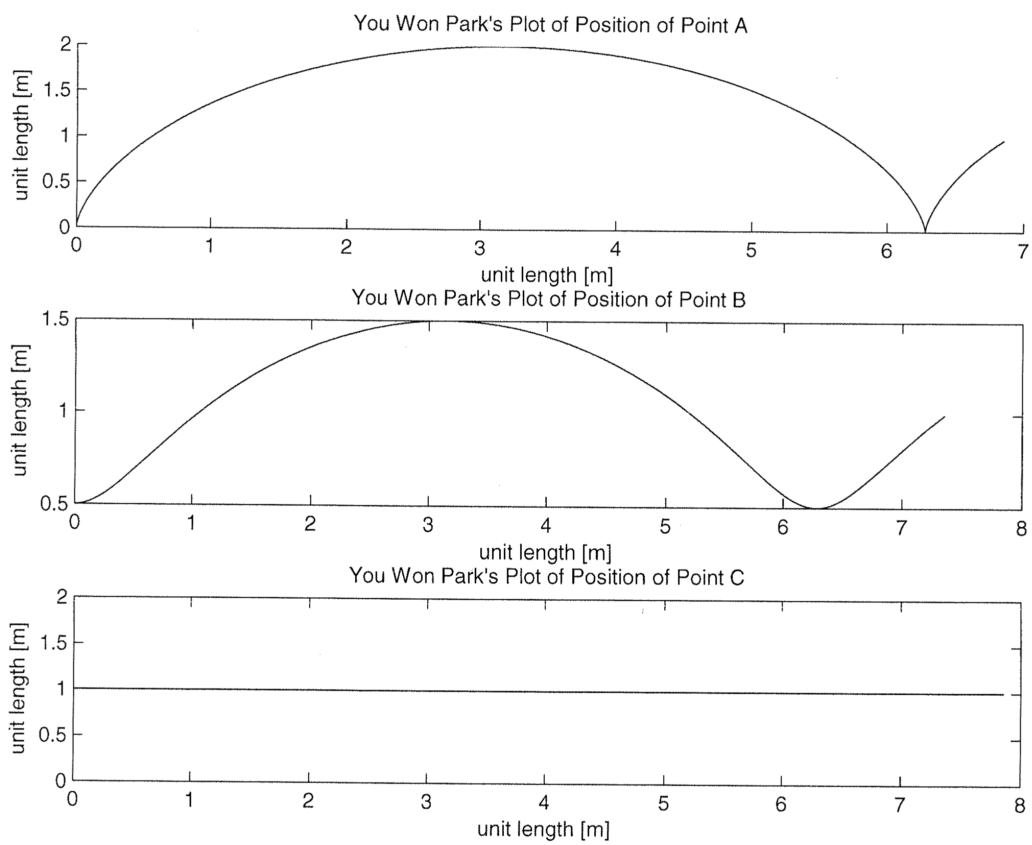
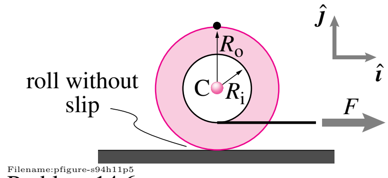
```
function prob1431
% You Won Park's solution to problem 14.31 in HW 17
% Due Mar. 26, 2009

% Constants, initial conditions
R = 1;      % Radius of disk [m]

% Angle interval
angspan = linspace(0,5*pi/2,1001);

% Point C coordinates (center of disk)
rc_x = R*angspan;   % x coord. of C
rc_y = R;           % y coord. of C

% Point A coordinates (ground contact)
ra_x = R*(angspan-sin(angspan));    % x coord. of A
ra_y = R*(1-cos(angspan));          % y coord. of A

% Point B coordinates (halfway)
rb_x = R*(angspan-.5*sin(angspan));     % x coord. of B
rb_y = R*(1-.5*cos(angspan));           % y coord. of B

% Plot positions of A,B,C
figure(1)
subplot(3,1,1)
hold on
plot(ra_x,ra_y,'k')     % Position of A
title('You Won Park''s Plot of Position of Point A')
xlabel('unit length [m]')
ylabel('unit length [m]')

subplot(3,1,2)
plot(rb_x,rb_y,'k')     % Position of B
title('You Won Park''s Plot of Position of Point B')
xlabel('unit length [m]')
ylabel('unit length [m]')

subplot(3,1,3)
plot(rc_x,rc_y,'k')     % Position of C
title('You Won Park''s Plot of Position of Point C')
xlabel('unit length [m]')
ylabel('unit length [m]')

end
```


You Won Park's Plot of Position of Point A
unit length [m]
unit length [m]
You Won Park's Plot of Position of Point B
unit length [m]
unit length [m]
You Won Park's Plot of Position of Point C
unit length [m]
unit length [m]

14.4.6 Spool Rolling without Slip and Pulled by a Cord. The light-weight spool is nearly empty but a lead ball with mass m has been placed at its center. A force F is applied in the horizontal direction to the cord wound around the wheel. Dimensions are as marked. Coordinate directions are as marked.

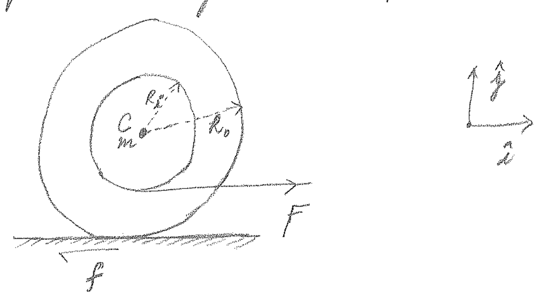
a) What is the acceleration of the center of the spool?

b) What is the horizontal force of the ground on the spool?



Problem 14.6

14.39 Spool rolling w/o slip



* We need to determine the acceleration of the center of mass $\vec{a}_{cm}$, and the horizontal force $\vec{f}$ acting on the lowest pt. as shown.

* Newton's II Law immediately yields:

LMB: $F\,\hat{i} - f\,\hat{i} = m\,\vec{a}_{cm}$ —(i)

& AMB: $R_i(-\hat{j}) \times F(\hat{i}) + R_o(-\hat{j}) \times f(-\hat{i}) = I^c \vec{\alpha}$ —(ii)

* Since the spool is "massless", $I^c \to 0$

$\Rightarrow$ (ii) yields $R_i F = R_o f$

Substituting in (i) $\boxed{\vec{a}_{cm} = \frac{F}{m}\left(1 - \frac{R_i}{R_o}\right)\hat{i}}$

Also, $\boxed{-\left(\frac{R_i}{R_o}\right)F\,\hat{i} = \vec{f}}$

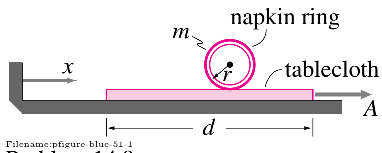
14.4.9 A napkin ring lies on a thick velvet tablecloth. The thin ring (of mass m, radius r) rolls without slip as a mischievous child pulls the tablecloth (mass M) out with acceleration A. The ring starts at the right end ($x = d$). You can make a reasonable physical model of this situation with an empty soda can and a piece of paper on a flat table.

a) What is the ring's acceleration as the tablecloth is being withdrawn?

b) How far has the tablecloth moved to the right from its starting point $x = 0$ when the ring rolls off its left-hand end?

c) Clearly describe the subsequent motion of the ring. Which way does it end up rolling at what speed?

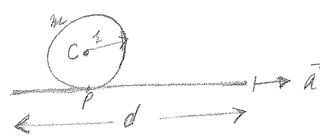
d) Would your answer to the previous question be different if the ring slipped on the cloth as the cloth was being pulled out?



Problem 14.9

14.4.2 "Napkin ring" problem.

(a) The lowest pt on the ring will have the same acceleration as the cloth.



$\alpha = \dot{\omega}$

$$\vec{a}_p = a\hat{i} = a_{cm}\hat{i} + \alpha\hat{k}\times r(-\hat{j}) \quad \text{— (1)}$$

Since friction acts at the lowest pt, then

$$\vec{M}_p = \dot{\vec{H}}_{/p} = r\hat{j}\times m\,a_{cm}\hat{i} + I^c_{zz}\alpha\hat{k} = \vec{0}$$

$$\Rightarrow \quad m\,a_{cm}\,r = I^c_{zz}\,\alpha$$

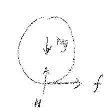
$$\cancel{m}\,a_{cm}\,\cancel{r} = \cancel{m}\,r^{\cancel{2}}\,\alpha$$

$$\Rightarrow \quad \boxed{a_{cm} = r\alpha}$$

Using (1) $\boxed{a_{cm} = \frac{a}{2}}$

FBD

on cloth



on table (before rolling)

(b) The ring moves 'd' in time 't' relative to the cloth with acceleration $a/2$

$$\Rightarrow \quad d = \frac{1}{2}\frac{a}{2}t^2 \quad \Rightarrow \quad t = 2\sqrt{d/a}$$

In the same time, the cloth moves $\frac{1}{2}a\left(2\sqrt{d/a}\right)^2 = \boxed{2d}$

(c) When the ring leaves the cloth, the lowest pt has velocity $\vec{v}_p = 2\sqrt{ad}\,\hat{\imath}$ $(v = v_0 + at)$

Now two cases are possible:

(i) Table is smooth: In this all velocities (angular, linear) are conserved. The ring coasts with the same velocity it has when it leaves the cloth.

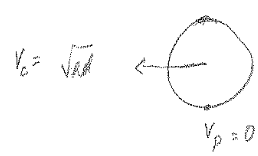
(ii) Table has friction: In this case, friction will act until lowest pt. comes to rest. Since lowest pt. is moving to the right — friction will be in the ~~ta~~ $-\hat{\imath}$ direction. Friction will cause a clockwise torque about C.M.

Assuming friction (μ), one can calculate the time 't_2' when lowest pt will come to rest: $t_2 = \dfrac{2\sqrt{ad}}{\mu g}$ $(v = at)$

Now turn attention to the center-of-mass: As it leaves the cloth, the center-of-mass will have velocity $v_c = \sqrt{ad}\ \hat{\imath}$.

On the table, because of friction, it will deaccelerate. By the time the lowest pt. comes to rest (in time 't_s'), the cm will have velocity $\sqrt{ad} - \frac{2\sqrt{ad}\,\mu g}{\cancel{\mu g}} = -\sqrt{ad}\ \hat{\imath}$

At t_s, we have a situation like this:



Now on there is no work from friction. So it stays rolling like above. (Obviously $v_c = \omega R$).

(d) Of course, if there is sliding initially on the cloth, we don't expect the same v_c as above. But it remains ~~tr~~ true the ring will roll in the same direction.

14.4.23 A disk rolls in a cylinder. For all of the problems below, the disk rolls without slip and rocks back and forth due to gravity.

a) **Sketch.** Draw a neat sketch of the disk in the cylinder. The sketch should show all variables, coordinates and dimension used in the problem.

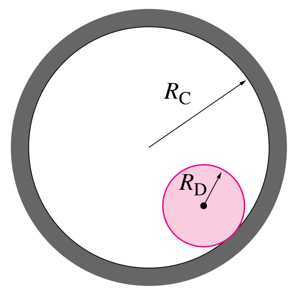
b) **FBD.** Draw a free body diagram of the disk.

c) **Momentum balance.** Write the equations of linear and angular momentum balance for the disk. Use the point on the cylinder which touches the disk for the angular momentum balance equation. Leave as unknown in these equations variables which you do not know.

d) **Kinematics.** The disk rolling in the cylinder is a *one*-degree-of-freedom system. That is, the values of only *one* coordinate and its derivatives are enough to determine the positions, velocities and accelerations of all points. The angle that the line from the center of the cylinder to the center of the disk makes from the vertical can be used as such a variable. Find all of the velocities and accelerations needed in the momentum balance equation in terms of this variable and it's derivative. [Hint: you'll need to think about the rolling contact in order to do this part.]

e) **Equation of motion.** Write the angular momentum balance equation as a single second order differential equation.

f) **Simple pendulum?** Does this equation reduce to the equation for a pendulum with a point mass and length equal to the radius of the cylinder, when the disk radius gets arbitrarily small? Why, or why not?



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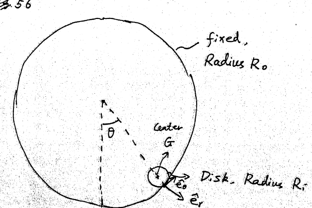
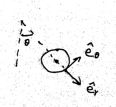
Problem 14.23: A disk rolls without slip inside a bigger cylinder.

14
~~13~~.56





G is doing a circular motion with a Radius $(R_o - R_i)$

$$\vec{V}_G = \dot{\theta}(R_o - R_i)\hat{e}_\theta$$

$$\vec{a}_G = \ddot{\theta}(R_o - R_i)\hat{e}_\theta - \dot{\theta}^2(R_o - R_i)\hat{e}_r$$

No slip condition

$\Rightarrow$ Angular velocity of the disk

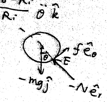
$$\vec{V}_G = -\omega R_i \hat{e}_\theta \qquad (\omega > 0 \text{ if counter clockwise})$$

$$\omega = -\frac{R_o - R_i}{R_i}\dot{\theta}$$

Angular acceleration

$$\dot{\omega} = -\frac{R_o - R_i}{R_i}\ddot{\theta}, \quad \dot{\vec{\omega}} = \dot{\omega}\hat{k} = -\frac{R_o - R_i}{R_i}\ddot{\theta}\hat{k}$$

Take AMB about the contact point E.



$$\dot{\vec{H}}_{/E} = \vec{r}_{G/E} \times m\vec{a}_G + I_G\dot{\vec{\omega}}$$

$$= (-R_i\hat{e}_r) \times m(\ddot{\theta}(R_o - R_i)\hat{e}_\theta - \dot{\theta}^2(R_o - R_i)\hat{e}_r) + \frac{1}{2}mR_i^2\left(-\frac{R_o - R_i}{R_i}\ddot{\theta}\right)\hat{k}$$

$$= -m\ddot{\theta}R_i(R_o - R_i)\hat{k} - \frac{1}{2}m(R_o - R_i)R_i\ddot{\theta}\hat{k}$$

$$= -\frac{3}{2} m R_i (R_o - R_i) \ddot{\theta} \hat{k}$$

$$\Sigma \vec{M}_{/E} = \vec{r}_{G/E} \times (-mg\hat{j})$$

$$= (-R_i \hat{e}_r) \times (-mg\hat{j}) = mg R_i \sin\theta \hat{k}$$

AMB : $\Sigma \vec{M}_{/E} = \dot{\vec{H}}_{/E} \Rightarrow mgR_i \sin\theta \hat{k} = -\frac{3}{2} mR_i (R_o - R_i) \ddot{\theta} \hat{k}$

$$\Rightarrow \frac{3}{2} m R_i (R_o - R_i) \ddot{\theta} + mg R_i \sin\theta = 0$$

$$\Rightarrow \boxed{\ddot{\theta} + \frac{2g}{3(R_o - R_i)} \sin\theta = 0}$$

If the disk radius gets arbitrarily small, then $R_i = 0$

The equation of motion becomes

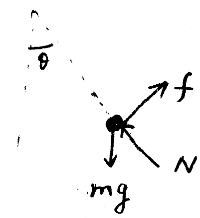
$$\ddot{\theta} + \frac{2g}{3R_o} \sin\theta = 0$$

This equation does not reduce to the simple pendulum equation, which should be $\ddot{\theta} + \frac{g}{R_o} \sin\theta = 0$

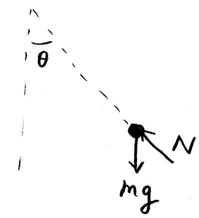
Reason: As the radius of the disk, R_i, goes to 0, our problem is NOT the same as a simple pendulum.
To enforce no slip condition, there must be friction acting the the object. However, for a simple pendulum, the reaction force should be in normal direction.
The difference can be illustated in the FBD's below

[redacted]

Our problem when $R_i \to 0$:



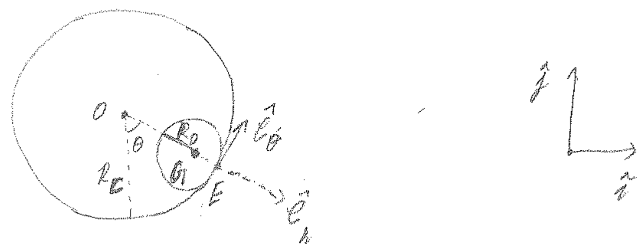
Simple pendulum.



The existence of friction force in our problem makes it different from simple pendulum problem

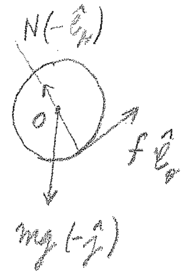
14.56

(a)



(b) During rolling, there is no kinetic friction but there is static friction.

FBD:



(c) LMB: $\quad mg(-\hat{j}) + f(\hat{e}_\theta) + N(-\hat{e}_r) = m\,\vec{a}_G$

AMB: $\sum \vec{M}_{/E} = \dot{\vec{H}}_{/E} = \vec{r}_{G/E} \times m\,\vec{a}_G + I\dot{\omega}\,\hat{k}$

(d) Kinematics: G is moving in a circle of radius $(R_C - R_D)$

$$\vec{v}_G = \dot{\theta}\,(R_C - R_D)\,\hat{e}_\theta \qquad \text{— (i)}$$

$$\Rightarrow \quad \vec{a}_G = \ddot{\theta}\,(R_C - R_D)\,\hat{e}_\theta - \dot{\theta}^2\,(R_C - R_D)\,\hat{e}_r \qquad \text{— (ii)}$$

(Remember: $\dot{\theta} \neq \omega$. Here, ω refers to rotation of the disc.)

Rolling w/o slip:

$$\vec{v}_G = -\omega(\hat{k}) \times R_D(\hat{e}_r) = -\omega R_D \hat{e}_\theta$$

using (i): $\omega = -\dfrac{R_C - R_D}{R_D}\dot{\theta}$

(e) Now substitute $\vec{a}_G$ from (ii) in AMB,

& $\vec{r}_{G/E} = R_D(-\hat{e}_R)$

& $\sum M_{/E} = \vec{r}_{G/E} \times mg(-\hat{j}) = mgR_D \sin\theta\, \hat{k}$

(after some algebra ....)

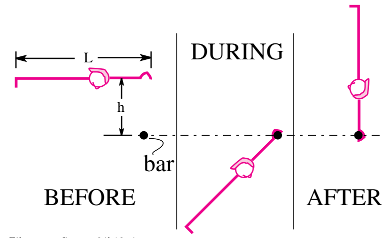
$$\boxed{\ddot{\theta} + \frac{2g}{3(R_c - R_D)}\sin\theta = 0}$$

(f) If $R_D \to 0$, $\ddot{\theta} = -\dfrac{2}{3}\dfrac{g}{R_c}\sin\theta$

which is not the same as $\ddot{\theta} = -\dfrac{g}{R_c}\sin\theta$, the equation for simple pendulum.

This is because even at small R_c, there is rolling! The function for $\ddot{\theta}$ will not abruptly jump by continuously varying R_D.

14.5.8 An acrobat modeled as a rigid body with uniform rigid mass m of length l. She falls without rotation in the position shown from height h where she was stationary. She then grabs a bar with a firm but slippery grip. What is h so that after the subsequent motion the acrobat ends up in a stationary handstand? [Hint: What quantities are preserved in what parts of the motion?]



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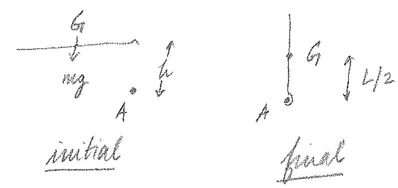
Problem 14.8

14.65



* NOTE: Cannot simply use conservation of energy because there is loss upon impact. However, angular momentum is conserved during impact.

$$\vec{H}_{/A}^{-} = m\sqrt{2gh}\,\frac{L}{2}\,\hat{k} \qquad \text{(just before impact)}$$

$$\vec{H}_{/A}^{+} = I_A^{zz}\,\omega_1\,\hat{k} \qquad \text{(just after impact)}$$

$$= \frac{mL^2}{3}\,\omega_1\,\hat{k}$$

$\Rightarrow$ (just after impact) $\omega_1 = \frac{3}{2}\frac{\sqrt{2gh}}{L}$

Since the pivot is "slippery", energy is conserved during the swing:

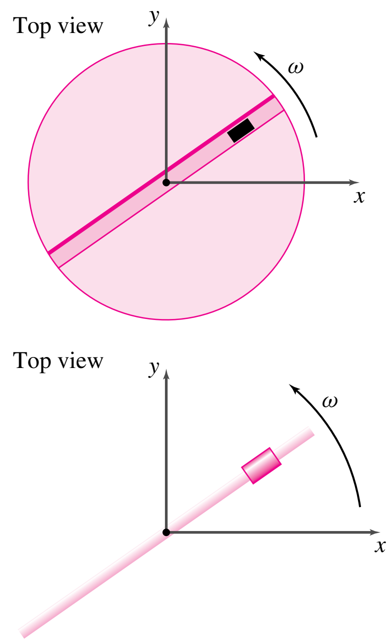
$$E_i = E_f$$

$$\frac{1}{2} I_A^{zz}\omega_1^2 = mg\frac{L}{2}$$

$$\Rightarrow \frac{mL^2}{3}\left(\frac{3}{2}\frac{\sqrt{2gh}}{L}\right)^2 = \frac{mgL}{2}$$

$$\boxed{h = \frac{2}{3}l}$$

15.1.5 Picking apart the polar coordinate formula for velocity. This problem concerns a small mass m that sits in a slot in a turntable. Alternatively you can think of a small bead that slides on a rod. The mass always stays in the slot (or on the rod). Assume the mass is a little bug that can walk as it pleases on the rod (or in the slot) and you control how the turntable/rod rotates. Name two situations in which one of the terms is zero but the other is not in the two term polar coordinate formula for velocity, $\dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$. You should thus gain some insight into the meaning of each of the two terms in that formula.



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Problem 15.5

15.5

$\vec{V} = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$

If $\dot{R} = 0$

$\vec{V} = R\dot{\theta}\hat{e}_\theta$

which means velocity only depends on R and the angular speed of the turnable/rod in $\hat{e}_\theta$

If $R = 0$ or the small mass in center of rotation.

$\vec{V} = \dot{R}\hat{e}_R$

which means velocity only depends on the rate of change of R in the direction of $\hat{e}_R$

15.1.6 Picking apart the polar coordinate formula for acceleration. Reconsider the configurations in problem 15.1.5. This time, name four situations in which all of the terms, but one, in the four term polar coordinate formula for acceleration, $\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$, are zero. Each situation should pick out a different term. You should thus gain some insight into the meaning of each of the four terms in that formula.

15.6

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{e}_\theta$$

If $R=0$, $\dot{\theta}=0$, $\dot{R}=0$, $\ddot{\theta}=0$

$\vec{a} = \ddot{R}\hat{e}_R$

which means when mass is in center of rotation, angular speed is zero and angular acceleration is also zero, the acceleration of mass only depends on $\ddot{R}$.

If $\ddot{R}=0$, $\dot{R}=0$, $\ddot{\theta}=0$

$\vec{a} = -R\dot{\theta}^2\hat{e}_R$

which means when R stays constant and angular acceleration is zero, the acceleration of mass only depends on angular velocity and R.

If $\ddot{R}=0$, $R=0$, $\ddot{\theta}=0$

$\vec{a} = 2\dot{R}\dot{\theta}\hat{e}_\theta$

which means when mass is in center of rotation, $\ddot{R}=0$, and angular velocity is zero, the acceleration of mass only depends on rate of change of R and angular velocity

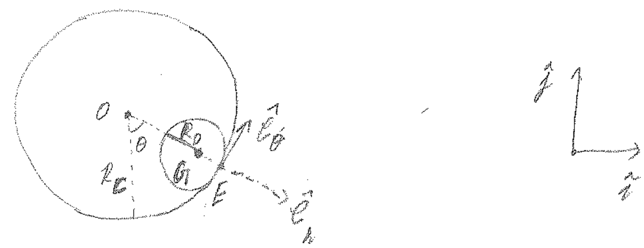
If $\ddot{R}=0$, $\dot{\theta}=0$, $\dot{R}=0$

$\vec{a} = R\ddot{\theta}\hat{e}_\theta$

which means when R stays constant and angular velocity is zero, the acceleration of mass only depends on angular acceleration

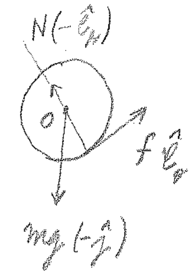
14.56

(a)



(b) During rolling, there is no kinetic friction but there is static friction.

FBD:



(c) LMB: $mg(-\hat{j}) + f(\hat{e}_\theta) + N(-\hat{e}_r) = m\vec{a}_G$

AMB: $\sum \vec{M}_{/E} = \dot{\vec{H}}_{/E} = \vec{r}_{G/E} \times m\vec{a}_G + I\dot{\omega}\hat{k}$

(d) Kinematics: G is moving in a circle of radius $(R_C - R_D)$

$$\vec{V}_G = \dot{\theta}(R_C - R_D)\hat{e}_\theta \qquad \text{— (i)}$$

$$\Rightarrow \vec{a}_G = \ddot{\theta}(R_C - R_D)\hat{e}_\theta - \dot{\theta}^2(R_C - R_D)\hat{e}_r \qquad \text{— (ii)}$$

(Remember: $\dot{\theta} \neq \omega$. Here, ω refers to rotation of the disc.)

Rolling w/o slip :

$$\vec{V}_G = -\omega(\hat{k}) \times R_D(\hat{e}_r) = -\omega R_D \hat{e}_\theta$$

Using (i) : $\omega = -\dfrac{R_C - R_D}{R_D}\dot{\theta}$

(e) Now substitute $\vec{a}_G$ from (ii) in AMB,

& $\vec{r}_{G/E} = R_D(-\hat{e}_R)$

& $\sum M_{/E} = \vec{r}_{G/E} \times mg(-\hat{j}) = mg R_D \sin\theta\, \hat{k}$

(after some algebra ....)

$$\boxed{\ddot{\theta} + \frac{2g}{3(R_C - R_D)}\sin\theta = 0}$$

(f) If $R_D \to 0$, $\ddot{\theta} = -\dfrac{2}{3}\dfrac{g}{R_C}\sin\theta$

which is not the same as $\ddot{\theta} = -\dfrac{g}{R_C}\sin\theta$,

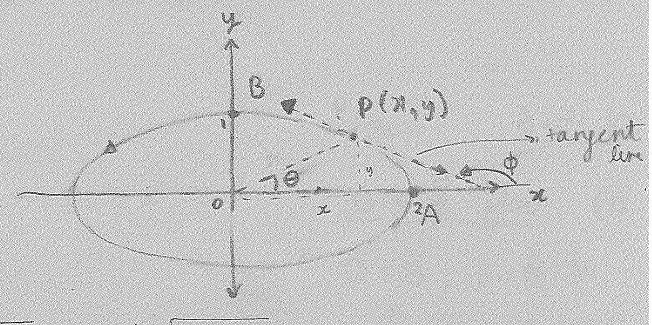
the equation for simple pendulum.

This is because even at small R_C, there is rolling! The function for $\ddot{\theta}$ will not abruptly jump by continuously varying R_D.

15.1.10 A particle travels at non-constant speed on an elliptical path given by $y^2 = b^2(1 - \frac{x^2}{a^2})$. Carefully sketch the ellipse for particular values of a and b. For various positions of the particle on the path, sketch the position vector $\vec{r}(t)$; the polar coordinate basis vectors $\hat{e}_r$ and $\hat{e}_\theta$; and the path coordinate basis vectors $\hat{e}_n$ and $\hat{e}_t$. At what points on the path are $\hat{e}_r$ and $\hat{e}_n$ parallel (or $\hat{e}_\theta$ and $\hat{e}_t$ parallel)?

15.10 $\quad y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$

- let $a = 2$ $\quad b = 1$
  then ellipse is as shown



for a point P(x, y) as shown

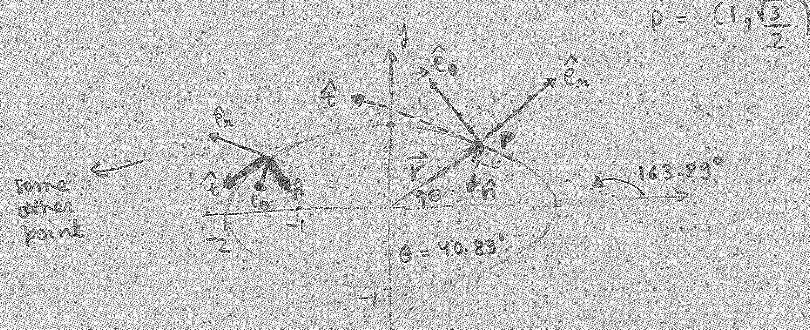
- $y = b\sqrt{1 - \frac{x^2}{a^2}} = \frac{b}{a}\sqrt{a^2 - x^2} = \frac{1}{2}\sqrt{4 - x^2}$
- $\tan\theta = \frac{y}{x} = \frac{1}{2x}\sqrt{4 - x^2}$ = slope of $\hat{e}_r$
- slope of tangent line = slope of $\hat{t}$ = $\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{2}\frac{-2x}{\sqrt{4-x^2}} = \frac{-x}{2\sqrt{4-x^2}} = \tan\phi$

sketch

- let $P = \left(1, \frac{1}{2}\sqrt{4 - 1^2}\right) = \left(1, \frac{\sqrt{3}}{2}\right)$
- $\tan\theta = \frac{\sqrt{3}}{2} \Longrightarrow \theta = 40.89^\circ$
- $\tan\phi = \frac{-1}{2\sqrt{4-1}} = \frac{-1}{2\sqrt{3}} \Longrightarrow \phi = 163.89^\circ$

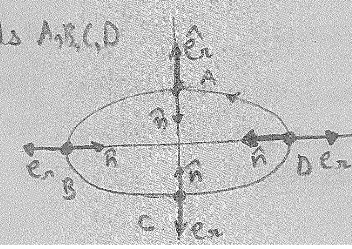
now we can draw

$P = \left(1, \frac{\sqrt{3}}{2}\right)$



$\hat{e}_r \parallel \hat{e}_n$ or $\hat{e}_r \perp \hat{e}_t$

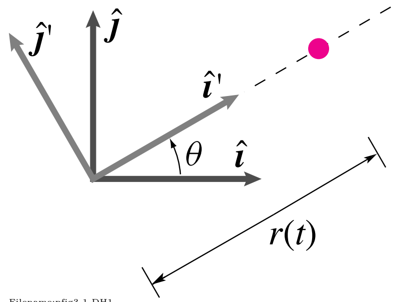
- intuitively they are parallel at following 4 points A, B, C, D
  (in mind: just try to span on the whole of ellipse
  $\vec{e}_r$ and $\hat{e}_t$ and see where they are ⊥)
  ie see when line to origin is ⊥ to tangent)



15.2.5 Given that $\vec{r}(t) = ct^2\hat{\imath}'$ and that $\theta(t) = d\,\sin(\lambda t)$, find $\vec{v}(t)$

a) in terms of $\hat{\imath}$ and $\hat{\jmath}$,

b) in terms of $\hat{\imath}'$ and $\hat{\jmath}'$.



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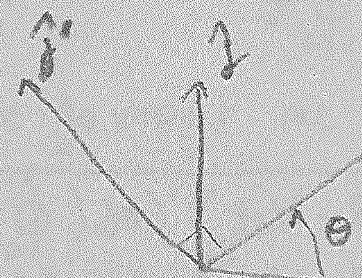
Problem 15.5

Q15.15

b) $\vec{r}(t) = ct^2\,\hat{\imath}'$



$\vec{v}(t) = \dot{\vec{r}}(t) = 2ct\,\hat{\imath}' + ct^2\,\dot{\hat{\imath}}'$

$= 2ct\,\hat{\imath}' + ct^2\,\dot{\theta}\,\hat{\jmath}'$

$$\boxed{\vec{V} = 2ct\,\hat{\imath}' + d\lambda ct^2\cos(\lambda t)\,\hat{\jmath}'}$$

a) now $\hat{\imath}' = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{\jmath}' = -\sin\theta\,\hat{\imath} \quad \cos\theta\,\hat{\jmath}$

using these in above we get

$$\boxed{\vec{V} = (2ct\cos\theta - d\lambda ct^2\sin\theta)\,\hat{\imath} + (2c}$$

where $\theta = d\sin(\lambda t)$

15.3.2 Actual path of bug trying to walk a straight line. A straight line is inscribed on a horizontal turntable. The line goes through the center. Let ϕ be angle of rotation of the turntable which spins at constant rate $\dot{\phi}_0$. A bug starts on the outside edge of the turntable of radius R and walks towards the center, passes through it, and continues to the opposite edge of the turntable. The bug walks at a constant speed v_A, as measured by how far her feet move per step, on the line inscribed on the table. Ignore gravity.

a) **Picture.** Make an accurate drawing of the bug's path as seen in the room (which is not rotating with the turntable). In order to make this plot, you will need to assume values of v_A and $\dot{\phi}_0$ and initial values of R and ϕ. You will need to write a parametric equation for the path in terms of variables that you can plot (probably x and y coordinates). You will also need to pick a range of times. Your plot should include the instant at which the bug walks through the origin. Make sure your x and y- axes are drawn to the same scale. A computer plot would be nice.

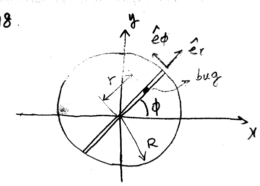
b) Calculate the radius of curvature of the bug's path as it goes through the origin.

c) Accurately draw (say, on the computer) the osculating circle when the bug is at the origin on the picture you drew for (a) above.

d) **Force.** What is the force on the bugs feet from the turntable when she starts her trip? Draw this force as an arrow on your picture of the bug's path.

e) **Force.** What is the force on the bugs feet when she is in the middle of the turntable? Draw this force as an arrow on your picture of the bug's path.

15.18



Assume at time t, the distance between the bug and the center is r(t), and the table has turned an angle of $\phi(t)$

Initially, $r(t=0) = R$
$\phi(t=0) = 0$

a). The position of bug at time t is

$$\begin{cases} x(t) = r\cos\phi \\ y(t) = r\sin\phi \end{cases} \quad \text{where} \quad \begin{cases} r(t) = R - V_A t \\ \phi(t) = \dot{\phi}_0 t \end{cases}$$

so
$$\begin{cases} x(t) = (R - V_A t)\cos(\dot{\phi}_0 t) \\ y(t) = (R - V_A t)\sin(\dot{\phi}_0 t) \end{cases}$$

Let $R = 1\,m$, $V_A = 0.2\,m/s$, $\dot{\phi}_0 = 1\,rad/s$. If we choose a sequence of number, then we can get a sequence of (x, y) using the parametric equation above. and can plot the trajectory. See attached Matlab code.

b). The velocity of the bug is

$$\vec{V} = \dot{\vec{r}} = \frac{d}{dt}(r(t)\,\hat{e}_r) = \dot{r}(t)\,\hat{e}_r + r(t)\,\dot{\hat{e}}_r$$

$$= -V_A\,\hat{e}_r + (R - V_A t)\,\vec{\omega}\times\hat{e}_r$$

$$= -V_A\,\hat{e}_r + (R - V_A t)\,\dot{\phi}_0\,\hat{e}_\phi$$

($\hat{e}_r, \hat{e}_\phi$ are polar coordinate unit vector shown in the picture)

The acceleration is

$$\vec{a} = \dot{\vec{v}} = -2V_A \dot{\phi}_o \hat{e}_\phi - (R - V_A t)\dot{\phi}_o^2 \hat{e}_r$$

when the bug goes through the center. $r = R - V_A t = 0$, $t = \frac{R}{V_A}$

$\therefore$ at that instant

$$\vec{v} = -V_A \hat{e}_r, \quad \vec{a} = -2V_A \dot{\phi}_o \hat{e}_\phi$$

compare this to the path coordinates expression

$$\vec{v} = v\hat{e}_t, \quad \vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

where ρ is the radius of curvature.

we get when the bug is at the origin

$$\begin{cases} v = |\vec{v}| = V_A \\ \hat{e}_t = -\hat{e}_r \\ \dot{v} = 0 \quad (\text{since } \vec{a} \perp \vec{v} \text{ at that instant}) \\ \frac{v^2}{\rho} = 2V_A \dot{\phi}_o \\ \hat{e}_n = -\hat{e}_\phi \end{cases}$$

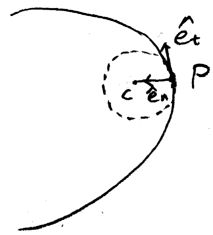
$$\Rightarrow \quad \rho = \frac{v^2}{2V_A \dot{\phi}_o} = \frac{V_A^2}{2V_A \dot{\phi}_o} = \frac{V_A}{2\dot{\phi}_o}$$

$\therefore$ The radius of curvature at origin $\boxed{\rho = \frac{V_A}{2\dot{\phi}_o}}$

c). To draw the osculating circle, we need to the center and radius. The radius is given in b).

Now we want to figure out the center.

Generally, Let's say c is the center of the osculating circle at point P.



$\therefore \quad \vec{r}_{c/p} = \rho \hat{e}_n$

In our case, P is at the origin, $\hat{e}_n = -\hat{e}_\phi$

$\therefore \quad (x_c - x_P)\hat{\imath} + (y_c - y_P)\hat{\jmath} = -\rho \hat{e}_\phi$

where $x_P = y_P = 0$

$\rho = \frac{V_A}{2\dot{\phi}_o}$, $\hat{e}_\phi = -\sin\phi \hat{\imath} + \cos\phi \hat{\jmath}$ (when the bug is at the origin, $\phi = \dot{\phi}_o t = \dot{\phi}_o \frac{R}{V_A}$)

$= -\sin(\dot{\phi}_o \frac{R}{V_A})\hat{\imath} + \cos(\dot{\phi}_o \frac{R}{V_A})\hat{\jmath}$

$$\Rightarrow \begin{cases} x_c = \frac{V_A}{2\dot{\phi}_o}\sin(\dot{\phi}_o \frac{R}{V_A}) \\ y_c = -\frac{V_A}{2\dot{\phi}_o}\cos(\dot{\phi}_o \frac{R}{V_A}) \end{cases}$$

With the position of c, we can then draw the circle. See Matlab code.

d). At the beginning, $t=0$, using the expression derived in b),

$\vec{a}_1 = -2V_A\dot{\phi}_o \hat{e}_\phi - R\dot{\phi}_o^2 \hat{e}_r$

and $\hat{e}_r = \hat{\imath}$, $\hat{e}_\phi = \hat{\jmath}$ at this time,

$\therefore \quad \vec{a}_1 = -2V_A\dot{\phi}_o \hat{\jmath} - R\dot{\phi}_o^2 \hat{\imath}$

Use LMB

$$\boxed{\vec{F}_1 = m\vec{a}_1 = -m(R\dot{\phi}_o^2 \hat{\imath} + 2V_A\dot{\phi}_o \hat{\jmath})}$$

is the force acting on the bug at the beginning.

e). When the bug is at the origin, $t = \frac{R}{V_A}$

$$\vec{a}_2 = -2V_A\dot{\phi}_0\,\hat{e}_\phi$$

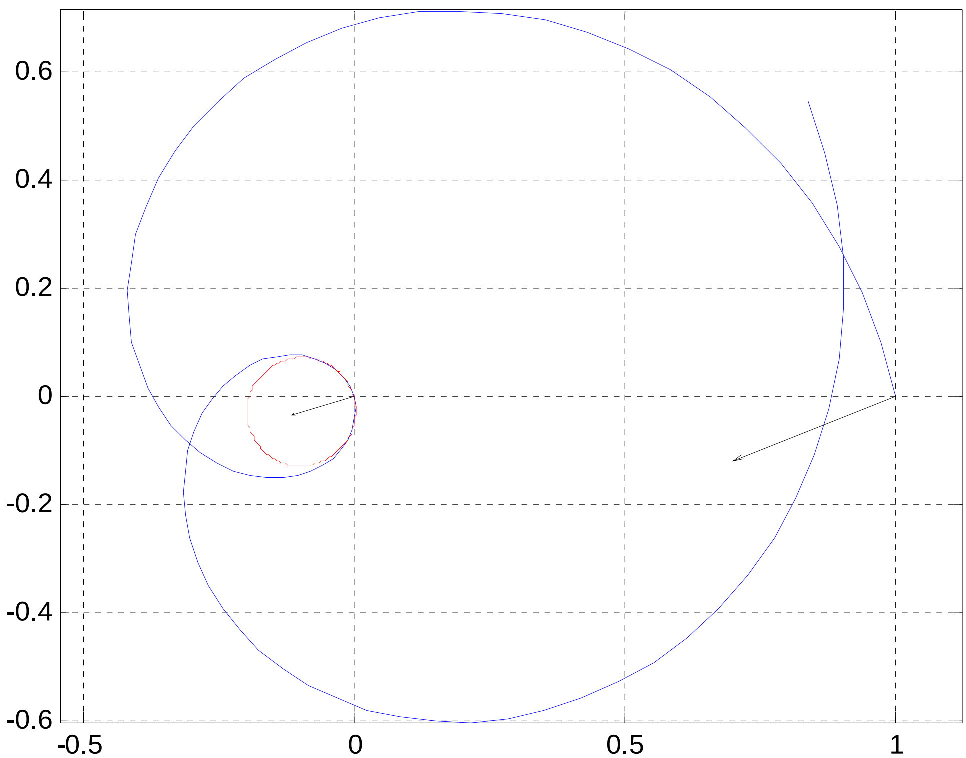
$$= -2V_A\dot{\phi}_0\left(-\sin\left(\dot{\phi}_0\frac{R}{V_A}\right)\hat{\imath} + \cos\left(\dot{\phi}_0\frac{R}{V_A}\right)\hat{\jmath}\right)$$

$$= 2V_A\dot{\phi}_0\sin\left(\dot{\phi}_0\frac{R}{V_A}\right)\hat{\imath} - 2V_A\dot{\phi}_0\cos\left(\dot{\phi}_0\frac{R}{V_A}\right)\hat{\jmath}$$

So the force on the bug at the origin is

$$\vec{F}_2 = m\vec{a}_2 = 2mV_A\dot{\phi}_0\left(\sin\left(\dot{\phi}_0\frac{R}{V_A}\right)\hat{\imath} - \cos\left(\dot{\phi}_0\frac{R}{V_A}\right)\hat{\jmath}\right)$$

```
function path1518()
%%%    draw path
R=1;    % radius of the turntable
va=0.2; % velocity of the bug on the turntable
phidot=1; % angular velocity of the turntable
t=[0:0.1:10];
x=(R-va*t).*cos(phidot*t);
y=(R-va*t).*sin(phidot*t);
plot(x,y);
axis equal;

grid on;

%%%% draw osculating circle when bug goes through the center
rau=va/(2*phidot);    % radius of curvature of the path at the origin
xc= va*sin(phidot*R/va)/(2*phidot);
yc= -va*cos(phidot*R/va)/(2*phidot); % position of the center
                                     %draw the circle;
theta=[0:0.01:2*pi];
circle1=xc+rau*cos(theta);
circle2=yc+rau*sin(theta);
hold on;
plot(circle1,circle2,'r');

%%%% draw force vector
m=1;            %mass of the bug
scale=0.3;      % scale for graphics

f1x=-m*R*phidot;
f1y=-m*2*va*phidot;
quiver(1,0,f1x,f1y,scale,'k'); % draw force at the beginning;

f2x=2*m*va*phidot*sin(phidot*R/va);
f2y=-2*m*va*phidot*cos(phidot*R/va);
quiver(0,0,f2x,f2y,scale,'k'); %draw force at the origin
```



0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.5
0
0.5
1

Q 15.18

a)

$\vec{R} = r\hat{e}_r$

$\vec{V} = \dot{r}\hat{e}_r + r\dot{\phi}_0\hat{e}_\theta$

let initially $\phi = 0$, $r = R$

$\dot{r} = -V_A$ initially $r = R$

(NOTE: by convention r is c
to be positive, here using
origin ---- but lets no wo
correct result) (?)

Hence. $r = R - V_A t$

$\phi = \dot{\phi}_0 t$

- writing equation in terms of x, y $(x = r\cos\theta,\ y = r\sin\theta)$

$$\boxed{\begin{aligned} x(t) &= (R - V_A t)\cos(\dot{\phi}_0 t) \\ y(t) &= (R - V_A t)\sin(\dot{\phi}_0 t) \end{aligned}}$$

- now we can plot these in MATLAB
  - with $R = 1\,m$ $\dot{\phi}_0 = 1$ rad/sec $V_A = 0.2$ m/s
  - time span $t = 0$ to $t = 10$ sec (it will cross origin at $t=$ when $R - V_A t = ($
- see the figure and code below.

$\boxed{\text{DEFINE / NOTE: } \hat{e}_r \hat{e}_\theta \text{ a frame moving with the turntable}}$

b) we have velocity of bug

- $\vec{V} = \underbrace{0}_{\text{velocity of origin of moving frame}} + \vec{\omega}\times\vec{r} + \vec{V}_{\text{rel to moving frame}}$

and acceleration using 5 term formula

- $\vec{a} = \underbrace{0}_{\vec{a}\text{ of origin of moving frame}} - \underbrace{\omega^2}_{\dot{\phi}_0}\underbrace{\vec{r}}_{r\hat{e}_r} + \cancel{\vec{\alpha}}^{\,0}\times\vec{r} + 2\underbrace{\vec{\omega}}_{\dot{\phi}_0}\times\vec{V}_{\text{rel to moving frame}} + \vec{a}_{\text{rel moving frame}}$

$\alpha = 0$
$\dot{\phi} = \text{constant}$

- now

→ $\vec{V}_{\text{rel to moving frame}} = -V_A\,\hat{e}_r$

→ $\vec{a}_{\text{rel to moving frame}} = 0$ $(V_A = \text{constant})$

→ $\vec{\omega} = \dot{\phi}_0\,\hat{k}$

- doing the math

$\vec{V} = -V_A\hat{e}_r + r\dot{\phi}_0\,\hat{e}_\theta$ ——— (1)

$\vec{a} = -\dot{\phi}_0^2 r\,\hat{e}_r - 2\dot{\phi}_0 V_A\,\hat{e}_\theta$ ——— (2)

when bug goes through origin $(r=0)$

$\vec{V} = -V_A \hat{e}_r$ $\longrightarrow$ (3)

$\vec{a} = -2\dot{\phi}_0 V_A \hat{e}_\theta$ $\longrightarrow$ (4)

→ we know
- $\vec{V} = |\vec{V}|\hat{t}$ $\quad \therefore \hat{t} = -\hat{e}_r$
- $\vec{a} = \dot{V}\hat{t} + \frac{V^2}{\rho}\hat{n} = -2\dot{\phi}_0 V_A \hat{e}_\theta$

→ $\hat{n}$ is $\perp$ to $\hat{t}$ (ie $-\hat{e}_r$) $\therefore$ $\hat{n} = \hat{e}_\theta$ or $-\hat{e}_\theta$, lets figure out bel

$$\therefore \quad -2\dot{\phi}_0 V_A \hat{e}_\theta = -\dot{V}\hat{e}_r + \frac{V^2}{\rho}\hat{n}$$

$\}\cdot\hat{e}_r \Rightarrow \dot{V} = 0$

$\}\cdot\hat{e}_\theta \Rightarrow \underbrace{\frac{V^2}{\rho}}_{\text{always +ve}} = 2\dot{\phi}_0 V_A$ and $\hat{n} = -\hat{e}_\theta$

Finally

just when bug crosses origin. (only at that instant!)

$$\boxed{\begin{aligned}\hat{n} &= -\hat{e}_\theta \\ \rho &= \frac{V_A^2}{2\dot{\phi}_0 V_A} = \frac{V_A}{2\dot{\phi}_0}\end{aligned}} \longrightarrow \text{radius of curvature}$$

$\boxed{\rho = .1\ m}$ using $V_A = .2$, $\dot{\phi}_0 = 1$

c)
- radius of circle is ρ
- centre of oscul. circle is along $\hat{n}$, at distance ρ from the bug (the centre of turntable) (drawn below)

$\therefore$ centre $= -\rho\,\hat{e}_\theta$

- $-\hat{e}_\theta = \sin\phi\,\hat{\imath} - \cos\phi\,\hat{\jmath}$
- at centre of turntable
- $t=5$ $\quad \phi = 5$ radians
- $\sin\phi = -.9589$
- $\cos\phi = .2837$

$\boxed{\text{centre} = (-.09589\,m,\ -.2837\,m)}$

d) by LMB $\vec{F} = m\vec{a} = m\left(-\dot{\phi}_o^2 r\, \hat{e}_r - 2\dot{\phi}_o V_A\, \hat{e}_\theta\right)$ $\xrightarrow{\text{from (2)}}$

at start $t=0$, $r=R=1\,m$, $\dot{\phi}_o = 1$ rad/s, $V_A = .2$ m/s, $\phi = 0$

$\hat{e}_r = \cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath} = \hat{\imath}$

$\hat{e}_\theta = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath} = \hat{\jmath}$

also let $m = 1$ kg (too much for a bug)

$\therefore \boxed{\vec{F} = -1\,\hat{\imath} - .4\,\hat{\jmath}}$ N

e) again by LMB $\vec{F} = m\left(-\dot{\phi}_o^2 r\,\hat{e}_r - 2\dot{\phi}_o V_A\,\hat{e}_\theta\right)$

at centre $t=5$, $r=0$, $\dot{\phi}_o = 1$, $V_A = .2$, $\phi = 5$ rad

$\vec{F} = -2\times .2\,\hat{e}_\theta = \boxed{-.4\,\hat{e}_\theta}$ N

$\hat{e}_\theta = +.9589\,\hat{\imath} + .2837\,\hat{\jmath}$

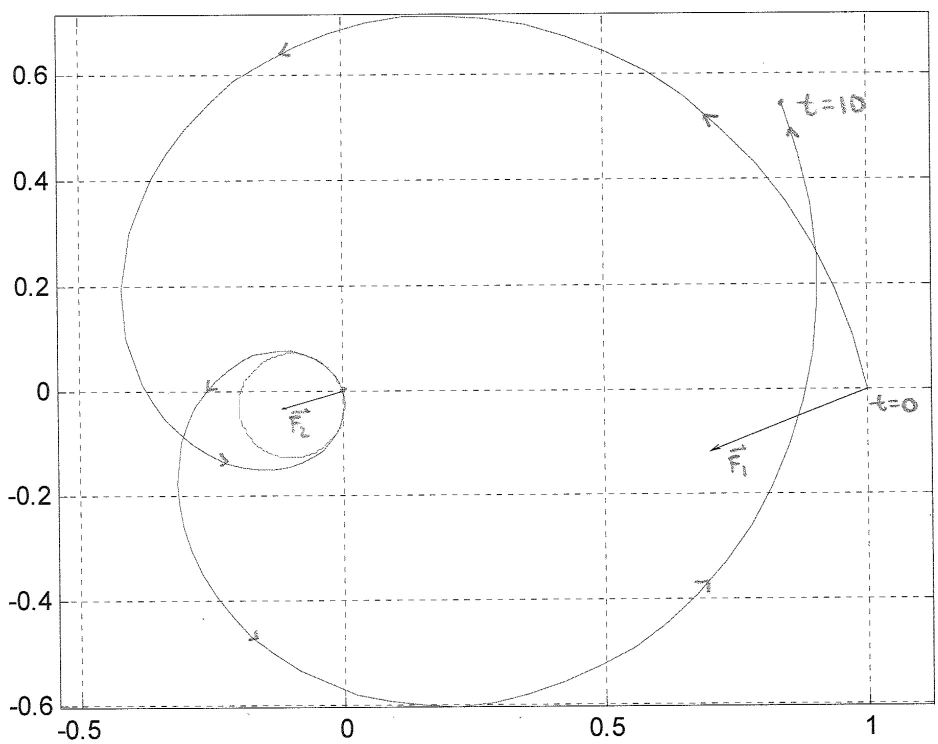
$\therefore \boxed{\vec{F} = -.3836\,\hat{\imath} - .1135\,\hat{\jmath} \text{ N}}$

both are ~~drwc~~ drawn in figure.

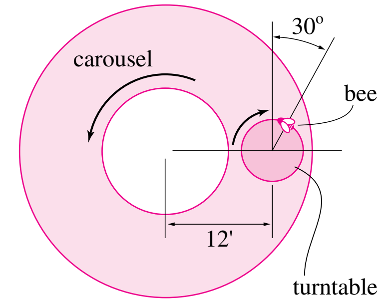
CODE

```
function path1518()
%%%    draw path
R=1;    % radius of the turntable
va=0.2; % velocity of the bug on the turntable
phidot=1; % angular velocity of the turntable
t=[0:0.1:10];
x=(R-va*t).*cos(phidot*t);
y=(R-va*t).*sin(phidot*t);
plot(x,y);
axis equal;

grid on;

%%%% draw osculating circle when bug goes through the center
rau=va/(2*phidot);    % radius of curvature of the path at the origin
xc= va*sin(phidot*R/va)/(2*phidot);
yc= -va*cos(phidot*R/va)/(2*phidot); % position of the center
                                       %draw the circle:    % R/va is time taken to reach origin
theta=[0:0.01:2*pi];
circle1=xc+rau*cos(theta);                                % phidot.R/va = angle of table when bug is
circle2=yc+rau*sin(theta);                                                 at origin
hold on;
plot(circle1,circle2,'r');

%%%% draw force vector
m=1;           %mass of the bug
scale=0.3;     % scale for graphics

f1x=-m*R*phidot;
f1y=-m*2*va*phidot;
quiver(1,0,f1x,f1y,scale,'k'); % draw force at the beginning:

f2x=2*m*va*phidot*sin(phidot*R/va);
f2y=-2*m*va*phidot*cos(phidot*R/va);
quiver(0,0,f2x,f2y,scale,'k'); %draw force at the origin
```

PLOTS

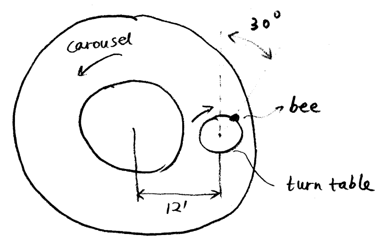


15.3.11 A honeybee, sensing that it can get a cheap thrill, alights on a phonograph turntable that is being carried by a carnival goer who is riding on a carousel. The situation is sketched below. The carousel has angular velocity of 5 rpm, which is increasing (accelerating) at 10 rev/min^2; the phonograph rotates at a constant 33 1/3 rpm. The honeybee is at the outer edge of the phonograph record in the position shown in the figure; the radius of the record is 7 inches. Calculate the magnitude of the acceleration of the honeybee.



Problem 15.11

15.27

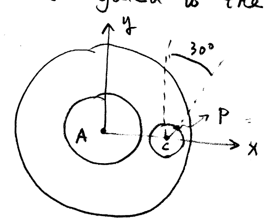


At this instant, the carousel has angular velocity of $\omega_c = 5\,rpm = 10\pi$ rad/mi

angular acceleration $\alpha_c = 10\ r/min^2 = 20\pi$ rad/min^2

The turntable rotates at $\omega_t = 33\frac{1}{3}\ rpm = 66\frac{2}{3}\pi$ rad/min, $\alpha_t = 0$

Use the moving frame glued to the carousel. Let's call it $\mathcal{B}$



where the bee is

$\vec{\omega}_{\mathcal{B}} = \omega_c \hat{k} = 10\pi$ rad/min

$\dot{\vec{\omega}}_{\mathcal{B}} = \alpha_c \hat{k} = 20\pi$ rad/min^2

$$\vec{a}_P = \vec{a}_A + \vec{a}_{P/\mathcal{B}} - \omega_{\mathcal{B}}^2 \vec{r}_{P/A} + \dot{\vec{\omega}}_{\mathcal{B}} \times \vec{r}_{P/A} + 2\vec{\omega}_{\mathcal{B}} \times \vec{v}_{P/\mathcal{B}}$$

i) $\vec{a}_A = 0$ since A is fixed

ii) $\vec{a}_{P/\mathcal{B}} = -\omega_t^2 \vec{r}_{P/C} = -(66\frac{2}{3}\pi)^2\, 7\,(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j})$ in/min^2

Since P rotates with the turntable at a constant angular velocity.

$\therefore\ \vec{a}_{P/\mathcal{B}} = (-1.5353\times10^5\,\hat{i} - 2.6952\times10^5\,\hat{j})$ in/min^2

iii) $-\omega_{\mathcal{B}}^2 \vec{r}_{P/A} = -\omega_{\mathcal{B}}^2 (\vec{r}_{P/C} + \vec{r}_{C/A})$

$$= -(10\pi)^2\left(\frac{7}{2}\hat{i} + \frac{7\sqrt{3}}{2}\hat{j} + 12\times 12\,\hat{i}\right)$$

$$= -1.4558\times 10^5\,\hat{i} - 5.9831\times 10^3\,\hat{j} \quad \text{in/min}^2.$$

iv) $\dot{\vec{\omega}}_B \times \vec{r}_{P/A}$

$$= 20\pi\,\hat{k} \times \left[\left(\frac{7}{2} + 144\right)\hat{i} + \frac{7\sqrt{3}}{2}\hat{j}\right]$$

$$= -380.898\,\hat{i} + 9.2677\times 10^3\,\hat{j} \quad \text{in/min}^2$$

v) $2\vec{\omega}_B \times \vec{v}_{P/B}$

$$\vec{v}_{P/B} = (-\omega_t\hat{k}) \times \vec{r}_{P/C}$$

$$\therefore\ 2\vec{\omega}_B \times \vec{v}_{P/B} = 2\,\omega_C\,\omega_t\,\vec{r}_{P/C}$$

$$= 2\times(10\pi)\times\left(66\tfrac{2}{3}\pi\right)\times\left(\frac{7}{2}\hat{i} + \frac{7\sqrt{3}}{2}\hat{j}\right)$$

$$= 4.6058\times 10^4\,\hat{i} + 7.9775\times 10^4\,\hat{j} \quad \text{in/min}^2$$

Sum all the five terms up.

$$\vec{a}_P = (-2.5343\times 10^5\,\hat{i} - 1.8646\times 10^5\,\hat{j}) \quad \text{in/min}^2$$

$\therefore$ the magnitude of acceleration

$$\boxed{\begin{aligned}|\vec{a}_P| &= 3.1464\times 10^5 \ \text{in/min}^2\\ &= 87.4 \ \text{in/sec}^2\end{aligned}}$$

15.4.1 Slider crank kinematics (No FBD required!). 2-D . Assume $R, \ell, \theta, \dot\theta, \ddot\theta$ are given. The crank mechanism parts move on the xy plane with the x direction being along the piston. Vectors should be expressed in terms of $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ components.

a) What is the angular velocity of the crank OA?

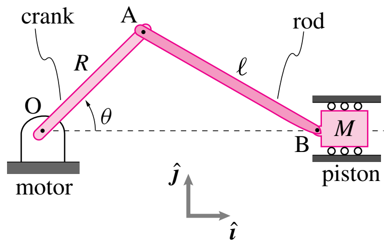
b) What is the angular acceleration of the crank OA?

c) What is the velocity of point A?

d) What is the acceleration of point A?

e) What is the angular velocity of the connecting rod AB? [Geometry fact: $\vec{r}_{AB} = \sqrt{\ell^2 - R^2\sin^2\theta}\ \hat{\imath} - R\sin\theta\,\hat{\jmath}$]

f) For what values of θ is the angular velocity of the connecting rod AB equal to zero (assume $\dot\theta \neq 0$)? (you need not answer part (e) correctly to answer this question correctly.)



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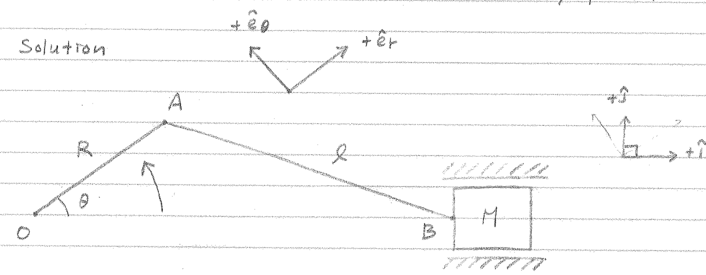
Problem 15.1

Sec 205

You Won Park
TAM 2030
10:10 AM Sec 205
TA: Pranav Bhounsule
HW 22, Apr. 14, 2009

15.29 Solution



$R, \ell, \theta, \dot\theta, \ddot\theta$ given

a) Angular velocity of OA?

$$\boxed{\vec{w}_{OA} = \dot\theta \hat{k}}$$

b) Angular acceleration of OA?

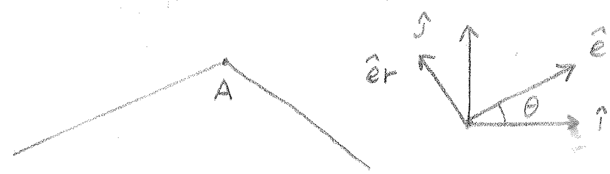
$$\boxed{\dot{\vec{w}}_{OA} = \ddot\theta \hat{k}}$$

c) Velocity of A?

$$\vec{v}_A = \dot\theta R\,\hat{e}_\theta = \boxed{\dot\theta R(-\sin\theta\,\hat\imath + \cos\theta\,\hat\jmath)}$$

d) Acceleration of A?

$$\vec{a}_A = \dot{\vec{v}}_A = \boxed{(-\ddot\theta R\sin\theta - \dot\theta^2 R\cos\theta)\hat\imath + (\ddot\theta R\cos\theta - \dot\theta^2 R\sin\theta)\hat\jmath}$$



e) Angular velocity of AB?

$$\vec{r}_{AB} = \sqrt{\ell^2 - R^2\sin^2\theta}\,\hat{\imath} - R\sin\theta\,\hat{\jmath}$$

$$\vec{v}_A = \vec{r}_{AB} \times \vec{\omega}_{AB}$$

$$= \left(\sqrt{\ell^2 - R^2\sin^2\theta}\,\hat{\imath} - R\sin\theta\,\hat{\jmath}\right) \times \omega_{AB}\hat{k}$$

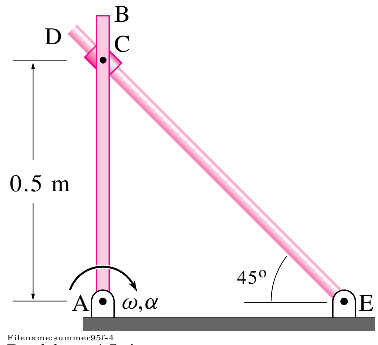
$$\left\{-R\dot{\theta}\sin\theta\,\hat{\imath} + R\dot{\theta}\cos\theta\,\hat{\jmath} = -\omega_{AB}\sqrt{\ell^2 - R^2\sin^2\theta}\,\hat{\jmath} - \omega_{AB}R\sin\theta\,\hat{\imath}\right\}$$

$$\{\}\cdot\hat{\jmath} \Rightarrow R\dot{\theta}\cos\theta = -\omega_{AB}\sqrt{\ell^2 - R^2\sin^2\theta}$$

$$\therefore \boxed{\vec{\omega}_{AB} = -\frac{R\dot{\theta}\cos\theta}{\sqrt{\ell^2 - R^2\sin^2\theta}}\hat{k}}$$

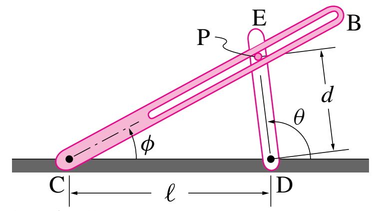
f) for $\boxed{\theta = \pm 90^\circ}$

15.4.4 The two rods AB and DE, connected together through a collar C, rotate in the vertical plane. The collar C is pinned to the rod AB but is free to slide on the frictionless rod DE. At the instant shown, rod AB is rotating clockwise with angular speed $\omega = 3\,\text{rad/s}$ and angular acceleration $\alpha = 2\,\text{rad/s}^2$. Find the angular velocity of rod DE.



Problem 15.4

15.4.10 The slotted link CB is driven in an oscillatory motion by the link ED which rotates about D with constant angular velocity $\dot{\theta} = \omega_D$. The pin P is attached to ED at fixed radius d and engages the slot on CB as shown. Find the angular velocity and acceleration $\dot{\phi}$ and $\ddot{\phi}$ of CB when $\theta = \pi/2$.



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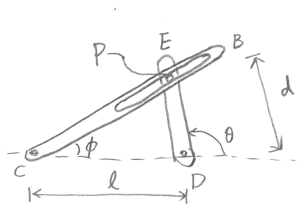
Problem 15.10

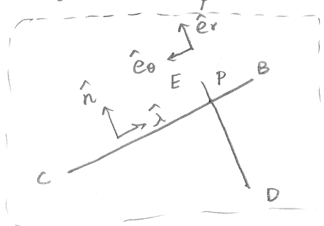
15.38



P is pinned on DE and can slide along CB, $\dot{\theta} = \omega_D$

Find $\dot{\phi}$, $\ddot{\phi}$ when $\theta = \frac{\pi}{2}$

i) Find $\dot{\phi}$



Build up two local coordinate system $\hat{e}_\theta, \hat{e}_r$ and $\hat{\lambda}, \hat{n}$.

Call the moving frame attaching to CB, $\mathcal{B}$

the moving frame attaching to DE, $\mathcal{D}$

(a)

$$\vec{V}_P = \vec{V}_P$$

$$\cancelto{0}{\vec{V}_C} + \vec{V}_{P/\mathcal{B}} + \vec{\omega}_{\mathcal{B}} \times \vec{r}_{P/C} = \cancelto{0}{\vec{V}_D} + \cancelto{0}{\vec{V}_{P/\mathcal{D}}} + \vec{\omega}_{\mathcal{D}} \times \vec{r}_{P/D}$$

$$\Rightarrow \quad V_{P/\mathcal{B}}\,\hat{\lambda} + \dot{\phi}\,\hat{k} \times r_{P/C}\,\hat{\lambda} = \omega_D\,\hat{k} \times d\,\hat{e}_r$$

when $\theta = \frac{\pi}{2}$, $\quad r_{P/C} = \frac{l}{\cos\phi}$

$$\Rightarrow \left\{ V_{P/\mathcal{B}}\,\hat{\lambda} + \dot{\phi}\,\frac{l}{\cos\phi}\,\hat{n} = \omega_D\, d\,\hat{e}_\theta \right\}$$

$$\{\,\}\cdot\hat{n} \quad \Rightarrow \quad \dot{\phi}\,\frac{l}{\cos\phi} = \omega_D\, d\,(\hat{e}_\theta \cdot \hat{n})$$

$$\{\,\}\cdot\hat{\lambda} \quad \Rightarrow \quad V_{P/\mathcal{B}} = \omega_D\, d\,(\hat{e}_\theta \cdot \hat{\lambda})$$

when $\theta = \frac{\pi}{2}$, $\quad \hat{e}_\theta \cdot \hat{n} = \sin\phi$

$$\hat{e}_\theta \cdot \hat{\lambda} = -\cos\phi$$

$\therefore \quad \dot{\phi} = \omega_D \frac{d}{\ell} \cos\phi \sin\phi \quad , \quad \vec{V}_{P/B} = -\omega_D d \cos\phi \hat{\lambda}$

$\because \quad \cos\phi = \frac{\ell}{\sqrt{d^2+\ell^2}} , \quad \sin\phi = \frac{d}{\sqrt{d^2+\ell^2}} \quad$ when $\theta = \frac{\pi}{2}$, $\Rightarrow \quad \boxed{\dot{\phi} = \omega_D \frac{d^2}{d^2+\ell^2}}$

(b) $\quad \vec{a}_P = \vec{a}_P$ $\quad$ (Note: $\dot{\vec{\omega}}_D = 0$ because $\dot{\theta} = \omega_D =$ constant)

$$\cancelto{0}{\vec{a}_C} + \vec{a}_{P/B} + \dot{\vec{\omega}}_B \times \vec{r}_{P/C} - \omega_B^2 \vec{r}_{P/C} + 2\vec{\omega}_B \times \vec{V}_{P/B} = \cancelto{0}{\vec{a}_D} + \cancelto{0}{\vec{a}_{P/D}} + \cancelto{0}{\dot{\vec{\omega}}_D \times \vec{r}_{P/D}} - \omega_D^2 \vec{r}_{P/D} + 2\vec{\omega}_D \times \cancelto{0}{\vec{V}_{P/D}}$$

$\Rightarrow$

$$a_{P/B}\hat{\lambda} + \ddot{\phi}\hat{k} \times \frac{\ell}{\cos\phi}\hat{\lambda} - \dot{\phi}^2 \frac{\ell}{\cos\phi}\hat{\lambda} + 2\dot{\phi}\hat{k} \times (-\omega_D d\cos\phi)\hat{\lambda} = -\omega_D^2 d\hat{e}_r$$

$$\Rightarrow \left\{ a_{P/B}\hat{\lambda} + \ddot{\phi}\frac{\ell}{\cos\phi}\hat{n} - \dot{\phi}^2\frac{\ell}{\cos\phi}\hat{\lambda} - 2\omega_D\dot{\phi}d\cos\phi\hat{n} = -\omega_D^2 d\hat{e}_r \right\}$$

$$\{\ \}\cdot\hat{n} \quad \Rightarrow \quad \ddot{\phi}\frac{\ell}{\cos\phi} - 2\omega_D\dot{\phi}d\cos\phi = -\omega_D^2 d(\hat{e}_r\cdot\hat{n})$$

$\because \hat{e}_r\cdot\hat{n} = \cos\phi$

$$\therefore \quad \ddot{\phi} = \frac{2\omega_D\dot{\phi}d\cos^2\phi}{\ell} - \omega_D^2\frac{d}{\ell}\cos^2\phi$$

$$= \frac{2\omega_D d}{\ell}\frac{\ell^2}{\ell^2+d^2}\omega_D\frac{d^2}{d^2+\ell^2} - \omega_D^2\frac{d}{\ell}\frac{\ell^2}{\ell^2+d^2}$$

$$= \omega_D^2\frac{d\ell(d^2-\ell^2)}{(\ell^2+d^2)^2}$$

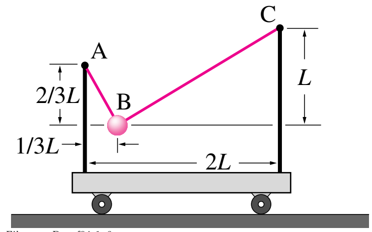
$\therefore$ Angular acceleration of CB when $\theta = \frac{\pi}{2}$, is $\ddot{\phi} = \omega_D^2\frac{d\ell(d^2-\ell^2)}{(\ell^2+d^2)^2}$

16.1.20 Due to forces not shown, the cart moves to the right with constant acceleration a_x. The ball B has mass m_B. At time $t = 0$, the string AB is cut. Find

a) the tension in string BC before cutting,

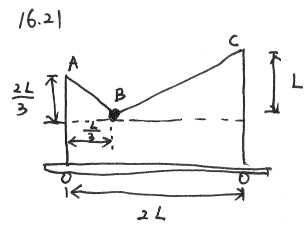
b) the absolute acceleration of the mass at the instant of cutting,

c) the tension in string BC at the instant of cutting.



Problem 16.20

16.21



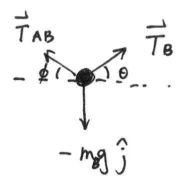
Cart moving forward with $\vec{a} = a_x \hat{i}$.

At time $t=0$, cut AB.

i) Before cutting, tension in BC ?

The ball B moves with the cart, $\vec{a}_B = a_x \hat{i}$

FBD :



$\vec{T}_{BC} = T_{BC}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$\vec{T}_{AB} = T_{AB}(-\cos\phi\,\hat{i} + \sin\phi\,\hat{j})$

LMB : $\{\vec{T}_{AB} + \vec{T}_{BC} - m_B g\hat{j} = m_B a_x \hat{i}\}$

$\{\}\cdot(\sin\phi\,\hat{i} + \cos\phi\,\hat{j})$

$\Rightarrow \quad T_{BC}(\cos\theta\sin\phi + \sin\theta\cos\phi) = m_B g\cos\phi + m_B a_x \sin\phi$

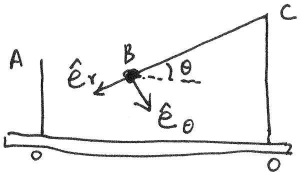
$\therefore \quad \cos\theta = \dfrac{\frac{5}{3}L}{\sqrt{(\frac{5}{3}L)^2 + L^2}} = \dfrac{5}{\sqrt{34}}, \quad \sin\theta = \dfrac{3}{\sqrt{34}}$

$\cos\phi = \dfrac{1}{\sqrt{5}}, \quad \sin\phi = \dfrac{2}{\sqrt{5}}$

$\Rightarrow \quad \boxed{T_{BC} = \dfrac{\sqrt{34}}{13} m_B (g + 2a_x)}$

~~Be~~ before cutting AB

After cutting AB, the tension in BC is not that before cutting.



Since BC is inextensible, ball B can only swing about C relative to the cart.

$$\therefore \quad \vec{a}_B = \vec{a}_{cart} + \vec{a}_{B/cart}$$

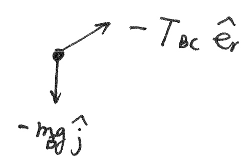
$$= \cancel{\vec{a}} \; a_x \hat{i} + \ddot{\theta} d \hat{e}_\theta - d \dot{\theta}^2 \hat{e}_r$$

where $d = |\vec{r}_{B/C}|$: distance between B and C.

Just after cutting, $\dot{\theta} = 0$

$$\therefore \quad \vec{a}_B = a_x \hat{i} + \ddot{\theta} d \hat{e}_\theta$$

FBD



LMB: $\quad -T_{BC}\hat{e}_r - mg\hat{j} = m\vec{a}_B$

$$\Rightarrow \quad \{ -T_{BC}\hat{e}_r - mg\hat{j} = m a_x \hat{i} + m\ddot{\theta} d \hat{e}_\theta \}$$

$\{\} \cdot \hat{e}_r \quad \Rightarrow \quad -T_{BC} = mg(\hat{j}\cdot\hat{e}_r) + m a_x (\hat{i}\cdot\hat{e}_r)$

$\{\} \cdot \hat{e}_\theta \quad \Rightarrow \quad m\ddot{\theta} d = -mg(\hat{j}\cdot\hat{e}_\theta) - m a_x (\hat{i}\cdot\hat{e}_\theta)$

$\hat{j}\cdot\hat{e}_r = -\sin\theta, \quad \hat{i}\cdot\hat{e}_r = -\cos\theta, \quad \hat{j}\cdot\hat{e}_\theta = -\cos\theta, \quad \hat{i}\cdot\hat{e}_\theta = \sin\theta$

$$\Rightarrow \quad T_{BC} = mg\sin\theta + m a_x \cos\theta = \frac{m}{\sqrt{34}}(3g + 5a_x)$$

and $\quad m\ddot{\theta} d = mg\cos\theta - m a_x \sin\theta = \dfrac{m}{\sqrt{34}}(5g - 3a_x)$

$\therefore$ Absolute acceleration of B : $\cancel{\vec{a}_B = a_x \hat{i}}$

$$\vec{a}_B = a_x \hat{\imath} + \ddot{\theta} d \, \hat{e}_\theta$$

$$= a_x \hat{\imath} + \ddot{\theta} d \, \cancel{\cos} \sin\theta \, \hat{\imath} + \ddot{\theta} d \, (-\cos\theta) \hat{\jmath}$$

$$= a_x \hat{\imath} + \frac{1}{\sqrt{34}} (5g - 3a_x) \frac{3}{\sqrt{34}} \hat{\imath} + \frac{1}{\sqrt{34}} (5g - 3a_x)\left(-\frac{5}{\sqrt{34}}\right) \hat{\jmath}$$

$$= \frac{1}{34} (25 a_x + 15 g) \hat{\imath} + \frac{1}{34} (15 a_x - 25 g) \hat{\jmath}$$

$$\therefore \boxed{\vec{a}_B = \frac{1}{34} (25 a_x + 15 g) \hat{\imath} + \frac{1}{34} (15 a_x - 25 g) \hat{\jmath}}$$

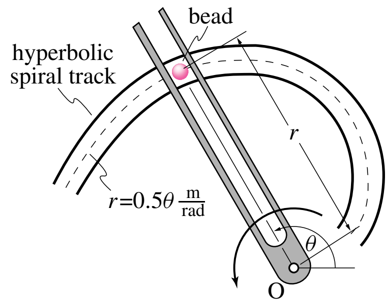
Tension in BC after cutting:

$$\boxed{T_{BC} = \frac{m}{\sqrt{34}} (3g + 5 a_x)}$$

16.1.21 The forked arm mechanism pushes the bead of mass 1 kg along a frictionless hyperbolic spiral track given by $r = 0.5\theta(\,\mathrm{m}/\,\mathrm{rad})$. The arm rotates about its pivot point at O with constant angular acceleration $\ddot{\theta} = 1\,\mathrm{rad}/\mathrm{s}^2$ driven by a motor (not shown). The arm starts from rest at $\theta = 0^\circ$.

a) Determine the radial and transverse components of the acceleration of the bead after 2 s have elapsed from the start of its motion.

b) Determine the magnitude of the net force on the mass at the same instant in time.



Problem 16.21

16.21 Given; $\ddot{\theta} = 1$ (constant); $\dot{\theta}(0) = \theta(0) = 0$

$r = 0.5\theta \Rightarrow r(t) = 0.5\theta(t)$

Find a_r, a_t, $|\vec{F}|$ at $t = 2s$

Lets solve for $r(t)$ and $\theta(t)$

$\ddot{\theta} = 1$

Integrate twice $\theta(t) = \frac{t^2}{2} + C_1 t + C_2$

But $\dot{\theta}(0) = \theta(0) = 0 \Rightarrow \boxed{\theta(t) = 0.5t^2}$

Also $r(t) = 0.5\,\theta(t) \Rightarrow \boxed{r(t) = 0.25t^2}$

(a) $\vec{a} = (\ddot{r} - \dot{\theta}^2 r)\hat{e}_r + (2\dot{r}\dot{\theta} + \ddot{\theta}r)\hat{e}_\theta$

$r(2) = 1$ $\quad \dot{r}(t) = 0.5t \Rightarrow \dot{r}(2) = 1$

$\ddot{r}(t) = 0.5 \Rightarrow \ddot{r}(2) = 0.5$

$\dot{\theta}(t) = t \Rightarrow \dot{\theta}(2) = 2$

$\ddot{\theta}(t) = 1 \Rightarrow \ddot{\theta}(2) = 1$

Thus $\vec{a} = (0.5 - 2^2\cdot 1)\hat{e}_r + (2\times 1\times 2 + 1\times 1)\hat{e}_\theta$

$\boxed{\vec{a} = -3.5\,\hat{e}_r + 5\hat{e}_\theta}$ (Answer)

(b) $\vec{F} = m\vec{a} = -3.5\hat{e}_r + 5\hat{e}_\theta$ {As $m = 1$ kg}

$|\vec{F}| = \sqrt{(-3.5)^2 + 5^2} = 6.1$

$\boxed{|\vec{F}| = 6.1\ \mathrm{N}}$

16.2.6 A thin uniform rod of mass m rests against a frictionless wall and on a frictionless floor. There is gravity.

a) Draw a free body diagram of the rod.

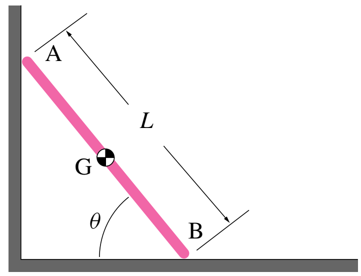
b) The rod is released from rest at $\theta = \theta_0 \neq 0$. Write the equation of motion of the rod.

c) Using the equation of motion, find the initial angular acceleration, $\dot{\omega}_{AB}$, and the acceleration of the center of mass, $\vec{a}_G$, of the rod.

d) Find the reactions on the rod at points A and B.

e) Find the acceleration of point B.

f) When $\theta = \frac{\theta_0}{2}$, find $\vec{\omega}_{AB}$ and the acceleration of point A.



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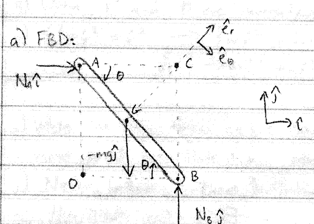
Problem 16.6

Johannes Feng
HW problems 16.30, 16.39, 16.41, 16.43, 16.48
Due 12/4/08
TAM 2030
Section 2 - 1:25
TA: Rong Long

(16.30) a) FBD:



b) $\theta(t=0) = \theta_0$

AMB$_{/C}$: $\sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}$

$$\vec{r}_{G/C} \times (-mg\hat{\jmath}) = \vec{r}_{G/C} \times m\vec{a}_G + I^G \dot{\omega}\hat{k}$$

$$\left(\frac{-L}{2}\cos\theta\,\hat{\imath} - \frac{L}{2}\sin\theta\,\hat{\jmath}\right) \times (-mg\hat{\jmath}) = \left(\frac{-L}{2}\cos\theta\,\hat{\imath} - \frac{L}{2}\sin\theta\,\hat{\jmath}\right) \times m\vec{a}_G - \frac{mL^2}{12}\ddot{\theta}\hat{k}$$

$$\frac{1}{2}mgL\cos\theta\,\hat{k} = \left(\frac{-L}{2}\cos\theta\,\hat{\imath} - \frac{L}{2}\sin\theta\,\hat{\jmath}\right) \times m\vec{a}_G - \frac{mL^2}{12}\ddot{\theta}\hat{k}$$

Need $\vec{a}_G$: use circular motion of G

$$\vec{a}_G = -\dot{\theta}^2\frac{L}{2}\hat{e}_r + \ddot{\theta}\frac{L}{2}\hat{e}_\theta = -\dot{\theta}^2\frac{L}{2}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) + \ddot{\theta}\frac{L}{2}(\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath})$$

$$= \left(-\dot{\theta}^2\frac{L}{2}\cos\theta + \ddot{\theta}\frac{L}{2}\sin\theta\right)\hat{\imath} + \left(-\dot{\theta}^2\frac{L}{2}\sin\theta - \ddot{\theta}\frac{L}{2}\cos\theta\right)\hat{\jmath}$$

Plug $\vec{a}_G$ into AMB$_{/C}$:

$$\left\{ \frac{1}{2}mgL\cos\theta\,\hat{k} = -\left(\frac{mL^2}{4}\left(\dot{\theta}^2\sin\theta\cos\theta + \ddot{\theta}\cos^2\theta\right) + \frac{mL^2}{4}\left(-\dot{\theta}^2\sin\theta\cos\theta + \ddot{\theta}\sin^2\theta\right)\right)\hat{k} - \frac{mL^2}{12}\ddot{\theta}\hat{k} \right\}$$

$$\{\}\cdot\hat{k} \to \frac{1}{2}mgL\cos\theta = \frac{-mL^2}{4}\ddot{\theta}\left(\cos^2\theta + \sin^2\theta\right) - \frac{mL^2}{12}\ddot{\theta}$$

$$6g\cos\theta = -3L\ddot{\theta} - L\ddot{\theta} = -4L\ddot{\theta}$$

$$\boxed{\ddot{\theta} = \frac{-3g}{2L}\cos\theta}$$

c) From part (b): $\boxed{\ddot{\theta}(t=0) = \frac{-3g}{2L}\cos\theta_0}$ (released from rest)

$$\vec{a}_G(t=0) = \left(-\cancel{\dot{\theta}_0^2}\frac{L}{2}\cos\theta_0 + \ddot{\theta}_0\frac{L}{2}\sin\theta_0\right)\hat{\imath} + \left(-\cancel{\dot{\theta}_0^2}\frac{L}{2}\sin\theta_0 - \ddot{\theta}_0\frac{L}{2}\cos\theta_0\right)\hat{\jmath}$$

$$\boxed{\vec{a}_G(t=0) = \frac{-3g}{4}\cos\theta_0\left(\sin\theta_0\,\hat{\imath} - \cos\theta_0\,\hat{\jmath}\right)}$$

d) LMB: $\sum \vec{F} = m a_G$

$$\left\{ N_A\hat{\imath} + (N_B - mg)\hat{\jmath} = m\vec{a}_G \right\}$$

$$\{\}\cdot\hat{\imath} \to \boxed{N_A = \frac{mL}{2}\left(-\dot{\theta}^2\cos\theta + \ddot{\theta}\sin\theta\right)}$$

$$\{\}\cdot\hat{\jmath} \to N_B - mg = \frac{mL}{2}\left(-\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta\right)$$

$$\boxed{N_B = m\left(\frac{L}{2}\left(-\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta\right) + g\right)}$$

e) $$\vec{a}_B = \vec{a}_G + \vec{a}_{B/G} = \left(-\dot{\theta}^2\frac{L}{2}\cos\theta + \ddot{\theta}\frac{L}{2}\sin\theta\right)\hat{\imath} + \left(-\dot{\theta}^2\frac{L}{2}\sin\theta - \ddot{\theta}\frac{L}{2}\cos\theta\right)\hat{\jmath} - \dot{\theta}^2\frac{L}{2}\hat{e}_\theta + \ddot{\theta}\frac{L}{2}\hat{e}_r$$

$$= \left(-\dot{\theta}^2\frac{L}{2}\cos\theta + \ddot{\theta}\frac{L}{2}\sin\theta\right)\hat{\imath} + \left(-\dot{\theta}^2\frac{L}{2}\sin\theta - \ddot{\theta}\frac{L}{2}\cos\theta\right)\hat{\jmath} - \dot{\theta}^2\frac{L}{2}(\sin\theta\,\hat{\imath} - \cos\theta\,\hat{\jmath}) + \ddot{\theta}\frac{L}{2}(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

$$= \boxed{\left(-\dot{\theta}^2\frac{L}{2}(\cos\theta - \sin\theta) + \ddot{\theta}\frac{L}{2}(\sin\theta + \cos\theta)\right)\hat{\imath} + \left(-\dot{\theta}^2\frac{L}{2}(\sin\theta + \cos\theta) + \ddot{\theta}\frac{L}{2}(\cos\theta + \sin\theta)\right)\hat{\jmath}}$$

f) Point A moves in circles around Point G

$\vec{V}_A = -\omega \frac{L}{2} \hat{e}_r = -\omega \frac{L}{2} (\cos\theta \hat{\imath} + \sin\theta \hat{\jmath})$

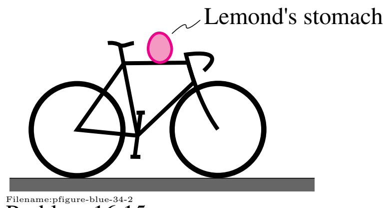
$\vec{V}_A = \vec{V}_B$ (rigid object)

$|\vec{a}_A| = |\vec{a}_B|$

16.2.15 Assume Greg Lemond's riding "tuck" was so good that you can neglect air resistance when you think about him and his bike. Further, you can regard his and the bike's combined mass (all 70 kg)) as concentrated at a point in his stomach somewhere. Greg's left foot has just fallen off the pedal so he is only pedaling with his right foot, which at the moment in question is at its lowest point in the motion. You note that, relative to the ground the right foot is only going 3/4 as fast as the bike (since it is going backwards relative to the bike), though you can't make out all the radii of his frictionless gears and rigid round wheels. Greg, ever in touch with his body, tells you he is pushing back on the pedal with a force of 70 N. You would like to know Greg's acceleration.

a) In your first misconceived experiment you set up a 70 kg bicycle in your laboratory and balance it with strings that cause no fore or aft forces. You tie a string to the vertically down right pedal and pull back with a force of 70 N. What acceleration do you measure?

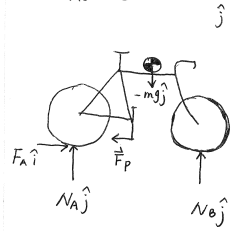
b) What is Greg's actual acceleration? [Hint: Greg's massless leg is pushing forward on his body with a force of 70 N] You can neglect the mass of the wheels and other transmission parts (chain, crank, etc).



Problem 16.15

16.39 (A)

Assume the wheels are massless and roll without slip.



If you pull the right pedal with $\vec{F}_P = -70\,N\,\hat{i}$, the FBD is shown on the left.

Since the wheels are massless, there is no friction on the front and a forward friction force on the back wheel.

The friction force $F_A\hat{i}$ on the rear wheel is static friction which does no work to the system. So we can use energy method.

Let the velocity of the bike be $\vec{V}_{bike} = V_b\hat{i}$

The velocity of the pedal is then $\vec{V}_{pedal} = \frac{3}{4}V_b\hat{i}$

$\therefore$ Power of $\vec{F}_P$

$$P_P = \vec{F}_P \cdot \vec{V}_{pedal} = -\frac{3}{4}V_b \times 70 = -52.5\,V_b$$

(V_b in terms of "m/s")

Since $\vec{F}_P$ is the only force that does work to this system,

$P_P = \dot{E}_K$ where $\dot{E}_K$ is rate of change of the kinetic energy of the system

In this case

$$E_K = \frac{1}{2}mV_b^2 \Rightarrow \dot{E}_K = mV_b\dot{V}_b$$

$$\Rightarrow -52.5\,V_b = 70\,V_b\dot{V}_b \Rightarrow \dot{V}_b = -0.75\ m/s^2$$

$\therefore$ The acceleration of the bike is $\boxed{\vec{a}_{bike} = \dot{V}_b\hat{i} = -0.75\ m/s^2\,\hat{i}}$

(B) If Greg is riding on the bike, he pushes the pedal with $\vec{F}_P = -70N\hat{i}$ and he also receive the reaction force from the pedal of $70N\hat{i}$.

Velocity of the pedal is $\vec{V}_{pedal} = \frac{3}{4}V_b\hat{i}$

velocity of ~~gr.~~ Greg is $\vec{V}_{Greg} = V_b\hat{i}$

Similary the total power to the system (Greg and bike) is

$$P = \vec{F}_P \cdot \vec{V}_{pedal} - \vec{F}_P \cdot \vec{V}_{Greg}$$

$$= -70N\hat{i} \cdot \left(\frac{3}{4}V_b\hat{i} - V_b\hat{i}\right) = 17.5\,V_b\,\hat{i}$$

(V_b in terms of m/s)

$$\therefore\ P = \dot{E}_K = m\,V_b\,\dot{V}_b$$

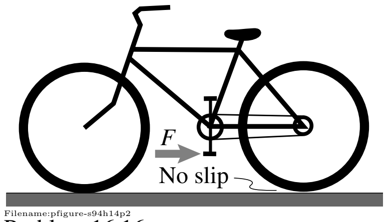
$$\Rightarrow\quad 17.5\,V_b = 70\,V_b\,\dot{V}_b \quad \Rightarrow \quad \dot{V}_b = 0.25\ m/s^2$$

∴ Greg's accelerative is

$$\boxed{\begin{aligned}\vec{a}_{Greg} &= \dot{\vec{V}}_{Greg} = \dot{V}_b\,\hat{i} \\ &= 0.25\ m/s^2\ \hat{i}\end{aligned}}$$

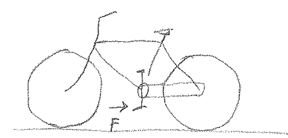
This result means Greg Lemond accelerates forward by pushing the pedal backwards.

16.2.16 Which way does the bike accelerate? A bicycle with all frictionless bearings is standing still on level ground. A horizontal force F is applied on one of the pedals as shown. There is no slip between the wheels and the ground. The bicycle is gently balanced from falling over sideways. It is heavy enough so that both wheels stay on the ground. Does the bicycle accelerate forward, backward, or not at all? Make any reasonable assumptions about the dimensions and mass. Justify your answer as clearly as you can, clearly enough to convince a person similar to yourself but who has not seen the experiment performed.



Problem 16.16

16.39 Which way bike accelerates?



Answer: It accelerates in the direction of force, i.e backward.

* One way of seeing it by power balance. (See p. 916 in book)

$$P = \dot{E}_K$$

$$\vec{F}\cdot\vec{v}_{pedal} = m\,\dot{v}_{bike}$$

$$\vec{F}\cdot(\vec{v}_{bike} + \vec{v}_{pedal/bike}) = m\,\dot{v}_{bike}$$

Assume $\leftarrow v_{bike}$

$\longrightarrow v_{pedal/bike}$

$$F\left(-v_{bike} + \frac{v_{bike}}{n}\right) = m\,\dot{v}_{bike}$$

$$F\,v_{bike}\left(-1 + \frac{1}{n}\right) = m\,\dot{v}_{bike}$$

Since $n > 1$, Left side is negative

$\Rightarrow \dot{v}_{bike} < 0$, hence bike accelerates backward.

* A more elegant way: Observe that about any point, the sum of moments of mg, N_1, N_2 vanish. (To see this, assume (for a while) that $F = 0$.)

Now take the moment about any point on the ground: The only contribution comes from F.

$$r \times F = r \times m\,\vec{a}_{cm} + \cancelto{0}{I\dot{\omega}\,k} \Rightarrow F \;\&\; a_{cm} \text{ are } \parallel.$$

BIKE PEDAL-FORCE SOLUTION: WHICH WAY DOES THE BIKE GO?

A Ruina, April 20, 2009

Question. You stand next to a normal bike on flat ground. You delicately balance the bike with the steering straight. Your friend stands next to the bike and pushes backwards (to the left on the figure) on the bottom pedal. Which way does the bike accelerate?

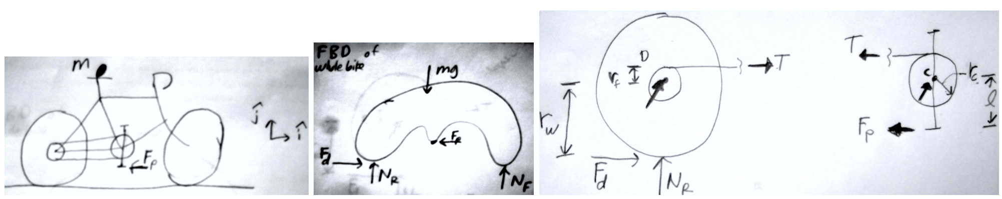
Assumptions. You apply negligible force. Your friend applies $-F_p\hat{\imath}$ to the pedal only. The rear wheel rolls. The only negligible mass is of the bike as a whole, m. All the bearings have negligible friction. The gearing of the bike is normal. The tension in the bottom part of the chain is negligible (not needed, but simplifies calculation). The key dimensions are

ℓ = the crank length. The crank is the piece from the middle of the bottom bracket to the pedal.

r_c = the radius of the chain wheel. The chain wheel is the gear that is in the front of the bike. It chain wheel is rigidly attached to the crank and rotates with the crank.

r_f = radius of the free-wheel sprocket. This is also called the cluster. It is the gear at the back of the bike. When there is tension on the upper chain, and the rear ratchet is thus engaged, the rear sprocket rotates with the rear wheel.

r_w = radius of the rear wheel.



Correct Answers.

(1) The bike accelerates to the left. Try it. It always goes backwards. The next answer is different and is also correct.

(2) The answer depends on the bike. Despite (1) above, the answer actually depends on what bike you try it on. For any bike you have ever seen or can buy, in any gear, answer (1) is correct. However, if the gearing is sufficiently low (how low? read solution below) then the bike will go forwards.

Reasoning.

A. The bike starts with no kinetic energy. It ends with some kinetic energy. Therefor some force had to do some positive work. The only force that has non-zero dot product with the velocity of the point it is touching is $-F_p\hat{\imath}$. Therefore *the pedal has to go backwards*. Does this mean the bike goes forwards or backwards? When a bike goes forwards the bottom pedal goes backwards relative to the bike. But relative to the ground it goes forwards. Otherwise people wouldn't ride bikes. The whole idea of a bike is that you go faster with a given leg motion than you do when you walk or run. So the bottom pedal goes the same direction as the bike. Since the pedal goes backwards, so does the bike.

B. Same as above, but lets calculate the pedal velocity and see if it really is in the same direction as the bike. Lets define ω_w and ω_c as the counterclockwise angular velocities of the rear wheel and crank,

2

respectively. Assume bike and pedal have absolute velocity $v_b\hat{\imath}$ and $v_b\hat{\imath}$, then

$$
\begin{aligned}
v_p &= v_b + v_{p/b} \qquad (1)\\
&= v_b + \omega_c \ell \qquad (2)\\
&= v_b + \omega_w \frac{r_f}{r_c}\ell \qquad (3)\\
&= v_b + (-v_b/r_w)\frac{r_f}{r_c}\ell \qquad (4)\\
&= v_b\left(1 - \frac{r_f\ell}{r_c r_w}\right). \qquad (5)
\end{aligned}
$$

So the pedal goes forward when the bike goes forward so long as $\frac{r_f\ell}{r_c r_w} < 1$. Thus the bike goes back when the pedal goes back with the same inequality. Such bikes all go back when you push back on the pedal. On the other hand, The bike would go forwards when you push back on the pedal if the bike has a 'very low gear'. You could get such low gears from having some combination of a big rear sprocket radius r_f, a big crank length ℓ, a small front chain ring radius r_c and a small rear wheel radius r_w.

C. Here's a different approach. Given F_p moment balance about C for the crank and chain wheel tells us the chain tension

$$T = \frac{\ell}{r_c}F_p.$$

Moment balance about the rear axle, for the rear wheel and freewheel gear tell us that the drive force F_d is

$$F_d = \frac{r_f}{r_w}T = \frac{r_f}{r_w}\underbrace{\left(\frac{\ell}{r_c}F_p\right)}_{T}.$$

Now look at linear momentum for the whole bike:

$$
\begin{aligned}
-F_p + F_d &= m a_b \qquad (6)\\
-F_p + \underbrace{\frac{r_f}{r_w}\frac{\ell}{r_c}F_p}_{F_d} &= m a_b \qquad (7)\\
\Rightarrow \quad a_b &= -\frac{F_p}{m}\left(1 - \frac{r_f}{r_w}\frac{\ell}{r_c}\right). \qquad (8)
\end{aligned}
$$

So, now by different reasoning, if $\frac{r_f\ell}{r_c r_w} < 1$ then a push back of F_p causes a negative acceleration a_b of the bike. That is, if $\frac{r_f\ell}{r_c r_w} < 1$ (which it is for all real bikes) then the bike accelerates backwards when you push backwards on the pedal.

16.3.1 Particle on a springy leash. A particle with mass m slides on a rigid horizontal frictionless plane. It is held by a string which is in turn connected to a linear elastic spring with constant k. The string length is such that the spring is relaxed when the mass is on top of the hole in the plane. The position of the particle is $\vec{r} = x\hat{\imath} + y\hat{\jmath}$. For each of the statements below, state the circumstances in which the statement is true (assuming the particle stays on the plane). Justify your answer with convincing explanation and/or calculation.

a) The force of the plane on the particle is $mg\hat{k}$.

b) $\ddot{x} + \frac{k}{m}x = 0$

c) $\ddot{y} + \frac{k}{m}y = 0$

d) $\ddot{r} + \frac{k}{m}r = 0$, where $r = |\vec{r}|$

e) $r =$ constant

f) $\dot{\theta} =$constant

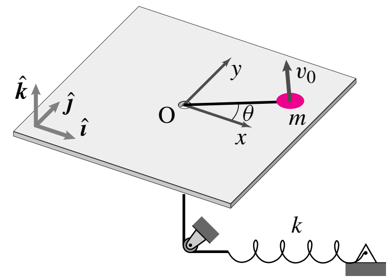
g) $r^2\dot{\theta} =$ constant.

h) $m(\dot{x}^2 + \dot{y}^2) + kr^2 =$ constant

i) The trajectory is a straight line segment.

j) The trajectory is a circle.

k) The trajectory is not a closed curve.



Problem 16.1: Particle on a springy leash.

16.40 (a) The normal reaction is always $mg\hat{k}$

(b) $\ddot{x} + \frac{k}{m}x = 0$ } always True.

(c) $\ddot{y} + \frac{k}{m}y = 0$

(d) $\ddot{r} + \frac{k}{m}r = 0$

(e) $\dot{r} = 0$ if particle is going in a circle with speed v_0 such that $kr = \frac{mv_0^2}{r}$
(Initial velocity $\vec{v}_0 \perp \vec{r}$)

(f) Same as (e)

(g) Same as (e) or (f)

(h) 2 * Energy = $mv^2 + kr^2$ always.

(i) $\dot{\theta} = 0$ for straight line. Particle will oscillate over the origin if $\vec{v}_0 = c\vec{r}$

(j) Same as (e), (f) or (g)

(k) Never

16.3.3 A particle with mass m is held by two long springs each with stiffness k so that the springs are relaxed when the mass is at the origin. Assume the motion is planar. Assume that the particle displacement is much smaller than the lengths of the springs.

a) Write the equations of motion in cartesian components.

b) Write the equations of motion in polar coordinates.

c) Express the conservation of angular momentum in cartesian coordinates.

d) Express the conservation of angular momentum in polar coordinates.

e) Show that (a) implies (c) and (b) implies (d) even if you didn't note them *a-priori*.

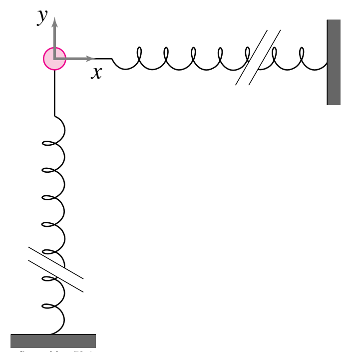
f) Express the conservation of energy in cartesian coordinates.

g) Express the conservation of energy in polar coordinates.

h) Show that (a) implies (f) and (b) implies (g) even if you didn't note them *a-priori*.

i) Find the general motion by solving the equations in (a). Describe all possible paths of the mass.

j) Can the mass move back and forth on a line which is not the x or y axis?



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Problem 16.3

16.42

(a) For small oscillations, the x component of force on the particle from vertical spring can be neglected. Hence, $\ddot{x} + \frac{k}{m} x = 0$

Similarly, $\ddot{y} + \frac{k}{m} y = 0$. $\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} = -\frac{k}{m}(x\hat{\imath} + y\hat{\jmath})$

(b) $-kr\,\hat{e}_r = m\left[(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta\right]$

(c) In 2-D, angular momentum about origin is $\vec{H}_0 = (xL_y - yL_x)\hat{k}$, where $L_x = \vec{L}\cdot\hat{\imath}$ is the x-component of $\vec{L}$.

$\sum M_0 = 0 \Rightarrow \frac{d}{dt}\vec{H}_0 = 0 \Rightarrow \frac{d}{dt}(x v_y - y v_x) = 0$

or, $\boxed{\frac{d}{dt}(x\dot{y} - y\dot{x}) = 0}$

(d) velocity in polar co-ord:

$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$\vec{H}_0 = r\hat{e}_r \times \vec{v} = r^2\dot{\theta}\hat{k}$

Conservation $\Rightarrow$ $\boxed{\frac{d}{dt}(r^2\dot{\theta}) = 0}$

(e) From (c), $x\ddot{y} + \cancel{\dot{x}\dot{y}} - \cancel{\dot{y}\dot{x}} - y\ddot{x} = 0$

$\Rightarrow \boxed{x\ddot{y} = y\ddot{x}}$

From (a) $\ddot{y} = -\frac{k}{m} y$

$\frac{kx}{m} = -\ddot{x}$] multiply : $\boxed{x\ddot{y} = \ddot{x}y}$ (same)

From (d) $2r\dot{r}\dot{\theta} + r^2\ddot{\theta} = 0$

$\Rightarrow 2\dot{r}\dot{\theta} + r\ddot{\theta} = 0$

which is obtained by finding $\{(b)\}\cdot\hat{e}_\theta$

(f) $\frac{d}{dt}\left(\frac{1}{2}m(\dot{x}^2+\dot{y}^2) + \frac{1}{2}k(x^2+y^2)\right) = 0$

(g) $\frac{d}{dt}\left[\frac{1}{2}m\left|\left(\dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta\right)\right|^2 + \frac{1}{2}kr^2\right] = 0$

$\frac{1}{2}m\left[\dot{r}^2 + (r\dot{\theta})^2\right] + \frac{1}{2}kr^2 = \text{const}$

(h) In (a), dot both sides with $\dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$

to get : $m(\ddot{x}\dot{x} + \ddot{y}\dot{y}) + k(\dot{x}x + \dot{y}y) = 0$

i.e. $\frac{d}{dt}\left[m(\dot{x}^2 + \dot{y}^2) + k(x^2+y^2)\right] = 0$

same as (f)

Now, dot both sides of (b) with $\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$ to get

$$k r \dot{r} + m\left[(\ddot{r}\dot{r} - \dot{r} r \dot{\theta}^2) + (r^2\dot{\theta}\ddot{\theta} + 2 r \dot{r} \dot{\theta}^2)\right] = 0$$

combine

or, $$\frac{d}{dt}\left[(k r^2) + m(\dot{r})^2 + m(r\dot{\theta})^2\right] = 0$$

Same as (g)

(i) $x = A_1 \sin(\omega t + B_1)$ $\quad y = A_2 \sin(\omega t + B_2)$

with $\omega = \sqrt{k/m}$. This describes an ellipse as you can reduce down to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Check!)

(j) If $B_1 = B_2 = 0$, $y = cx$ which is a straight line.

4 16.43) Solution

a) $\ddot{x} + \frac{k}{m}x = 0 \rightarrow -kx = m\ddot{x}$ ($\hat{\imath}$-direction)

$\ddot{y} + \frac{k}{m}y = 0 \rightarrow -ky = m\ddot{y}$ ($\hat{\jmath}$-direction)

combine: $-k(x\hat{\imath} + y\hat{\jmath}) = m(\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath})$

b) $\Sigma \vec{F} = ma = m[(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta]$

$\Sigma \vec{F} = -kr\hat{e}_r$

$$\boxed{-kr\hat{e}_r = m(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + m(r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta}$$

c) $\vec{H}^+ = \vec{H}^-$

$\vec{H}^+ = x^+\dot{y}^+ + y^+\dot{x}^+$ $\Rightarrow$ $x^+\dot{y}^+ + y^+\dot{x}^+ - (x^-\dot{y}^- + y^-\dot{x}^-) = 0$

$\vec{H}^- = x^-\dot{y}^- + y^-\dot{x}^-$ $\quad x^+\dot{y}^+ - x^-\dot{y}^- = \frac{d}{dt}x\dot{y}$

$y^+\dot{x}^+ - y^-\dot{x}^- = \frac{d}{dt}y\dot{x}$

$$\boxed{0 = \frac{d}{dt}(x\dot{y} + y\dot{x})}$$

d) $\vec{H}^+ = r^+\hat{e}_r \times (r^+\dot{\theta}^+)\hat{e}_\theta = (r^+)^2\dot{\theta}^+\,\hat{k}$

$\vec{H}^- = (r^-)^2\dot{\theta}^-\,k$ (likewise)

$\vec{H}^+ - \vec{H}^- = 0 = (r^+)^2\dot{\theta}^+ - (r^-)^2\dot{\theta}^- = \boxed{\frac{d}{dt}(r^2\dot{\theta}) = 0}$

e) $\{-k(x\hat{\imath} + y\hat{\jmath}) = m(\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath})\} \cdot (x\hat{\jmath} - y\hat{\imath})$

$= \underbrace{-kyx - kyx}_{0} = -my\ddot{x} + mx\ddot{y}$

$m(x\ddot{y} - y\ddot{x}) = 0 \Rightarrow \frac{d}{dt}(x\dot{y} + y\dot{x}) = 0$

$\{\text{equation in b}\} \cdot \hat{e}_\theta \Rightarrow 0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$\Rightarrow \frac{d}{dt}(r^2\dot{\theta}) = 0$

f) $E_T = \frac{1}{2}m\vec{v}^2 + \frac{1}{2}k\vec{r}^2 = \boxed{\frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}k(x^2 + y^2) = \text{const.}}$

g) $\frac{1}{2}m\vec{v}^2 + \frac{1}{2}k\vec{r}^2 = \frac{1}{2}m\left[\frac{d}{dt}(r\hat{e}_r)\right]^2 + \frac{1}{2}kr^2$

$\frac{d}{dt}r\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$$|\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta| = (\dot{r}^2 + (r\dot{\theta})^2)^{1/2}$$

$$\boxed{E_T = \frac{1}{2}m(\dot{r}^2 + (r\dot{\theta})^2) + \frac{1}{2}kr^2 = \text{constant}}$$

h) $\{-k(x\hat{\imath} + y\hat{\jmath}) = m(\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath})\} \cdot (\dot{x}\hat{\imath} + \dot{y}\hat{\jmath})$

$-kx\dot{x} - ky\dot{y} = mx\ddot{x} + my\ddot{y}$

$0 = mx\ddot{x} + my\ddot{y} + kx\dot{x} + ky\dot{y}$

$\int 0 = \text{const} = \int(mx\ddot{x} + my\ddot{y} + kx\dot{x} + ky\dot{y})dt$

$\text{const} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}k(x^2 + y^2)$

$\{\text{eq. from b}\} \cdot (r\dot{\theta}\hat{e}_\theta)$

$\Rightarrow 0 = mr^2\ddot{\theta}\dot{\theta} + m2\dot{r}\dot{\theta}r\dot{\theta}$

$0 = mr\dot{\theta}(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

i) $x = A\sin\left(\sqrt{\frac{k}{m}}t + B\right)$

$y = C\sin\left(\sqrt{\frac{k}{m}}t + D\right)$

This is the general form of an ellipse.

j) Consider $B = D = 0$

$x = A\sin\left(\sqrt{\frac{k}{m}}t\right)$

$y = C\sin\left(\sqrt{\frac{k}{m}}t\right)$

say $\sin\left(\sqrt{\frac{k}{m}}t\right) = n$ for a given value of t

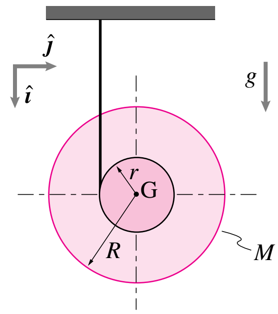
$x = An$

$y = Cn$

$y = \frac{1}{A}x \leftarrow$ linear equation

<u>Yes</u> this is possible

16.3.8 A model for a yo-yo consists of a thin disk of mass M and radius R and a light drum of radius r, rigidly attached to the disk, around which a light inextensible cable is wound. Assuming that the cable unravels without slipping on the drum, determine the acceleration a_G of the center of mass.



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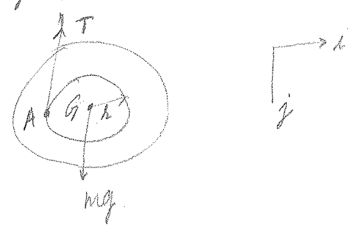
Problem 16.8

16.47 Yo-yo.

FBD



* Newton's IInd Law for vertical direction;

"LMB" $\quad mg - T = m a_G \quad$ — (i)

* Since the yoyo is rotating as it falls, we can also write the torque equation:

"AMB" $\quad \sum \vec{M}_{/G} = I_{cm} \alpha \hat{k}$

$$-Tr\,\hat{k} = \frac{mR^2}{2} \alpha \hat{k} \quad \text{— (ii)}$$

* Observe that the no-slip constraint at pt. A is the same as that for rolling, i.e.

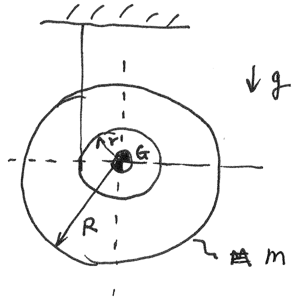
$$a_G = r\alpha \quad \text{— (iii)}$$

Solve simultaneously for a_G:

$$\boxed{\vec{a}_G = \frac{g}{1 - \frac{R^2}{2r^2}} \hat{j}}$$

16.48

$\hat{j} \perp \hat{i}$



Note:

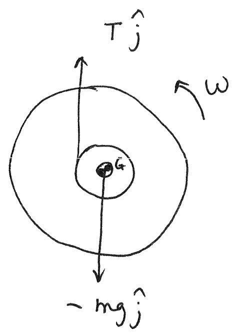
You can use the given coordinate system "$\hat{i}$, $\hat{j}$ (downward)"

But you have to be very clear about sign conventions. To avoid confusion, we just use the normal system $\hat{j} \perp \hat{i}$

The cable unravels without slip, Find acceleration G.

FBD:

$\hat{j} \perp \hat{i}$



LMB:

$$T\hat{j} - mg\hat{j} = m\vec{a}_G \cancel{= m a_G \hat{j}}$$

$$\Rightarrow \vec{a}_G = \left(\frac{T}{m} - g\right)\hat{j}$$

$\therefore$ The acceleration of G is in $\hat{j}$ direction.

Let $\vec{a}_G = -a_G\hat{j}$

then, $a_G = g - \frac{T}{m}$ $\quad \cdots$ ①

AMB about G:

$$-Tr\hat{k} = I^G\dot{\omega}\hat{k} \quad \cdots ②$$

$\dot{\omega}$ is the angular acceleration and is counterclockwise.

No slip condition:

$$a_G = -\dot{\omega}r \quad \cdots ③$$

②, ③ $\Rightarrow$ $T = -\frac{I^G\dot{\omega}}{r} = -\frac{I^G\left(-\frac{a_G}{r}\right)}{r} = \frac{I^G a_G}{r^2} \cdots$ ④

plug ④ in ①

$$\therefore a_G = g - \frac{I^G a_G}{mr^2}$$

$$\Rightarrow a_G = \frac{mr^2 g}{mr^2 + I^G}$$

$\because$ The drum of radius r is massless. And assume the disk of radius R is uniform

$\Rightarrow \quad I^G = \frac{1}{2} m R^2$

$$\therefore \quad \boxed{a_G = \frac{mr^2 g}{mr^2 + \frac{1}{2} m R^2} = \frac{2r^2}{2r^2 + R^2} g}$$

Of course G is accelerating downwards, i.e. $\boxed{\vec{a}_G = -\frac{2r^2}{2r^2+R^2} g\hat{j}}$

16.3.10 Assume the rod in the figure for problem 15.1.5 has polar moment of inertia $Iozz$. Assume it is free to rotate. The bead is free to slide on the rod. Assume that at $t = 0$ the angular velocity of the rod is 1 rad/s, that the radius of the bead is one meter and that the radial velocity of the bead, dR/dt, is zero.

a) Draw separate free body diagrams of the bead and rod.

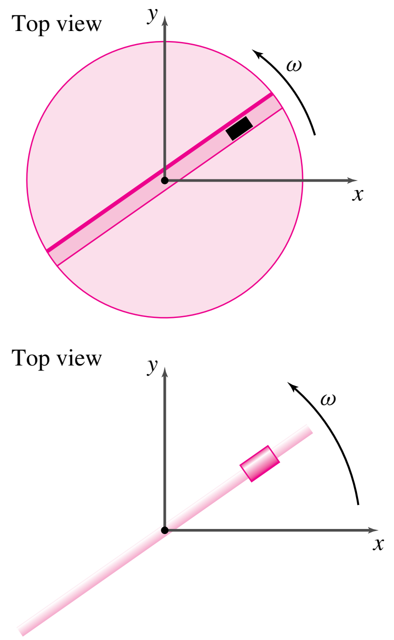
b) Write equations of motion for the system.

c) Use the equations of motion to show that angular momentum is conserved.

d) Find one equation of motion for the system using: (1) the equations of motion for the bead and rod and (2) conservation of angular momentum.

e) Write an expression for conservation of energy. Let the initial total energy of the system be, say, E_0.

f) As t goes to infinity does the bead's distance go to infinity? Its speed? The angular velocity of the turntable? The net angle of twist of the turntable?



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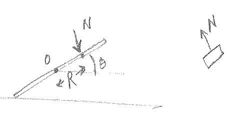
Problem 16.10: **Coupled motion of bead and rod and turntable.**

16.49

(a)



We'd like to "know" $\theta(t)$ & $R(t)$

(b) Write Newton's law for the bead: "LMB"

$$N\hat{e}_\theta = m\left[(\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\hat{e}_\theta\right]$$

from which it follows: $\boxed{\ddot{R} - R\dot{\theta}^2 = 0}$ (i)

& $\boxed{N = m(R\ddot{\theta} + 2\dot{R}\dot{\theta})}$ (ii)

Now write AMB for rod about O:

$$\sum M_{/O} = \dot{\vec{H}}_{rod/O}$$

$$R\hat{e}_R \times -N\hat{e}_\theta = I_{zz}\ddot{\theta}\hat{k}$$

$$\Rightarrow \boxed{mR^2\ddot{\theta} + 2\dot{R}R\dot{\theta}m + I_{zz}\ddot{\theta} = 0} \text{ (iii)}$$

Together, (i) & (iii) are coupled equations which could, in principle, be solved for $\theta(t)$, $R(t)$.

(c) $\vec{H}_{/O} = H_{rod/O} + H_{bead/O}$

$$\vec{H}_{/O} = I_{zz}\dot{\theta}\hat{k} + mR^2\dot{\theta}\hat{k} = (I_{zz} + mR^2)\dot{\theta}\hat{k}$$

$\frac{d}{dt}(\vec{H}_{/O}) = 0$ yields equation (iii)

(d) Take $\dot{\theta} = \dfrac{H_{/o}}{I_{zz} + mR^2}$ from (c)

& plug into (i) to get:

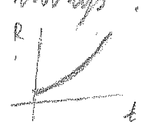
$$\boxed{\ddot{R} - R\left(\frac{H_{/o}}{I_{zz}+mR^2}\right)^2 = 0}$$

(e) Total energy = rotational energy of (rod + bead) + translational energy of bead

$$E_0 = \frac{1}{2}\left(I_{zz} + mR^2\right)\omega^2 + \frac{1}{2}m\dot{R}^2$$

or,
$$\boxed{E_0 = \frac{1}{2}H_{/o}\,\omega + \frac{1}{2}m\dot{R}^2}$$

(f) * From eq^n in (d), observe that $\ddot{R} > 0$ always. This implies that $R(t)$ is concave upward & increases initially



: In fact, $R \to \infty$ as $t \to \infty$.

However, note that $\ddot{R} \sim \dfrac{1}{R^3}$ for large R.

In particular, as $R \to \infty$, $\ddot{R} \to 0 \Rightarrow \dot{R} \to$ const.

$\therefore$ We deduce that distance of bead will approach ∞, & its speed will approach a constant

* Now consider $\vec{H}_{/O}$ = constant from (c)

$H_{/O} = (I_{zz} + mR^2)\dot{\theta} \sim mR^2\dot{\theta}$ at large R

Since $H_{/O}$ is constant & $R \to \infty$, the only possibility is $\dot{\theta} \to 0$ (i.e. $\omega \to 0$)

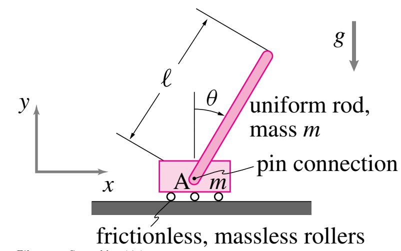
Therefore $\theta \to$ constant.

16.3.16 As shown in the figure, a block of mass m rolls without friction on a rigid surface and is at position x (measured from a fixed point). Attached to the block is a uniform rod of length ℓ which pivots about one end which is at the center of mass of the block. The rod and block have equal mass. The rod makes an angle θ with the vertical. Use the numbers below for the values of the constants and variables at the time of interest:

$$\begin{aligned} \ell &= 1\,\text{m} \\ m &= 2\,\text{kg} \\ \theta &= \pi/2 \\ d\theta/dt &= 1\,\text{rad/s} \\ d^2\theta/dt^2 &= 2\,\text{rad/s}^2 \\ x &= 1\,\text{m} \\ dx/dt &= 2\,\text{m/s} \\ d^2x/dt^2 &= 3\,\text{m/s}^2 \end{aligned}$$

a) What is the kinetic energy of the system?

b) What is the linear momentum of the system (momentum is a vector)?



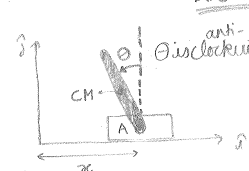
Problem 16.16

HW-26

16.55

Dont need. FBD Here
(no stamp)

NOTE
anti-
θ is clockwise



a) $K.E = K.E_{block} + K.E_{rod}$

$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I_{cm} \dot{\theta}^2 + \frac{1}{2} m V_{CM}^2$

where $\vec{V}_{CM} = \vec{V}_A + \dot{\theta}\hat{k} \times \vec{r}_{CM/A}$

$= \dot{x}\hat{\imath} + \dot{\theta}\hat{k} \times \left(-\frac{\ell}{2}\sin\theta\,\hat{\imath} + \frac{\ell}{2}\cos\theta\,\hat{\jmath}\right)$

$= \left(\dot{x} - \dot{\theta}\frac{\ell}{2}\cos\theta\right)\hat{\imath} - \dot{\theta}\frac{\ell}{2}\sin\theta\,\hat{\jmath}$ —— (1)

$V_{CM}^2 = \dot{x}^2 + \dot{\theta}^2\frac{\ell^2}{4} - 2\dot{x}\dot{\theta}\frac{\ell}{2}\cos\theta$

$K.E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2}\left(\frac{1}{12} m\ell^2\right)\dot{\theta}^2 + \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{\theta}^2 \frac{\ell^2}{4} - \frac{1}{2} m \dot{x}\dot{\theta}\ell\cos\theta$

$= m\dot{x}^2 + \frac{1}{6} m\ell^2\dot{\theta}^2 - \frac{1}{2} m \dot{x}\dot{\theta}\ell\cos\theta$

$\frac{1}{2}\cdot\frac{1}{12} \cdot \frac{1}{2}\cdot\frac{1}{4}\frac{3}{3}$

Put in values

$= 2.2^2 + \frac{1}{6}\cdot 2\cdot 1^2 \cdot 1^2 - \frac{1}{2} 2\cdot 2\cdot 1\cdot 1 \cos\left(\frac{\pi}{2}\right)$

$8 + \frac{1}{3}$

$= \boxed{25/3}$

b) linear momentum

$\vec{p} = \underbrace{m\vec{V}_A}_{block} + \underbrace{m\vec{V}_{CM}}_{rod} = m\dot{x}\hat{\imath} + m\left(\dot{x} - \dot{\theta}\frac{\ell}{2}\cos\theta\right)\hat{\imath} - m\dot{\theta}\frac{\ell}{2}\sin\theta\,\hat{\jmath}$

$= m\left[\left(2\dot{x} - \frac{\dot{\theta}\ell}{2}\cos\theta\right)\hat{\imath} - \frac{\dot{\theta}\ell}{2}\sin\theta\,\hat{\jmath}\right]$

put in values

$= 2\left[(2\cdot 2 - 0)\hat{\imath} - 1\cdot\frac{1}{2}\sin\frac{\pi}{2}\,\hat{\jmath}\right] = \boxed{8\hat{\imath} - 1\hat{\jmath}}$

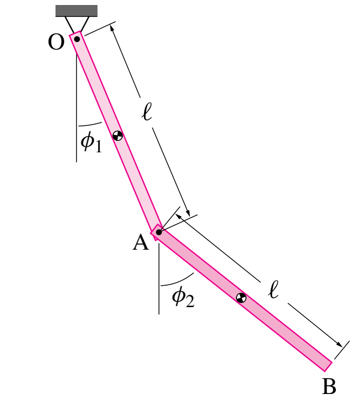
note: in the problem θ is measured clockwise

→ thats fine just change $\theta \to -\theta$ in all the equations.

$\therefore$ T = same

$\vec{p} = 8\hat{\imath} + 1\hat{\jmath}$

16.3.28 Double pendulum. The double pendulum shown is made up of two uniform bars, each of length ℓ and mass m. The pendulum is released from rest at $\phi_1 = 0$ and $\phi_2 = \pi/2$. Just after release what are the values of $\ddot{\phi}_1$ and $\ddot{\phi}_2$? Answer in terms of other quantities.



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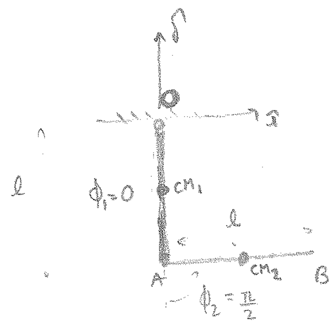
Problem 16.28

HW- 27

Q 16.67

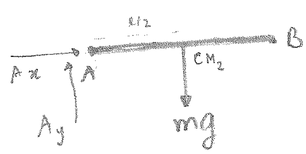
note $I_c = \frac{1}{12} m\ell^2$



initial configuration

rod 2 (just at t = 0)

FBD



$\dot{\phi}_2 = 0$ initially $\quad \ddot{\phi}_2 = ?$

by AMB about A (the moment of A_x A_y will be 0)

$$\left(\frac{\ell}{2}\hat{\imath} \times -mg\,\hat{\jmath}\right) = \frac{1}{12} m\ell^2 \ddot{\phi}_2 \hat{k} + \frac{\ell}{2}\hat{\imath} \times \left(m\vec{a}_{CM_2}\right) \quad \text{— (A)}$$

now

$$\vec{a}_{CM_2} = \vec{a}_A - \cancel{\dot{\phi}_2^2}^{\,0} \vec{r}_{CM_2/A} + \ddot{\phi}_2 \hat{k} \times \vec{r}_{CM_2/A} \longrightarrow (1)$$

$$\vec{a}_A = -\cancel{\dot{\phi}_1^2}\, \vec{r}_{A/O} + \ddot{\phi}_1 \hat{k} \times \vec{r}_{A/O} \longrightarrow (2)$$

0 (initially)

$$\vec{a}_A = (\ddot{\phi}_1 \hat{k}) \times (-\ell\hat{\jmath}) = \ddot{\phi}_1 \ell\, \hat{\imath} \longrightarrow (3)$$

$$\therefore \vec{a}_{CM_2} = \ddot{\phi}_1 \ell + \ddot{\phi}_2 \hat{k} \times \frac{\ell}{2}\hat{\imath} = \ddot{\phi}_1 \ell\, \hat{\imath} + \ddot{\phi}_2 \frac{\ell}{2}\hat{\jmath} \longrightarrow (4)$$

plugging stuff into AMB/A for rod 2 (A)

$$-\frac{mg}{2}\ell\,\hat{k} = \frac{1}{12}m\ell^2\ddot{\phi}_2\hat{k} + \frac{m\ell}{2}\hat{\imath}\times\left(\ddot{\phi}_1\ell\,\hat{\imath} + \ddot{\phi}_2\frac{\ell}{2}\hat{\jmath}\right)$$

$$= \frac{1}{12}m\ell^2\ddot{\phi}_2\hat{k} + \frac{m\ell^2}{4}\ddot{\phi}_2\hat{k}$$

$\therefore \{\ \}\cdot\hat{k}$

$$\ddot{\phi}_2 = -\frac{mg\frac{\ell}{2}}{\frac{m\ell^2}{4}+\frac{1}{12}m\ell^2} = -\frac{g}{\ell}\left(\frac{\frac{1}{2}}{\frac{1}{4}+\frac{1}{12}}\right)$$

$$\boxed{\ddot{\phi}_2 = -\frac{3}{2}\frac{g}{\ell}}$$

by LMB of rod 2

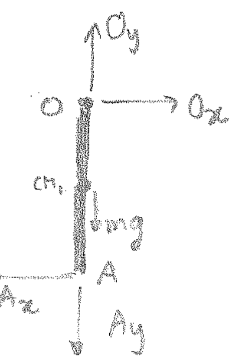
$$A_x\hat{\imath} + A_y\hat{\jmath} - mg\hat{\jmath} = m\left(\ddot{\phi}_1\ell\,\hat{\imath} + \ddot{\phi}_2\frac{\ell}{2}\hat{\jmath}\right)$$

$$\therefore A_x = m\ddot{\phi}_1\ell$$

$$A_y = mg + \frac{m}{2}\ddot{\phi}_2\ell$$

AMB/O of rod 1

FBD



$-mg\hat{\jmath}$ and A_y give zero moment

$$\therefore \vec{r}_{A/O}\times -A_x\hat{\imath} = \frac{1}{12}m\ell^2\ddot{\phi}_1\hat{k} + \left(-\frac{\ell}{2}\hat{\jmath}\right)\times m(\vec{a}_{cm_1})$$

$$-\ell A_x\hat{k} = \frac{1}{12}m\ell^2\ddot{\phi}_1\hat{k} - \frac{\ell}{2}\hat{\jmath}\times m\left(-\dot{\phi}_1^2\vec{r}_{cm_1/O} + \ddot{\phi}_1\hat{k}\times\vec{r}_{cm_1/O}\right)$$

from above

0 initially

$$-\ell^2 m\ddot{\phi}_1\hat{k} = \frac{1}{12}m\ell^2\ddot{\phi}_1\hat{k} - \frac{\ell}{2}\hat{\jmath}\times m\left(\ddot{\phi}_1\frac{\ell}{2}\hat{\imath}\right)$$

$$-m\ell^2\ddot{\phi}_1\hat{k} = \frac{1}{12}m\ell^2\ddot{\phi}_1\hat{k} + \frac{m\ell^2}{4}\ddot{\phi}_1\hat{k} = \left(\frac{1}{12}+\frac{1}{4}\right)m\ell^2\ddot{\phi}_1\hat{k} = \frac{1}{3}m\ell^2\ddot{\phi}$$

$\{\ \}\cdot\hat{k}$

$$-m\ell^2\ddot{\phi}_1 = \frac{1}{3}m\ell^2\ddot{\phi}_1 \Rightarrow \ddot{\phi}_1\left(\frac{1}{3}m\ell^2 + m\ell^2\right) = 0$$

$$\Rightarrow \boxed{\ddot{\phi}_1 = 0}$$

16.3.30 Consider a rigid spoked wheel with no rim. Assume that when it rolls a spoke hits the ground and doesn't bounce. The body just swings around the contact point until the next spoke hits the ground. The uniform spokes have length R. Assume that the mass of the wheel is m, and that the polar moment of inertia about its center is I (use $I = mR^2/2$ if you want to get a better sense of the solution). Assume that just before collision number n, the angular velocity of the wheel is ω_{n-}, the kinetic energy is T_{n-}, the potential energy (you must clearly define your datum) is U_n-. Just after collision n the angular velocity of the wheel is ω_{n+}. The Kinetic Energy is T_{N+}, the potential energy (you must clearly define your datum) is U_{n+}. The wheel has k spokes (pick $k = 4$ if you have trouble with abstraction). This problem is not easy. It can be answered at a variety of levels. The deeper you get into it the more you will learn.

a) What is the relation between ω_{n-} and T_{n-}?

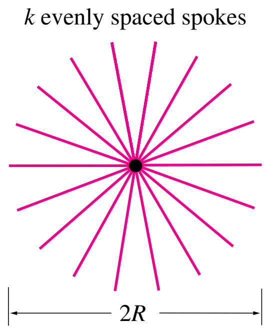
b) What is the relation between ω_{n-} and ω_{n+}?

c) Assume 'rolling' on level ground. What is the relation between ω_{n+} and ω_{n+1}?

d) Assume rolling down hill at slope θ. What is the relation between ω_{n+} and ω_{n+1}?

e) Can it be true that $\omega_{n+} = \omega_{(n+1)+}$? About how fast is the wheel going in this situation?

f) As the number of spokes m goes to infinity, in what senses does this wheel become like an ordinary wheel?



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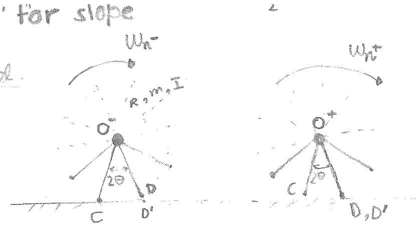
Problem 16.30

NOTE I use 'α' for slope

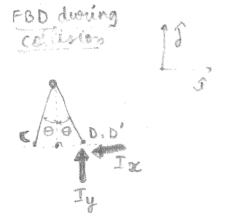
16.69 rimmless wheel.



n^{th} collision$^-$ (before) n^{th} collision$^+$ (after)

- In the figure D' is point on ground where spoke D will collide & stick
- Notice Impulse $\vec{I}$ is not perpendicular to floor
- Datum for P.E we can choose to be ground
- Angle bet spokes $2\theta = \frac{2\pi}{k}$
- ω is negative (clockwise)

FBD during collision



Impulse $= -I_x\hat{\imath} + I_y\hat{\jmath}$

- LMB balance about D'

before

$$\vec{H}^-_{/D'} = I\vec{\omega}_n^- + \vec{r}_{O/D'} \times m\vec{V}_{O^-}$$

$$= I\vec{\omega}_n^- + \vec{r}_{O/D'} \times m(\vec{\omega}_{n^-} \times \vec{r}_{O/C})$$

$$= -I\omega_n^-\hat{k} + (-R\sin\theta\,\hat{\imath} + R\cos\theta\,\hat{\jmath}) \times m(-\omega_n^-\hat{k} \times (R\sin\theta\,\hat{\imath} + R\cos\theta\,\hat{\jmath}))$$

$$= -I\omega_{n^-}\hat{k} - mR^2\cos 2\theta\,\omega_{n^-}\hat{k}$$

$$= -(I + mR^2\cos 2\theta)\,\omega_{n^-}\hat{k}$$

after

$$\vec{H}^+_{/O'} = I\vec{\omega}^+_n \hat{k} + \vec{r}_{O/O'} \times m(\underbrace{\vec{\omega}^+_n \times \vec{r}_{O/O'}}_{\vec{v}_{O^+}})\hat{k}$$

= or simply $I_{O'}\vec{\omega}$

$$= (I + mR^2)\,\omega^+_n\,\hat{k}$$

by AMB/O' (because angular impulse about O' = 0)

$$\vec{H}^+ = \vec{H}^-$$

$$\therefore \boxed{\omega^+_n = \frac{(I + mR^2\cos 2\theta)}{(I + mR^2)}\,\omega^-_n}$$

a) $T_{n^-} = \frac{1}{2} I_c \omega^-_n$ } C is a hinge point b/c collision

$$\boxed{T_{n^-} = \frac{1}{2}(I + mR^2)\,\omega^-_n}$$

b)

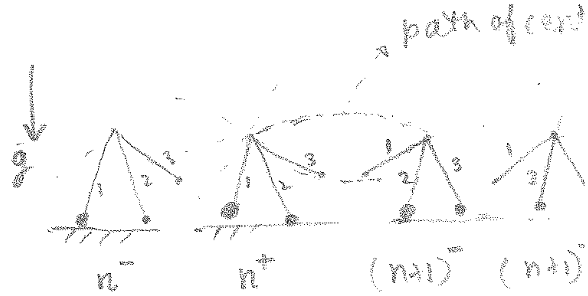
$$\boxed{\omega^+_n = \left(\frac{I + mR^2\cos 2\theta}{I + mR^2}\right)\omega^-_n = \left(1 - \frac{2\sin^2\theta}{\left(\frac{I}{mR^2} + 1\right)}\right)\omega^-_n}$$

using $\cos 2\theta = 1 - 2\sin^2\theta$

c) notice

$$T_{n^+} + U_{n^+} = T_{(n+1)^-} + U_{(n+1)^-}$$

- using energy conservation bet two collisions. (during collision its not conserved)



- for level ground $U_{n^+} = U_{(n+1)^-}$ = because height of centre is same

3

$\therefore \quad T_{n^+} = T_{(n+1)^-}$

using part (a)

$$\omega_{n^+} = \omega_{(n+1)^-}$$

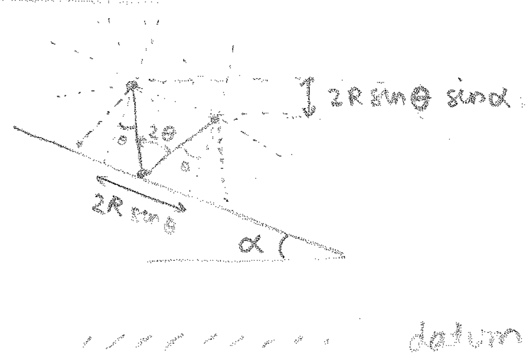
further using part (b)

$$\omega_{n^+} = \omega_{(n+1)^-} = \frac{\omega_{(n+1)^+}}{\left(1 - \frac{2\sin^2\theta}{1+I/mR^2}\right)}$$

$$\therefore \boxed{\omega_{(n+1)^+} = \left(1 - \frac{2\sin^2\theta}{1+I/mR^2}\right)\omega_{n^+}}$$

d) again we can use energy cons. bet. n^+ and $(n+1)^-$ but now with datum at base of hill

$$U_{n^+} \neq U_{(n+1)^-}$$



$$\therefore T_{(n+1)^-} = T_{n^+} + U_{n^+} - U_{(n+1)^-}$$

$$T_{(n+1)^-} = T_{n^+} + mg(2R\sin\theta\sin\alpha)$$

using part (a)

$$\omega_{(n+1)^-} = \omega_{n^+} + \frac{mg(2R\sin\theta\sin\alpha)}{\frac{1}{2}(I+mR^2)}$$

using part (b)

$$\boxed{\omega_{(n+1)^+} = \left(1 - \frac{2\sin^2\theta}{1+I/mR^2}\right)\left(\omega_{n^+} + \frac{4mgR}{I+mR^2}\sin\theta\sin\alpha\right)}$$

e) 1) from part c we can see [for level ground]

$$\omega_{(n+1)^+} = \omega_{n^+} \Rightarrow \text{only if } \frac{2\sin^2\theta}{1+\frac{I}{mR^2}} = 0$$

$$\Rightarrow \text{ie } \theta = 0, \pi, 2\pi \ldots$$

when there are no number of spokes

not possible, it will make the wheel like a rod

won't roll any more

[for level ground]

$$\boxed{\omega^+_{(n+1)} = \omega^+_n \text{ if } \theta = 0}$$

ie number of spokes are ∞ → [Valid for any ω]

e) 2) from part d we see [for slope α]

$$\omega^+_{n+1} = \omega^+_n + \underbrace{\frac{4mgR\sin\theta\sin\alpha}{I+mR^2} - \frac{2\sin^2\theta}{1+I/mR^2}\omega^+_n - \frac{8mgR\sin^3\theta\sin\alpha}{(I+mR^2)\left(1+\frac{I}{mR^2}\right)}}_{\text{term II}}$$

$[\omega^+_{n+1} = \omega^+_n]$ [if] term II $= 0$

$$\text{ie} \quad \omega^+_n = \frac{\left(\frac{4mgR\sin\theta\sin\alpha}{I+mR^2}\right)\left(1 - \frac{2\sin^2\theta}{1+\frac{I}{mR^2}}\right)}{\left(\frac{2\sin^2\theta}{1+I/mR^2}\right)}$$

$$\boxed{\omega^+_n = \frac{4g\sin\theta\sin\alpha}{R\left(1+\frac{I}{mR^2}\right)}\left(\frac{1+I/mR^2}{2\sin^2\theta} - 1\right)}$$

b) when spokes $\to \infty$

$\theta \to 0$

$\omega_n^+ \to \omega_n^-$

$\therefore$ wheel rolls with constant ω on ground

(accelerates on slope)

SIMPLIFICATIONS $I = \frac{1}{2} mR^2 \quad \therefore \frac{I}{mR^2} = \frac{1}{2}$

then,

- $\omega_n^+ = \left(1 - \frac{4}{3}\sin^2\theta\right)\omega_n^-$

and it will continue rolling ie $(\omega_{n+1}^+ = \omega_n^+)$

$\longrightarrow$ never on level ground (unless $\theta = 0$ ie its a disc)

$\longrightarrow$ on slope α if

$$\omega_n^+ = \frac{8}{3} g/R \sin\theta \sin\alpha \left(\frac{3}{4\sin^2\theta} - 1\right)$$

$$= g/R \sin\theta \sin\alpha \left(\frac{2}{\sin^2\theta} - \frac{8}{3}\right)$$

$$= \omega_{n+1}^+ = \omega_{n+m}^+$$

ie at start of slope we want $\omega = g/R \sin\theta \sin\alpha \left(\frac{2}{\sin^2\theta} - \frac{8}{3}\right)$

$\omega > 0$ otherwise it won't roll down

$\therefore \quad \frac{2}{\sin^2\theta} > \frac{8}{3}$ ie $\sin\theta < \frac{\sqrt{3}}{2}$ $\quad \boxed{\theta < 60}$

we need this otherwise it will stop after one rub on any slope