Problems for Chapter 9

Unconstrained 1D dynamics

9.1 Force and motion in 1D

Preparatory Problems

9.1 Give three examples of unconstrained 1D motion of real life objects where you can use the particle idealization for dynamic calculations.

9.2 A car is going downhill on a constant slope straight road. You consider the car as a particle for finding out its speed at the end of the road. For specifying initial velocity, which point on the car would you consider?

9.3 The acceleration of a particle is given as a function of time, $a(t)$. Is this information sufficient to find the speed of the particle at the end of, say, T seconds?

9.4 If a particle has constant acceleration, its linear momentum (a) remains constant, (b) changes linearly with time, or (c) changes quadratically with time. Which one is true?

9.5 In a motorcycle race on a straight track, the speed of a motorcyclist at the 200 mmark is recorded. Given that the rider started from rest position, you can find the acceleration of the motorcyle from the given information, provided the acceleration (a) is constant, (b) varies linearly with time, or (c) is a sinusoidal function of time.

9.6 The force acting on a particle is given as a function of time. If you plot the force function and find the area under the graph, you can determine (a) the net displacement of the particle, (b) average velocity of the particle, or (c) the change in linear momentum of the particle.

9.7 If the linear momentum of a body remains constant in time, it must have (a) a constant force acting on it, (b) no net force acting on it, or (c) a sinusoidal force acting on it.

9.8 The distance between two points in a bicycle race is 10 km. How many minutes does a bicyclist take to cover this distance if he/she maintains a constant speed of 15 mph.

9.9 A 5 kNconstant force acts on an object of mass 1 kgfor 5 seconds. If the object was initially at rest, find the final speed of the object.

9.10 Given that $\dot{x} = k_1 + k_2 t$, $k_1 = 1\,\text{ft/s}$, $k_2 = 1\,\text{ft/s}^2$, and $x(0) = 1\,\text{ft}$, what is the displacement at the end of 10 seconds?

More-Involved Problems

9.11 Find $x(3\,\text{s})$ given that

$$\dot{x} = x/(1\,\text{s}) \quad \text{and} \quad x(0) = 1\,\text{m}$$

or, expressed slightly differently,

$$\dot{x} = cx \quad \text{and} \quad x(0) = x_0,$$

where $c = 1\,\text{s}^{-1}$ and $x_0 = 1\,\text{m}$. Make a sketch of x versus t.

9.12 A ball of mass m is dropped from rest at a height h above the ground. Find the position and velocity as a function of time. Neglect air friction. When does the ball hit the ground? What is the velocity of the ball just before it hits?

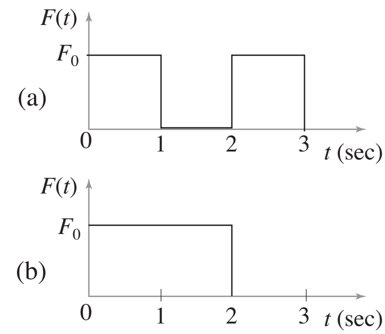
9.13 The speed of a particle varies sinusoidally as $v = A\sin[(3\,\text{rad/s})t]$, where $A = 0.5\,\text{m/s}$. Let the initial position of the particle be $x(0) = 0$. Find the position of the particle at $t = \pi/2\,\text{s}$.

9.14 The speed of a particle is directly proportional to its position and is given as $\dot{x} = x/\,\text{s}$. If the initial position, $x(0\,\text{s}) = 1\,\text{m}$, how far would the particle be from the origin in 5 seconds?

9.15 Consider a force $F(t)$ acting on a cart for a short duration. In case (a), the force acts in two impulses of one second duration each as shown in Fig. 9.15. In case (b), the force acts continuously for two seconds. Given that the mass of the cart is 10 kg, $v(0) = 0$, and $F_0 = 10\,\text{N}$, for each force profile,

a) Find the speed of the cart at the end of 3 seconds, and
b) Find the distance travelled by the cart in 3 seconds.

Comment on your answers for the two cases.



problem 9.15:

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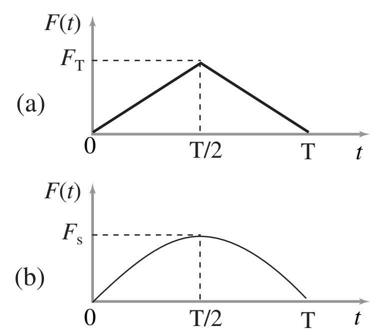
9.16 A car of mass m is accelerated by applying a triangular force profile shown in Fig. 9.16(a). Find the speed of the car at $t = T$ seconds. If the same speed is to be achieved at t=T seconds with a sinusoidal force profile, $F(t) = F_s \sin\frac{\pi t}{T}$, find the required force magnitude F_s.



problem 9.16:

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9.17 A particle of mass $m = 1\,\text{kg}$ is acted upon by a short duration force given by

$$F(t) = \begin{cases} F_0 t & 0 \le t \le 1\,\text{s} \\ F_0(2-t) & 1\,\text{s} < t \le 2\,\text{s} \end{cases}$$

where $F_0 = 5\,\text{N}$. If the particle starts from rest, find the speed of the particle as a function time. Sketch the given force profile as a function of time and draw the corresponding speed $v(t)$ as a function of time. What is the speed of the particle at $t = 2$ s?

9.18 A ball of mass m is dropped vertically from rest at a height h above the ground. Air resistance causes a drag force on the ball directly proportional to the speed v of the ball, $F_d = bv$. Find the velocity and position of the ball as a function of time. Find the velocity as a function of position. Gravity is non-negligible, of course.

9.19 In quadratic drag problems, the deceleration is proportional to the square of velocity, *i.e.*, $a = \frac{dv}{dt} = -kv^2$. Assume that a particle with initial velocity $v(0) = v_0$. experiences quadratic drag.

a) How long does it take for the particle to reduce its speed to half of its initial speed (*i.e.*, find t such that $v(t) = \frac{1}{2}v_0$)?
b) Find the position of the particle as a function of velocity. How far does the particle move from its initial position when its velocity drops to half its initial value?

9.20 A sinusoidal force acts on a 1 kg mass as shown in the figure and graph below. The mass is initially still; i. e.,

$$x(0) = v(0) = 0$$

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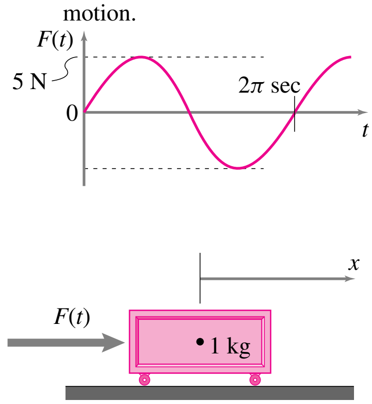
a) What is the velocity of the mass after 2π seconds?
b) What is the position of the mass after 2π seconds?
c) Plot position x versus time t for the motion.



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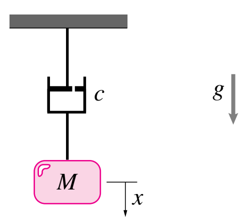
9.21 A motorcycle accelerates from 0 mph to 60 mph in 5 seconds. Find the average acceleration in m/s^2. How does this acceleration compare with g, the acceleration of an object falling near the earth's surface?

9.22 A car moves on a straight road with an initial velocity $v_0 = 30\,m/s$. Let its position $x = 0$ at $t = 0$. For the first 5 s it has no acceleration, and thereafter it brakes with a retarding force that gives it a constant acceleration $a_x = -10m/s^2$. Calculate the velocity and the x-coordinate of the car when $t = 8$ s and when $t = 12$ s, and find the maximum distance travelled by the car.

9.23 A grain of sugar falling through honey has a negative acceleration proportional to the difference between its velocity and its 'terminal' velocity (which is a known constant v_t). Write this sentence as a differential equation, defining any constants you need. Solve the equation assuming some given initial velocity v_0.

9.24 The mass-dashpot system shown below is released from rest at $x = 0$. Determine an equation of motion for the particle of mass m that involves only $\dot{x}$ and x (a first-order ordinary differential equation). The damping coefficient of the dashpot is c.



problem 9.24:

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9.25 Due to gravity, a particle falls in air with a drag force proportional to the speed squared.

1. Write $\sum \vec{F} = m\vec{a}$ in terms of variables you clearly define,
2. find a constant speed motion that satisfies your differential equation,
3. pick numerical values for your constants and for the initial height. Assume the initial speed is zero
   a) set up the equation for numerical solution,
   b) solve the equation on the computer,
   c) make a plot with your computer solution and show how that plot supports your answer to (2).

9.26 A force pulls a particle of mass m towards the origin according to the law (assume same equation works for $x > 0, x < 0$)

$$F = Ax + Bx^2 + C\dot{x}$$

Assume $\dot{x}(0) = 0$.
Using numerical solution, find values of A, B, C, m, and x_0 so that

1. the mass never crosses the origin,
2. the mass crosses the origin once,
3. the mass crosses the origin many times.

[Hint: Vary one parameter at a time and choose a different set of parameter values for each case.]

9.2 Energy methods in 1D

Preparatory Problems

9.27 A mass m is at position x moving at velocity v and being acted upon by force F. For each of the quantities below:

i give the symbol used for the quantity
ii describe the quantity in words
iii give a formula to evaluate the quantity in terms of some or all of m, x, v and F and any other variables you may need.
iv Give the standard units for the quantity in the SI system.
v Give the standard units for the quantity in the English system.

a) Power
b) Kinetic energy
c) Work
d) Potential energy

9.28 Write an equation relating the two words in each of these pairs. If any conditions or descriptions of the situation are needed, give them. If you know more than one equation (or form for a given equation), give all that you know. All should be given in the context of this section: 1D motion.

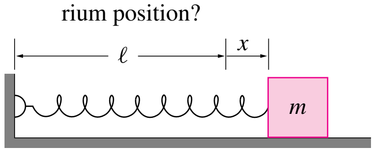
a) work and power
b) work and kinetic energy
c) power and kinetic energy
d) work and potential energy
e) potential energy and kinetic energy

9.29 A force $F = F_0 \sin(ct)$ acts on a particle with mass $m = 3\,\text{kg}$ which has position $x = 3\,\text{m}$, velocity $v = 5\,\text{m/s}$ at $t = 2\,\text{s}$. $F_0 = 4\,\text{N}$ and $c = 2/\,\text{s}$. At $t = 2\,\text{s}$ evaluate (give numbers and units):
a) a,
b) E_K,
c) P,
d) $\dot{E}_K$,
e) the rate at which the force is doing work.

9.30 A force only depends on position according to $F = C_0 + C_1 x$ where C_0 and C_1 are constants. What is the work done by this force when the point to which it is applied moves from x_1 to x_2. Answer in terms of some or all of C_0, C_1, x_1 and x_2.

9.31 Find the potential E_P associated with each of these force fields.
a) $F = 0$.
b) $F = F_0$ (=constant).
c) $F = kx$.
d) $F = A \sin(x/x_0)$.
e) $F = c/x^2$.

9.32 Consider a spring-mass system with $m = 2\,\text{kg}$ and $k = 5\,\text{N/m}$. The mass is pulled to the right a distance $x = x_0 = 0.5\,\text{m}$ from the unstretched position and released from rest. No external forces act on the mass.
a) What are the initial potential and kinetic energy of the system?
b) What is the potential and kinetic energy of the system as the mass passes through the static equilibrium (unstretched spring) position?
c) What is the speed of the mass when it passes through the static equilibrium position?



problem 9.32:

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9.33 A mass m is held in place by a spring whose restoring force is $T(x) = kx$. Derive the equation of motion of the system (that is, find the acceleration a in terms of x).

9.34 The peak propulsion force on a 4-wheel-drive car is about μmg where $\mu \approx 1$ for rubber on road (a bit more for fancy racing tires). Assume a car starts from rest at position zero. Answer the following questions with symbols and with numbers (using $\mu = 1$, $m = 1000\,\text{kg}$, and $g = 10\,\text{m/s}^2$).
a) What is the minimum distance required to reach $v_1 = 60\,\text{mph}$?
b) What is the extra distance required to get from $v_1 = 60\,\text{mph}$ up to $v_2 = 70\,\text{mph}$?
c) What is the peak power used by the engine in getting up to $v_1 = 60\,\text{mph}$?

9.35 A car (mass $m = 1000\,\text{kg}$) traveling at speed $v_0 30\,\text{m/s}$ crashes into a brick wall and comes to a stop as the front end of the car compresses a distance $d = 1\,\text{m}$. Answer with symbols and numbers. Assume constant deceleration during the crash. Neglect the mass of crushing region of the car.
a) What is the total energy dissipated in the crash?
b) What is the force of the car on the wall?
c) What is the force of the wall on the car?
d) What is the deceleration of the car passengers (assuming they are strapped in and move with the bulk of the car). Answer in g's?
e) Assuming an $m_p = 50\,\text{kg}$ person, what is the force of the seat belts on the person (answer in body weight).
f) If a parent was holding a 15 kg child on his lap, what force would he need to hold on to the child through the crash (answer in N and in number of child body weights).

More-Involved Problems

9.36 A 10 year old ($m = 90\,\text{lbf}$) jumps off an $h = 10\,\text{ft}$ wall and accelerates down with $g = 32\,\text{ft/s}^2$. She bends her legs a distance $d = 1\,\text{ft}$ to brake her fall and bring her body to a stop. Neglect the mass of her legs. Assume constant deceleration as she brakes the fall.
a) What is the total distance her body falls?
b) What is the potential energy lost?
c) How much work must be absorbed by her legs?
d) What is the force of her legs on her body (answer in symbols, numbers and numbers of body weight)?

9.37 In traditional archery, when pulling an arrow back the force increases approximately linearly up to the peak 'draw force' F_{draw} that varies from about $F_{draw} = 25\,\text{lbf}$ for a bow made for a small person to about $F_{draw} = 75\,\text{lbf}$ for a bow made for a big strong person. The distance the arrow is pulled back, the draw length ℓ_{draw}, varies from about $\ell_{draw} = 2\,\text{ft}$ for a small adult to about 30 inch for a big adult. An arrow has mass of about 300 grain (1 grain $\approx 64.8 milli$ gm, so an arrow has mass of about $19.44 \approx 20\,\text{gm} \approx 3/4$ ounce). Give all answers in symbols and numbers.
a) What is the range of speeds you can expect an arrow to fly?
b) What is the range of heights an arrow might go if shot straight up?

9.38 A big person ($m = 100\,\text{kg}$) jumps on a trampoline which we model as a linear spring with stiffness k. You know that the trampoline deflects $d_0 = 20\,\text{cm}$ under the stationary weight mg of the person (use $g = 10\,\text{m/s}^2$). Assume there is no dissipation and the person is jumping repeatedly a height $h = 1\,\text{m}$ above the unloaded surface of the trampoline. Give all answers with symbols and numbers.
a) What is the stiffness k of the spring (answer in terms of some or all of m, g and d_0).
b) What is the maximum deflection of the trampoline during these jumps?
c) What is the peak force of the trampoline on the jumper? (answer in symbols, Newtons, and numbers of body weights).

9.39 For the car of problem 9.34 what is the average power required to reach speed v_1? There are two plausible ways to calculate this power:

$$\bar{P}_1 = \int_0^x P(x')\,dx'/x \text{ and}$$
$$\bar{P}_2 = \int_0^t P(t')\,dt'/t\,.$$

Use both. Do the two methods give the same answer? If so, why, if not, why not?

9.40 For problem 9.35 which answers would change in which way if the deceleration was not exactly constant during the crash? That is, for which quantities would be bigger, which smaller, which the same, for which would the answer depend on the nature of the non-constant acceleration?

9.41 The earth's gravitational pull on a mass m is $F = -\frac{mgR^2}{r^2}$, where mg is the pull at the surface of the earth and R is the radius of the earth. Assume a ballistic rocket is shot straight with a launch velocity of v_0 (measured in a 'fixed' not-rotating-with-the-earth frame). Assume the rocket goes in a straight radial line as the earth turns underneath it (relative to the surface of the earth this rocket would be launched somewhat to the West to cancel the earths rotation). Assume the period of active thrust is negligibly short (hence the word *ballistic*: "relating to or characteristic of the motion of objects moving under their own momentum and the force of gravity").

a) Solve for v as a function of r (and some or all of m, g, R and v_0.
b) Find the maximum height the rocket reaches.
c) Find the 'escape velocity' v_{escape}, the minimum launch speed needed for the rocket to never return.
d) On one graph plot height (r or $r - R$) vs t for a v_0 just below v_{escape} and for v_0 just greater than v_{escape}. If you use numerical methods to make this plot use $g = 10\,\text{m/s}^2$, $R = 6400\,\text{km}$, and $m = 1\,\text{kg}$. Make sure your axis are such that you can see a clear qualitative difference between the two cases.

9.42 The power available to a very strong accelerating cyclist over short periods of time (up to, say, about 1 minute) is about 1 horsepower. Assume a rider starts from rest and uses this constant power. Assume a mass (bike + rider) of 150 lbm, a realistic drag force of $.006\,\text{lbf}/(\text{ft/s})^2 v^2$. Neglect other drag forces.

1. What is the peak speed of the cyclist?
2. Using analytic or numerical methods make a plot of speed vs. time.
3. What is the acceleration as $t \to \infty$ in this solution?
4. What is the acceleration as $t \to 0$ in your solution?

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Also see several problems in the harmonic oscillator section.

9.3 Elementary vibration analysis

Preparatory Problems

9.43 The basic model.

a) Draw a spring (k) mass (m) system in a configuration where the spring is stretched.
b) On the drawing indicate the variable x.
c) Draw a free body diagram of the mass.
d) Write the equation of linear momentum balance for the mass.
e) Rearrange the momentum balance equation to get the Harmonic-oscillator equation in standard form.
f) Write the general solution to the harmonic oscillator equation in 2 different ways (one as a sum of a sine and cosine function and one as a phase shifted sine or cosine function).
g) What is the natural frequency of this system?
h) What is the period?
i) What is the frequency (or circular frequency)?
j) Find the solution for the special case that the mass is released from rest at $x(0) = x_0$.
   - give the analytic expression.
   - plot the position vs time for at least one whole cycle of motion.
   - with the same time scale, plot velocity vs time (what is the peak velocity).
   - with the same time scale, plot both the potential and kinetic energies vs time.
k) Find the solution for the special case that the mass is launched at v_0 from the rest position (just the analytic form, no need to repeat all the parts just above).

9.44 Does the function $x = C_1 e^{\lambda t} + C_2 e^{-\lambda t}$ satisfy the harmonic oscillator equation $\ddot{x} + \lambda^2 x = 0$ for any, possibly special, values of C_1 and C_2? Show that it does or does not.

9.45 Given that $\ddot{x} = -(1/\text{s}^2)x$, $x(0) = 1\,\text{m}$, and $\dot{x}(0) = 0$ find:

a) $x(\pi\ \text{s}) = ?$
b) $\dot{x}(\pi\ \text{s}) = ?$

9.46 Given that $\ddot{x} + x = 0$, $x(0) = 1$, and $\dot{x}(0) = 0$, find the value of x at $t = \pi/2$ s.

9.47 Given that $\ddot{x} + \lambda^2 x = C_0$, $x(0) = x_0$, and $\dot{x}(0) = 0$, find the value of x at $t = \pi/\lambda$ s.

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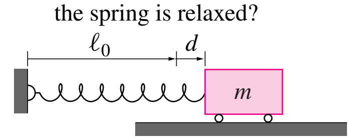
9.48 A mass m is connected to a spring k and released from rest with the spring stretched a distance d from its static equilibrium position. It then oscillates back and forth repeatedly crossing the equilibrium. How much time passes from release until the mass moves through the equilibrium position for the second time? Neglect gravity and friction. Answer in terms of some or all of m, k, and d.

9.49 A spring with rest length ℓ_0 is attached to a mass m which slides frictionlessly on a horizontal ground as shown. At time $t = 0$ the mass is released with no initial speed with the spring stretched a distance d. *[Remember to define any coordinates or base vectors you use.]*

a) What is the acceleration of the mass just after release?
b) Find a differential equation which describes the horizontal motion of the mass.
c) What is the position of the mass at an arbitrary time t?
d) What is the speed of the mass when it passes through the position where the spring is relaxed?



problem 9.49:

Filename:s97f1

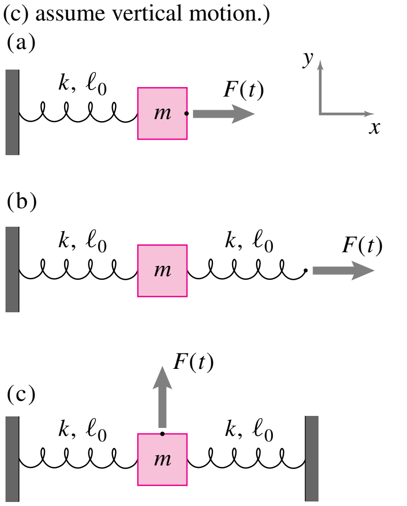
9.50 Reconsider the spring-mass system from problem 9.49.

a) Find the potential and kinetic energy of the spring mass system as functions of time.

b) Assigning numerical values to the various variables, use a computer to make a plot of the potential and kinetic energy as a function of time for several periods of oscillation. Are the potential and kinetic energy ever equal at the same time? If so, at what position $x(t)$?

c) Make a plot of kinetic energy versus potential energy. What is the phase relationship between the kinetic and potential energy?

9.51 For the three spring-mass systems shown in the figure, find the equation of motion of the mass in each case. All springs are massless and are shown in their relaxed states. Ignore gravity. (In problem (c) assume vertical motion.)



problem 9.51:

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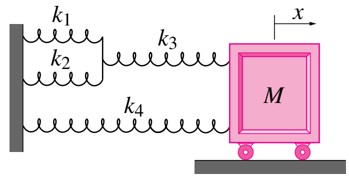
More-Involved Problems

9.52 A spring and mass system is shown in the figure.

a) First, as a review, let k_1, k_2, and k_3 equal zero and k_4 be nonzero. What is the natural frequency of this system?

b) Now, let all the springs have nonzero stiffness. What is the stiffness of a single spring equivalent to the combination of k_1, k_2, k_3, k_4? What is the frequency of oscillation of mass M?



problem 9.52:

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9.53 Mass hanging from a spring. A mass m is hanging from a spring with constant k which has the length l_0 when it is relaxed (i. e., when no mass is attached). It only moves vertically.

a) Draw a Free Body Diagram of the mass.

b) Write the equation of linear momentum balance.

c) Reduce this equation to a standard differential equation in x, the position of the mass.

d) Verify that one solution is that $x(t)$ is constant at $x = l_0 + mg/k$.

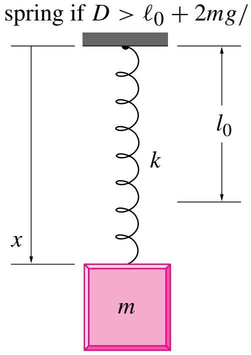
e) What is the meaning of that solution? (That is, describe in words what is going on.)

f) Define a new variable $\hat{x} = x - (l_0 + mg/k)$. Substitute $x = \hat{x} + (l_0 + mg/k)$ into your differential equation and note that the equation is simpler in terms of the variable $\hat{x}$.

g) Assume that the mass is released from an an initial position of $x = D$. What is the motion of the mass?

h) What is the period of oscillation of this oscillating mass?

i) Why might this solution not make physical sense for a long, soft spring if $D > \ell_0 + 2mg/k$)?



problem 9.53:

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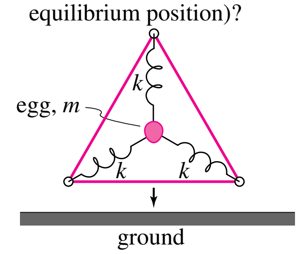
9.54 One of the winners in an egg-drop contest was a structure in which rubber bands held the egg at the center of it. Here is a model. Consider the egg to be a particle of mass m and the springs to be linear with spring constants k. Consider only a two-dimensional version of the winning design as shown in the figure. Assume the frame hits the ground on one of the straight sections. Assume small motions (deflection $\ll$ side-length) and that the springs do not buckle.

a) what will be the frequency of vibration of the egg after impact?

b) What is the maximum vertical deflection of the egg (relative to its equilibrium position)?



problem 9.54:

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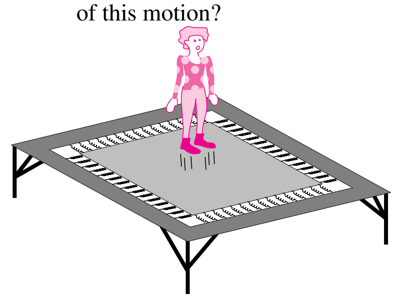
9.55 A person jumps on a trampoline. The trampoline is modeled as having an effective vertical undamped linear spring with stiffness $k = 200\,\mathrm{lbf/\,ft}$. The person is modeled as a rigid mass $m = 150\,\mathrm{lbm}$. $g = 32.2\,\mathrm{ft/s^2}$.

a) What is the period of motion if the person's motion is so small that her feet never leave the trampoline?

b) What is the maximum amplitude of motion for which her feet never leave the trampoline?

c) (harder) If she repeatedly jumps so that her feet clear the trampoline by a height $h = 5\,\mathrm{ft}$, what is the period of this motion?



problem 9.55: A person jumps on a trampoline.

Filename:pfigure3-trampoline

9.56 A mass moves on a frictionless surface. It is connected to a dashpot with damping coefficient b to its right and a spring with constant k and rest length ℓ to its left. At the instant of interest, the mass is moving to the right and the spring is stretched a distance x from its position where the spring is unstretched. There is gravity.

a) Draw a free body diagram of the mass at the instant of interest.
b) Derive the equation of motion of the mass.

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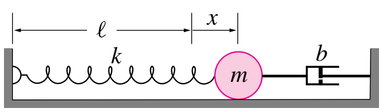

problem 9.56:

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9.57 The equation of motion of an unforced mass-spring-dashpot system is, $m\ddot{x} + c\dot{x} + kx = 0$, as discussed in the text. For a system with $m = 0.4\,\text{kg}$, $c = 10\,\text{kg/s}$, and $k = 5\,\text{N/m}$,

a) Find whether the system is underdamped, critically damped, or overdamped.
b) Sketch a typical solution of the system.
c) Make an accurate plot of the response of the system (displacement vs time) for the initial conditions $x(0) = 0.1\,\text{m}$ and $\dot{x}(0) = 0$.

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9.58 Experiments conducted on free oscillations of a damped oscillator reveal that the amplitude of oscillations drops to 25% of its peak value in just 3 periods of oscillations. The period os oscillation is measured to be 0.6 s and the mass of the system is known to be 1.2 kg. Find the damping coefficient and the spring stiffness of the system.

9.59 You are required to design a mass-spring-dashpot system that, if disturbed, returns to its equilibrium position the quickest. You are given a mass, $m = 1\,\text{kg}$, and a damper with $c = 10\,\text{kg/s}$. What should be the stiffness of the spring? Your solution needs to include your definition of "'quickest".

9.4 Coupled motion in 1D

The primary emphasis of this section is setting up correct differential equations (without sign errors) and solving these equations on the computer. Experts note: normal modes are not covered here.

Preparatory Problems

9.60 Write the following set of coupled second order ODE's as a system of first order ODE's.

$$\begin{aligned}\ddot{x}_1 &= k_2(x_2 - x_1) - k_1 x_1\\ \ddot{x}_2 &= k_3 x_2 - k_2(x_2 - x_1)\end{aligned}$$

9.61 See also problem 9.62. The solution of a set of a second order differential equations is:

$$\begin{aligned}\xi(t) &= A\sin\omega t + B\cos\omega t + \xi^*\\ \dot{\xi}(t) &= A\omega\cos\omega t - B\omega\sin\omega t,\end{aligned}$$

where A and B are constants to be determined from initial conditions. Assume A and B are the only unknowns and write the equations in matrix form to solve for A and B in terms of $\xi(0)$ and $\dot{\xi}(0)$.

9.62 Solve for the constants A and B in Problem 9.61 using the matrix form, if $\xi(0) = 0$, $\dot{\xi}(0) = 0.5$, $\omega = 0.5\,\text{rad/s}$ and $\xi^* = 0.2$.

9.63 A set of first order linear differential equations is given:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 + kx_1 + cx_2 &= 0\end{aligned}$$

Write these equations in the form $\dot{\vec{x}} = [\mathbf{A}]\vec{x}$, where $\vec{x} = \begin{Bmatrix} x_1\\ x_2\end{Bmatrix}$.

9.64 Write the following pair of coupled ODE's as a set of first order ODE's.

$$\begin{aligned}\ddot{x}_1 + x_1 &= \dot{x}_2\sin t\\ \ddot{x}_2 + x_2 &= \dot{x}_1\cos t\end{aligned}$$

9.65 The following set of differential equations can not only be written in first order form but in matrix form $\dot{\vec{x}} = [\mathbf{A}]\vec{x} + \vec{c}$. In general things are not so simple, but this *linear* case is prevalent in the analytic study of dynamical systems.

$$\begin{aligned}\dot{x}_1 &= x_3\\ \dot{x}_2 &= x_4\\ \dot{x}_3 + 5\Omega^2 x_1 - 4\Omega^2 x_2 &= 2\Omega^2 v_1^*\\ \dot{x}_4 - 4\Omega^2 x_1 + 5\Omega^2 x_2 &= -\Omega^2 v_1^*\end{aligned}$$

9.66 Write each of the following equations as a system of first order ODE's.

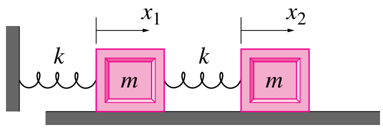
a) $\ddot{\theta} + \lambda^2\theta = \cos t$,
b) $\ddot{x} + 2p\dot{x} + kx = 0$,
c) $\ddot{x} + 2c\dot{x} + k\sin x = 0$.

9.67 A train is moving at constant absolute velocity $v\hat{\imath}$. A passenger, idealized as a point mass, is walking at an absolute absolute velocity $u\hat{\imath}$, where $u > v$. What is the velocity of the passenger relative to the train?

• • •

9.68 Two equal masses, each denoted by the letter m, are on an air track. One mass is connected by a spring to the end of the track. The other mass is connected by a spring to the first mass. The two spring constants are equal and represented by the letter k. In the rest (springs are relaxed) configuration, the masses are a distance ℓ apart. Motion of the two masses x_1 and x_2 is measured relative to this configuration.

a) Draw a free body diagram for each mass.
b) Write the equation of linear momentum balance for each mass.
c) Write the equations as a system of first order ODEs.
d) Pick parameter values and initial conditions of your choice and simulate a motion of this system. Make a plot of the motion of, say, one of the masses vs time,
e) Explain how your plot does or does not make sense in terms of your understanding of this system. Is the initial motion in the right direction? Are the solutions periodic? Bounded? etc.

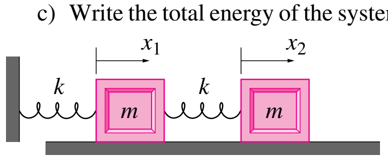


problem 9.68:

Filename:pfigure-s94f1p4

9.69 Two equal masses, each denoted by the letter m, are on an air track. One mass is connected by a spring to the end of the track. The other mass is connected by a spring to the first mass. The two spring constants are equal and represented by the letter k. In the rest configuration (springs are relaxed) the masses are a distance ℓ apart. Motion of the two masses x_1 and x_2 is measured relative to this configuration.

a) Write the potential energy of the system for arbitrary displacements x_1 and x_2 at some time t.
b) Write the kinetic energy of the system at the same time t in terms of $\dot{x}_1$, $\dot{x}_2$, m, and k.
c) Write the total energy of the system.

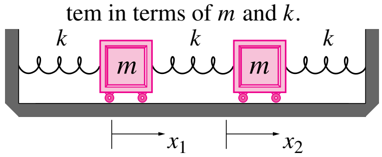


problem 9.69:

Filename:pfigure-twomassenergy

9.70 Normal Modes. Three equal springs (k) hold two equal masses (m) in place. There is no friction. x_1 and x_2 are the displacements of the masses from their equilibrium positions.

a) How many independent normal modes of vibration are there for this system?
b) Assume the system is in a normal mode of vibration and it is observed that $x_1 = A\sin(ct) + B\cos(ct)$ where A, B, and c are constants. What is $x_2(t)$? (The answer is not unique. You may express your answer in terms of any of A, B, c, m and k.)
c) Find all of the frequencies of normal-mode-vibration for this system in terms of m and k.

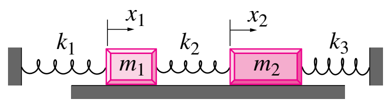


problem 9.70:

Filename:pfigure-f93f2

9.71 A two degree of freedom mass-spring system, made up of two unequal masses m_1 and m_2 and three springs with unequal stiffnesses k_1, k_2 and k_3, is shown in the figure. All three springs are relaxed in the configuration shown. Neglect friction.

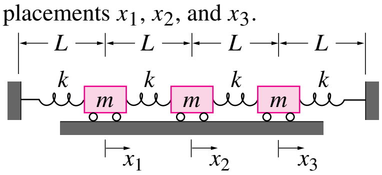
a) Derive the equations of motion for the two masses.
b) Does each mass undergo simple harmonic motion?



problem 9.71:

Filename:pfigure-s94h5p1

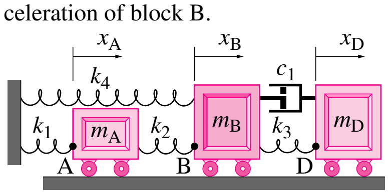
9.72 For the three-mass system shown, draw a free body diagram of each mass. Write the spring forces in terms of the displacements x_1, x_2, and x_3.



problem 9.72:

Filename:s92f1p1

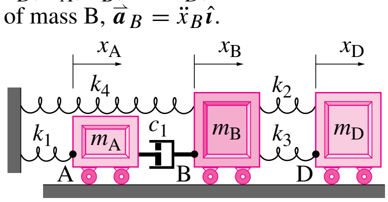
9.73 The springs shown are relaxed when $x_A = x_B = x_D = 0$. In terms of some or all of $m_A, m_B, m_D, x_A, x_B, x_D, \dot{x}_A, \dot{x}_B, \dot{x}_C$, and k_1, k_2, k_3, k_4, c_1, and F, find the acceleration of block B.



problem 9.73:

Filename:pfigure-s95q3

9.74 A system of three masses, four springs, and one damper are connected as shown. Assume that all the springs are relaxed when $x_A = x_B = x_D = 0$. Given k_1, k_2, k_3, k_4, c_1, m_A, m_B, m_D, x_A, x_B, x_D, $\dot{x}_A$, $\dot{x}_B$, and $\dot{x}_D$, find the acceleration of mass B, $\vec{a}_B = \ddot{x}_B\hat{\imath}$.

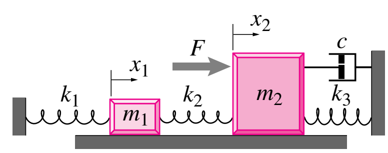


problem 9.74:

Filename:pfigure-s95f1

9.75 Equations of motion. Two masses are connected to fixed supports and each other with the three springs and dashpot shown. The force F acts on mass 2. The displacements x_1 and x_2 are defined so that $x_1 = x_2 = 0$ when the springs are unstretched. The ground is frictionless. The governing equations for the system shown can be written in first order form if we define $v_1 \equiv \dot{x}_1$ and $v_2 \equiv \dot{x}_2$.

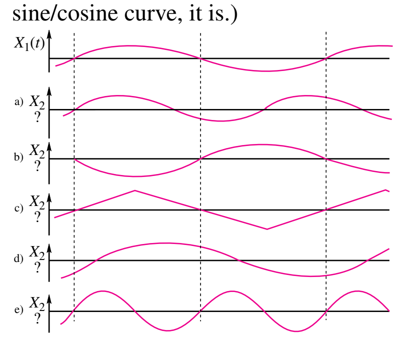
a) Write the governing equations in a neat first order form. Your equations should be in terms of any or all of the constants m_1, m_2, k_1, k_2,k_3, C, the constant force F, and t. Getting the signs right is important.
b) Write computer commands to find and plot $v_1(t)$ for 10 units of time. Make up appropriate initial conditions.
c) For constants and initial conditions of your choosing, plot x_1 vs t for enough time so that decaying erratic oscillations can be observed.



problem 9.75:

Filename:p-f96-f-3

9.76 $x_1(t)$ and $x_2(t)$ are measured positions on two points of a vibrating structure. $x_1(t)$ is shown. Some candidates for $x_2(t)$ are shown. Which of the $x_2(t)$ could possibly be associated with a normal mode vibration of the structure? Answer "could" or "could not" next to each choice. (If a curve looks like it is meant to be a sine/cosine curve, it is.)



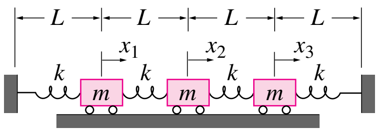
problem 9.76:

Filename:pfigure-blue-144-1

9.77 For the three-mass system shown, one of the normal modes is described with the eigenvector (1, 0, -1). Assume $x_1 = x_2 = x_3 = 0$ when all the springs are fully relaxed.

a) What is the angular frequency ω for this mode? Answer in terms of L, m, k, and g. (Hint: Note that in this mode of vibration the middle mass does not move.)

b) Make a neat plot of x_2 versus x_1 for one cycle of vibration with this mode.



problem 9.77:

Filename:pfigure-blue-160-2

9.78 The three beads of masses m, $2m$, and m connected by massless linear springs of constant k slide freely on a straight rod. Let x_i denote the displacement of the i^{th} bead from its equilibrium position at rest.

a) Write expressions for the total kinetic and potential energies.

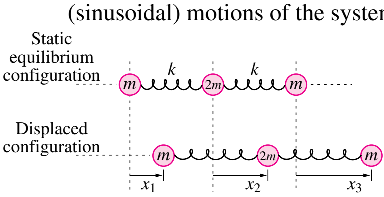
b) Write an expression for the total linear momentum.

c) Draw free body diagrams for the beads and use Newton's second law to derive the equations for motion for the system.

d) Verify that total energy and linear momentum are both conserved.

e) Show that the center of mass must either remain at rest or move at constant velocity.

f) What can you say about vibratory (sinusoidal) motions of the system?



problem 9.78:

Filename:pfigure-blue-161-1

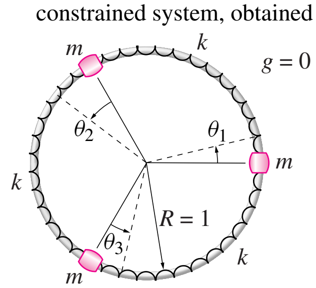
9.79 The system shown below comprises three identical beads of mass m that can slide frictionlessly on the rigid, immobile, circular hoop. The beads are connected by three identical linear springs of stiffness k, wound around the hoop as shown and equally spaced when the springs are unstretched (the strings are unstretched when $\theta_1 = \theta_2 = \theta_3 = 0$.)

a) Determine the natural frequencies and associated mode shapes for the system. (Hint: you should be able to deduce a 'rigid-body' mode by inspection.)

b) If your calculations in (a) are correct, then you should have also obtained the mode shape $(0, 1, -1)^T$. Write down the most general set of initial conditions so that the ensuing motion of the system is simple harmonic in that mode shape.

c) Since $(0, 1, -1)^T$ is a mode shape, then by "symmetry", $(-1, 0, 1)^T$ and $(1, -1, 0)^T$ are also mode shapes (draw a picture). Explain how we can have three mode shapes associated with the same frequency.

d) Without doing any calculations, compare the frequencies of the constrained system to those of the unconstrained system, obtained in (a).

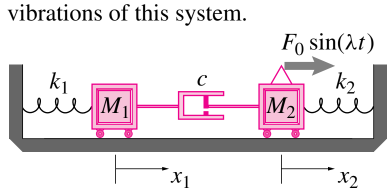


problem 9.79:

Filename:pfigure-blue-158-1

9.80 Equations of motion. Two masses are connected to fixed supports and each other with the two springs and dashpot shown. The displacements x_1 and x_2 are defined so that $x_1 = x_2 = 0$ when both springs are unstretched.

For the special case that $C = 0$ and $F_0 = 0$ clearly define two different set of initial conditions that lead to normal mode vibrations of this system.



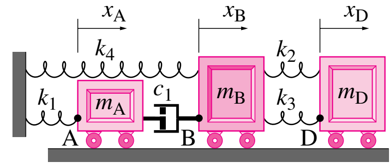
problem 9.80:

Filename:p-s96-p3-1

9.81 As in problem 9.74, a system of three masses, four springs, and one damper are connected as shown. Assume that all the springs are relaxed when $x_A = x_B = x_D = 0$.

a) In the special case when $k_1 = k_2 = k_3 = k_4 = k$, $c_1 = 0$, and $m_A = m_B = m_D = m$, find a normal mode of vibration. Define it in any clear way and explain or show why it is a normal mode in any clear way.

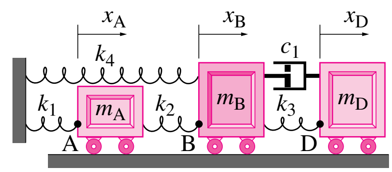
b) In the same special case as in (a) above, find another normal mode of vibration.



problem 9.81:

Filename:pfigure-s95f1a

9.82 As in problem **??**, a system of three masses, four springs, and one damper are connected as shown. In the special case when $c_1 = 0$, find the normal modes of vibration.



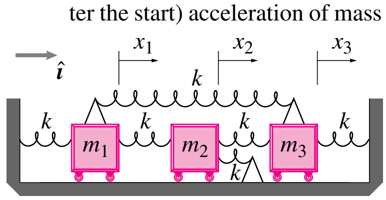
problem 9.82:

Filename:pfigure-s95q3a

9.83 Normal modes. All three masses have $m = 1\,\text{kg}$ and all 6 springs are $k = 1\,\text{N/m}$. The system is at rest when $x_1 = x_2 = x_3 = 0$.

a) Find as many different initial conditions as you can for which normal mode vibrations result. In each case, find the associated natural frequency. (we will call two initial conditions $[v]$ and $[w]$ different if there is no constant c so that $[v_1\ v_2\ v_3] = c[w_1\ w_2\ w_3]$. Assume the initial velocities are zero.)

b) For the initial condition
$[x_0] = [\,0.1\,\text{m}\ \ 0\ \ 0\,]$,
$[\dot{x}_0] = [\,0\ \ 2\,\text{m/s}\ \ 0\,]$
what is the initial (immediately after the start) acceleration of mass 2?

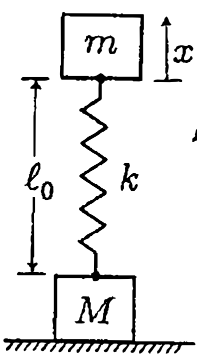


problem 9.83:

Filename:pfigure3-f95p2p2

9.84 Two blocks with masses M and m are connected by a spring with constant k and free length ℓ_0 that can sustain compression. Mass$M$ is resting on the ground at the start. There is gravity. The upwards vertical displacement of mass m is x, which is zero when the spring is at its rest length and M is on the ground.

a) For what value of x is the system in static equilibrium?
b) Find a differential equation governing the motion of the m assuming M remains on the ground.
c) Draw a free body diagram of M.
d) For what value of x is M on the verge of lifting off the ground.
e) Defining y as the height of the lower mass, write two coupled differential equations for the motion of m and M if both masses are in the air.
f) Find the value of $x < 0$ so that if the system is started from rest with that x and $y = 0$ that the ground reaction force on M just goes to zero.
g) Starting here, this problem is more of a project than a typical homework problem. Assume $x(t = 0)$ is less than the value computed above. Write a computer program that integrates the equations of motion until M lifts off and then switches to integrating the equations for the two masses in the air.
h) modify your program so that if M hits the ground again, it sticks until the ground reaction force goes to zero again.
i) By playing around, this way or that, see if you can find a special value for $x(t = 0)$ so that the bouncing continues indefinitely. (This is a perhaps surprising result, that a system with plastic collisions can continue to bounce indefinitely.)



problem 9.84:

Filename:pfigure-2blocksandspring

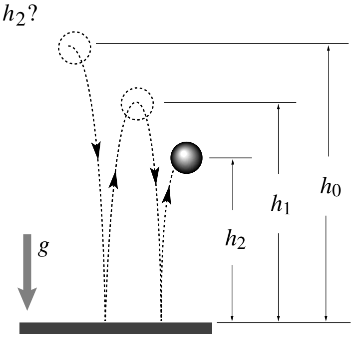
9.5 1D Collisions

Preparatory Problems

9.85 Before a collision two particles, $m_A = 1\,\text{kg}$ and $m_B = 2\,\text{kg}$, have velocities of $v_A^- = 10\,\text{m/s}$ and $v_B^- = 5\,\text{m/s}$. After the collision the velocity of A is $v_A^+ = 8\,\text{m/s}$.

a) What is the momentum of A before the collision?
b) What is the momentum of B before the collision?
c) What is the system momentum before the collision?
d) What is the momentum of A after the collision?
e) What is the system momentum after the collision?
f) What is the momentum of B after the collision?
g) What is the impulse that A applies to B during the collision?
h) What is the impulse that Be applies to A during the collision?
i) What is the kinetic energy of the system before the collision?
j) What is the kinetic energy of the system after the collision?
k) What is the coefficient of restitution?

9.86 A ball is dropped from a height of $h_0 = 10\,\text{m}$ onto a hard stationary surface. After the first bounce, it reaches a height of $h_1 = 6.4\,\text{m}$. What is the coefficient of restitution between the ball and ground? What is the height of the second bounce, h_2?



problem 9.86:

Filename:Danef94s1q7

9.87 A ball of mass m is dropped vertically from a height h. The only force acting on the ball in its flight is gravity. The ball strikes the ground with speed v^- and after collision it rebounds vertically with reduced speed v^+ directly proportional to the incoming speed, $v^+ = ev^-$, where $0 < e < 1$. What is the maximum height the ball reaches after one bounce, in terms of h, e, and g.

More-Involved Problems

9.88 Before a collision two particles, $m_A = 7\,\text{kg}$ and $m_B = 9\,\text{kg}$, have velocities of $v_A^- = 6\,\text{m/s}$ and $v_B^- = 2\,\text{m/s}$. The coefficient of restitution is $e = .5$. Find the impulse of mass A on mass B and the velocities of the two masses after the collision.

9.89 A ball of mass m is dropped from height h onto the solid hard ground where its coefficient of restitution is $e < 1$. The gravitational constant is g.

a) How many times does the ball bounce before it comes to a stop?
b) How long does it take from first release until it comes to a stop?
c) What is the total distance the ball travels before coming to a stop (add up and down distances)?

9.90 A basketball with mass m_b is dropped from height h onto the hard solid ground on which it has coefficient of restitution e_b. Just on top of the basketball, falling with it and then bouncing against it after the basketball hits the ground, is a small rubber ball with mass m_r that has a coefficient of restitution e_r with the basketball.

a) In terms of some or all of m_b, m_r, h, g, e_b and e_r how high does the rubber ball bounce?
b) assuming the coefficients of restitution are less than or equal to one, for given h, what mass and restitution parameters maximize the height of the bounce of the rubber ball and what is that height?

9.91 Show that it is necessary that $|e| \leq 1$ for the kinetic energy of the system to not increase.

9.6 Advanced vibration

Preparatory Problems

9.92 Given that $\ddot{\theta} + k^2\theta = \beta \sin \omega t$, $\theta(0) = 0$, and $\dot{\theta}(0) = \dot{\theta}_0$, find $\theta(t)$.

More-Involved Problems

9.93 A 3 kg mass is suspended by a spring ($k = 10\,\mathrm{N/m}$) and forced by a 5 N sinusoidally oscillating force with a period of 1 s. What is the amplitude of the steady-state oscillations (ignore the "homogeneous" solution)

9.94 A machine produces a steady-state vibration due to a forcing function described by $Q(t) = Q_0 \sin \omega t$, where $Q_0 = 5000N$. The machine rests on a circular concrete foundation. The foundation rests on an isotropic, elastic half-space. The equivalent spring constant of the half-space is $k = 2,000,000\,\mathrm{N{\cdot}m}$ and has a damping ratio $d = c/c_c = 0.125$. The machine operates at a frequency of $\omega = 4$ Hz.

1. What is the natural frequency of the system?
2. If the system were undamped, what would the steady-state displacement be?
3. What is the steady-state displacement given that $d = 0.125$?
4. How much additional thickness of concrete should be added to the footing to reduce the damped steady-state amplitude by 50%? (The diameter must be held constant.)

Answers to *'d problems

2.55) $r_x = \vec{r} \cdot \hat{\imath} = (3\cos\theta + 1.5\sin\theta)$ ft, $r_y = \vec{r} \cdot \hat{\jmath} = (3\sin\theta - 1.5\cos\theta)$ ft.

2.77) No partial credit.

2.78) To get chicken road sin theta.

2.83) $\vec{N}\frac{1000\,\mathrm{N}}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$.

2.86) $d = \sqrt{\frac{3}{2}}$.

2.90a) $\hat{\lambda}_{OB} = \frac{1}{\sqrt{50}}(4\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$.

b) $\hat{\lambda}_{OA} = \frac{1}{\sqrt{34}}(3\hat{\jmath} + 5\hat{k})$.

c) $\vec{F}_1 = \frac{5\,\mathrm{N}}{\sqrt{34}}(3\hat{\jmath} + 5\hat{k})$, $\vec{F}_2 = \frac{7\,\mathrm{N}}{\sqrt{50}}(4\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$.

d) $\angle AOB = 34.45\,\mathrm{deg}$.

e) $F_{1_x} = 0$

f) $\vec{r}_{DO} \times \vec{F}_1 = \left(\frac{100}{\sqrt{34}}\hat{\jmath} - \frac{60}{\sqrt{34}}\hat{k}\right)$ N·m.

g) $M_\lambda = \frac{140}{\sqrt{50}}$ N·m.

h) $M_\lambda = \frac{140}{\sqrt{50}}$ N·m.(same as (7))

2.92a) $\hat{n} = \frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$.

b) $d = 1$.

c) $\frac{1}{3}(-2, 19, 11)$.

2.94) $\ell/\sqrt{2}$

2.110) Yes.

2.122a) $\vec{r}_2 = \vec{r}_1 + \vec{F}_1 \times \hat{k}M_1/|\vec{F}_1|^2$, $\vec{F}_2 = \vec{F}_1$.

b) $\vec{r}_2 = \vec{r}_1 + \vec{F}_1 \times \hat{k}M_1/|\vec{F}_1|^2 + c\vec{F}_1$ where c is any real number, $\vec{F}_2 = \vec{F}_1$.

c) $\vec{F}_2 = \vec{0}$ and $\vec{M}_2 = \vec{M}_1$ applied at any point in the plane.

2.123a) $\vec{r}_2 = \vec{r}_1 + \vec{F}_1 \times \vec{M}_1/|\vec{F}_1|^2$, $\vec{F}_2 = \vec{F}_1$, $\vec{M}_2 = \vec{M}_1 \cdot \vec{F}_1\vec{F}_1/|\vec{F}_1|^2$. If $\vec{F}_1 = \vec{0}$ then $\vec{F}_2 = \vec{0}$, $\vec{M}_2 = \vec{M}_1$, and $\vec{r}_2$ is any point at all in space.

b) $\vec{r}_2 = \vec{r}_1 + \vec{F}_1 \times \vec{M}_1 / |\vec{F}_1|^2 + c\vec{F}_1$ where c is any real number, $\vec{F}_2 = \vec{F}_1$, $\vec{M}_2 = \vec{M}_1 \cdot \vec{F}_1 \vec{F}_1 / |\vec{F}_1|^2$. See above for the special case of $\vec{F}_1 = \vec{0}$.

2.124) $(0.5\,\mathrm{m},\ -0.4\,\mathrm{m})$

3.1a) The forces and moments that show on a free body diagram, the *external* forces and moments.

b) The forces and moments that show on a free body diagram, the *external* forces and moments. No "inertial" or "acceleration" forces show.

3.2) You don't.

3.12) Note, no couples show on any of the free body diagrams requested.

4.5) $T_1 = Nmg$, $T_2 = (N-1)mg$, $T_N = (1)mg$, and in general $T_n = (N+1-n)mg$

4.23) (a)$T_{AB} = 30\,\mathrm{N}$, (b) $T_{AB} = \frac{300}{17}\,\mathrm{N}$, (c)$T_{AB} = \frac{5\sqrt{26}}{2}\,\mathrm{N}$

4.59) $\theta \geq \tan^{-1}\left((1-\mu^2)/2\mu\right)$

4.62) For this device to hold, $\mu \geq 1$. (Demanding $\mu \geq 1$ is large for a practical device because typical rock friction has $\mu \approx 0.5$. The too-large number follows from the simplified geometry and numbers chosen for a homework problem.)

4.66) $T_{AB} = \sqrt{10}\mu mg/(3+\mu)$

4.66) Minimum tension if rope slope is μ (instead of $1/3$)

4.68a) $\frac{m}{M} = \frac{R\sin\theta}{R\cos\theta + r} = \frac{2\sin\theta}{1+2\cos\theta}$.

b) $T = mg = 2Mg\frac{\sin\theta}{1+2\cos\theta}$.

c) $\vec{F}_C = Mg\left[-\frac{2\sin\theta}{2\cos\theta+1}\hat{\imath}' + \hat{\jmath}'\right]$ (where $\hat{\imath}'$ and $\hat{\jmath}'$ are aligned with the horizontal and vertical directions)

c) $\tan\phi = \frac{\sin\theta}{2+\cos\theta}$. Needs somewhat involved trigonometry, geometry, and algebra.

d) $\tan\psi = \frac{m}{M} = \frac{2\sin\theta}{1+2\cos\theta}$.

4.69a) $\frac{m}{M} = \frac{R\sin\theta}{R\cos\theta r} = \frac{2\sin\theta}{2\cos\theta - 1}$.

b) $T = mg = 2Mg\frac{\sin\theta}{2\cos\theta-1}$.

c) $\vec{F}_C = \frac{Mg}{1-2\cos\theta}\left[\sin\theta\hat{\imath} + (\cos\theta - 2)\hat{\jmath}\right]$.

4.70a) $\frac{F_1}{F_2} = \frac{R_o + R_i\sin\phi}{R_o - R_i\sin\phi}$

b) For $R_o = 3R_i$ and $\mu = 0.2$, $\frac{F_1}{F_2} \approx 1.14$.

4.75) None are true. The tension is $100\,\mathrm{N}$.

4.90) Maximum overhang when $n \to \infty is\ell$.

4.93) Assuming no side-loads from floor the support from leg AB is $250\,\mathrm{N}$, $T_{AB} = -250\,\mathrm{N}$.

4.94) $T_{IE} = mg/2, T_{CH} = \sqrt{2}mg/2, T_{BH} = -mg/2, A_x = mg/2, A_y = mg/2, A_z = mg$

4.97g) $T_{EH} = 0$ as you can find a number of ways.

4.98a) Use axis EC.

b) Use axis AH.

c) Use $\hat{\jmath}$ axis through B.

d) Use axis DE.

e) Use axis EH.

f) Can't do in one shot.

4.99) $T_{AC} = -\sqrt{2}mg = -1000\sqrt{2}\,\mathrm{N} \approx -1410\,\mathrm{N}$ (the bar is in compression)

4.99) $T_{IP} = 0$

4.99) $T_{KL} = \sqrt{2}mg/6 = \left(1000\sqrt{2}/6\right) N \approx 408\,\mathrm{N}$ (the bar is in tension)

4.101) Hint: With reference to a free body diagram of the robot, use moment balance about axis BC.

5.9) $T_{AC} = -1000\,\mathrm{N}$, (AC is in compression)

5.10) $T_{AB} = 173\,\mathrm{N}$

5.13) 12 of the 15 bars are zero-force members; all but BD, DG, and GJ. The others carry no load but are needed for stability.

5.36) $T_{EB} = -11F/2$

5.36) $T_{HI} = -11bF/2a$

5.36) $T_{JK} = -35bF/2a$, (more than 3 times the compression of HI)

6.1) 1000 N

6.2) 0.08 cm

6.3) 1160 N

6.4) 5 cm

6.5) $k_e = 66.7\,\mathrm{N/cm}$, $\delta = 0.75\,\mathrm{cm}$

6.7) $k = 20\,\mathrm{N/cm}$

6.8) Middle spring: $\delta = 1$ cm; side-springs $\delta = 0.5$ cm

6.12) Surprise! This pendulum is in equilibrium for all values of θ.

6.37) 200 N

6.48) $N = (h(w + d)/d\ell)\, F_h$

6.55) Either by looking at part KAP or at part BAQ, if we think of moment balance about A we see that the cutting force has to fight about twice the torque in the gear mechanism as in the ungeared mechanism. For example KAP is aided in its cutting by the torque from the force at G.

6.56) The mechanism multiplies the force at B and C by a factor of 2 compared to having the handle hinged at A. The force at G also gets (a shade less than) this force but with half the lever arm. Together they give a force multiplication of (a shade less than) 2+1=3.

6.57) $F_P = 125\,\mathrm{N}$

6.57) $F_P = 125\,\mathrm{N}$

6.57) For the load at I, $F_P = 75\,\text{N}$. For the load at J, $F_P = 250N$.

6.57) With the welded handle there is just a simple lever and the mechanical advantage comes from the horizontal distance between the load and hinge A. For the 4 bar mechanism the force at C is the applied vertical load, no matter where it is applied. So the lever arm is the horizontal distance from A to C.

6.58) $F_A = 500\,\text{lbf}$

6.59d) reduce the dimension marked "2 inches". The smaller the less the friction needed.

e) As the "2 inch" dimension is reduced to zero, the needed coefficient of friction goes to zero and the forces squeezing the pipe go to infinity. This is bad because it can damage the pipe. It is also bad because a small pipe deformation will cause the hinge on the wrench to snap through, like a so called "toggle mechanism" and thus not grab at all.

6.60) $\vec{\boldsymbol{R}}_A = \vec{\boldsymbol{0}}$

6.60) $T = 200\,\text{lbf}$

6.62) $F_D = \ell_{EC}(\ell_{EH} - d)F/d\ell_{CD}$

6.62) $T_{CC'} = (\ell_{EH}/d - 1)\,(\ell_{EC}/\ell_{CD} + 1)\,F$

6.62) As $d \to 0$, $F_D \to \infty$. Two problems: the amount of motion goes to zero and the assumption of rigidity becomes non-negligibly inaccurate.

6.63) $F_N\,\left(b(a^2 + b^2)/a^2\right)\,F = 130F = 1300\,\text{lbf}$

6.63) The mechanism uses three tricks to multiply the force: a lever, a wedge, and a toggle. Each of these multiplies by about 5. Thus the nut-force F_N is on the order of $5^3 = 125$ times as big as F.

7.3) $(117\gamma/2)\,\text{m}^3 = 5.85 * 10^5\,\text{N}$

7.4) Water starts to spill at $h = 3r_{AB} = 3\,\text{m}$.

7.4) Assuming no friction at B, $\vec{\boldsymbol{F}}_A = 2.25 * 10^5\hat{\boldsymbol{\imath}}\,\text{N}$

7.9a) $\rho g \pi r^2 \ell$

b) $-\rho g \pi r^2 (h - \ell)$, note the minus sign, it now takes force to lift the can.

8.14) $F_{Ay} = -500\,\text{N}$, $M_A = -500/3\,\text{N·m}$

8.15) $V(\ell/2) = -w\ell/8$, $M(\ell/2) = w\ell^2/16$, $M_{max} = M(3\ell/8) = 9wl^2/128$

8.17b) [Hint: at every height y the cross sectional area must be big enough to hold the weight plus the wire below that point. From this you can set up and a differential equation for the cross sectional area A as a function of y. Find appropriate initial conditions and solve the equation. Once solved, the volume of wire can be calculated as $V = \int_0^1 0\,\text{mi} A(y)dy$ and the mass as ρV.]

9.11) $x(3\,\text{s}) = 20\,\text{m}$

9.15) (a) $v(3\,\mathrm{s}) = 2\,\mathrm{m/s}$ in each case. (b) $x(3\,\mathrm{s}) = 3\,\mathrm{m}$ for case (a), $x(3\,\mathrm{s}) = 4\,\mathrm{m}$ for case (b).

9.16) $F_s = \frac{\pi}{4} F_T$

9.48) Time span $= 3\pi\sqrt{m/k}/2$

9.51) (a) $m\ddot{x} + kx = F(t)$, (b) $m\ddot{x} + kx = F(t)$, and (c) $m\ddot{y} + 2ky - 2k\ell_0 \frac{y}{\sqrt{\ell_0^2+y^2}} = F(t)$

9.53b) $mg - k(x - \ell_0) = m\ddot{x}$

c) $\ddot{x} + \frac{k}{m}x = g + \frac{k\ell_0}{m}$

e) This solution is the static equilibrium position; i.e., when the mass is hanging at rest, its weight is exactly balanced by the upwards force of the spring at this constant position x.

f) $\ddot{\hat{x}} + \frac{k}{m}\hat{x} = 0$

g) $x(t) = \left[D - \left(\ell_0 + \frac{mg}{k}\right)\right] \cos\sqrt{\frac{k}{m}}\, t + (\ell_0 + \frac{mg}{k})$

h) period$=2\pi\sqrt{\frac{m}{k}}$.

i) If the initial position D is more than $\ell_0 + 2mg/k$, then the spring is in compression for part of the motion. A floppy spring would buckle.

9.55a) period$= \frac{2\pi}{\sqrt{\frac{k}{m}}} = 0.96\,\mathrm{s}$

b) maximum amplitude$=0.75\,\mathrm{ft}$

c) period$= 2\sqrt{\frac{2h}{g}} + \sqrt{\frac{m}{k}}\left[\pi + 2\tan^{-1}\sqrt{\frac{mg}{2kh}}\right] \approx 1.64\,\mathrm{s}$.

9.56) LHS of Linear Momentum Balance: $\sum \vec{\boldsymbol{F}} = -(kx + b\dot{x})\hat{\boldsymbol{\imath}} + (N - mg)\hat{\boldsymbol{\jmath}}$.

9.70a) Two normal modes.

b) $x_2 = const * x_1 = const * (A\sin(ct) + B\cos(ct))$,where $const = \pm 1$.

c) $\omega_1 = \sqrt{\frac{3k}{m}}, \omega_2 = \sqrt{\frac{k}{m}}$.

9.71b) If we start off by assuming that each mass undergoes simple harmonic motion at the same frequency but different amplitudes, we will find that this two-degree-of-freedom system has two natural frequencies. Associated with each natural frequency is a fixed ratio between the amplitudes of each mass. Each mass will undergo simple harmonic motion at one of the two natural frequencies only if the initial displacements of the masses are in the fixed ratio associated with that frequency.

9.73) $\vec{\boldsymbol{a}}_B = \ddot{x}_B\hat{\boldsymbol{\imath}} = \frac{1}{m_B}[-k_4x_B - k_2(x_B - x_A) + c_1(\dot{x}_D - \dot{x}_B) + k_3(x_D - x_B)]\hat{\boldsymbol{\imath}}$.

9.74) $\vec{\boldsymbol{a}}_B = \ddot{x}_B\hat{\boldsymbol{\imath}} = \frac{1}{m_B}[-k_4x_B - c_1(\dot{x}_B - \dot{x}_A) + (k_2 + k_3)(x_D - x_B)]$.

9.77a) $\omega = \sqrt{\frac{2k}{m}}$.

9.81a) One normal mode: $[1, 0, 0]$.

b) The other two normal modes: $[0, 1, \frac{1\pm\sqrt{17}}{4}]$.

9.87) $h_{max} = e^2 h$.

10.4a) $\vec{\boldsymbol{v}}(5\,\mathrm{s}) = (30\hat{\boldsymbol{\imath}} + 300\hat{\boldsymbol{\jmath}})\,\mathrm{m/s}$.

b) $\vec{\boldsymbol{a}}(5\,\mathrm{s}) = (6\hat{\boldsymbol{\imath}} + 120\hat{\boldsymbol{\jmath}})\,\mathrm{m/s^2}$.

10.5) $\vec{\boldsymbol{r}}(t) = \left(x_0 + \frac{u_0}{\Omega} - \frac{u_0}{\Omega}\cos(\Omega t)\right)\hat{\boldsymbol{\imath}} + (y_0 + v_0 t)\,\hat{\boldsymbol{\jmath}}$.

10.13) $\vec{\boldsymbol{v}} = 2t\,\mathrm{m/s^2}\hat{\boldsymbol{\imath}} + e^{\frac{t}{s}}\,\mathrm{m/s}\hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{a}} = 2\,\mathrm{m/s^2}\hat{\boldsymbol{\imath}} + e^{\frac{t}{s}}\,\mathrm{m/s^2}\,\hat{\boldsymbol{\jmath}}$.

10.48) $T_3 = 13\,\mathrm{N}$

10.61) Equation of motion: $-mg\,\hat{\boldsymbol{\jmath}} - b(\dot{x}^2+\dot{y}^2)\left(\frac{\dot{x}\hat{\boldsymbol{\imath}}+\dot{y}\hat{\boldsymbol{\jmath}}}{\sqrt{\dot{x}^2+\dot{y}^2}}\right) = m(\ddot{x}\hat{\boldsymbol{\imath}}+\ddot{y}\hat{\boldsymbol{\jmath}})$.

10.62a) System of equations:

$$
\begin{aligned}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{v}_x &= -\frac{b}{m} v_x \sqrt{v_x^2 + v_y^2} \\
\dot{v}_y &= -g - \frac{b}{m} v_y \sqrt{v_x^2 + v_y^2}
\end{aligned}
$$