CHAPTER 12

Constrained straight-line motion

Here is an introduction to kinematic constraint in its simplest context, systems that are constrained to move without rotation in a straight line. In one dimension pulley problems provide the main example. Two and three dimensional problems are covered, such as finding structural support forces in accelerating vehicles and the slowing or incipient capsize of a braking car or bicycle. Angular momentum balance is introduced as a needed tool but without the complexities of rotatioinal kinematics.

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In the previous chapters you learned that it is straightforward to write the equations of motion for a particle, or for a collection of a few particles, if you have a model for the forces on the particles in terms of their positions, velocities, and time. Putting aside the philosophical objection that the microscopic physics is not well represented by Newtonian particles, we now address another class of issues.

Figure 12.1: A truck or car running on straight level road is in straight-line motion, neglecting, of course, the wheel rotation, the bouncing, the moving engine parts, and the wandering eyes of the passengers.

Filename:TruckStraightLine5030

- Sometimes, often actually, the simplest model of mechanical interaction is not a law for force as a function of position, velocity and time, but just a geometric restriction on the relative positions or velocities of points. The reasons for this geometric, instead of force-based, approach are two-fold:
  - Sometimes the minute details of the motion are not of interest and therefore not worth tracking (*e.g.*, the vibrations of a solid, or relative motions of atoms in a solid are not of interest), and
  - Often one does not know an accurate force law (*e.g.*, at the microscopic level one does not know the details of atomic interactions; or, at the machine level, one may not know exactly the relations between the small play in an axle and the force on the axle, even though one knows that the axle restricts the relative motion of a train with its wheels and the ground).

Much mechanical modeling involves the replacement of force-interaction rules with assumptions about the geometry of the motions. Idealization the force interaction as causing a definite geometric restriction on motion is called *kinematic constraint*.

The utility of free body diagrams, the principle of action and reaction, the linear and angular momentum balance equations, and the balance of energy apply to all systems, no matter how they are or are not constrained. But, if objects are kinematically constrained the methods in mechanics have a slightly different flavor. It is easiest to get the idea if we start with systems that have simple constraints and that move in simple ways. In this short chapter, we will discuss the mechanics of things where every point in the body has the same velocity and acceleration as every other point (so called *parallel motion*) and furthermore where every point moves in a straight line.

Example: **Train on Straight Level Tracks**
Consider a train on straight level tracks. If we focus on the body of the train, we can approximate the motion as parallel straight-line motion. All parts move the same amount, with the same velocities and accelerations in the same fixed direction.

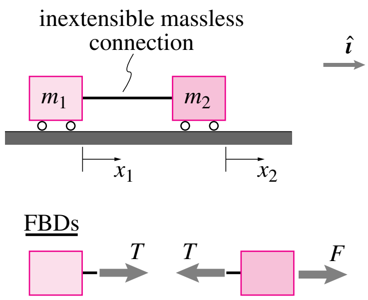

Figure 12.2: A schematic of one car pulling another, or of a boat pulling a barge. Also shown are FBDs of the bodies separately. Because our analysis is only in one spatial dimension, forces with no component in $\hat{\imath}$ direction are not shown.

Filename:tfigure-boatpullsbarge

We start with 1-D mechanics and constraint with string and pulleys, and then move on to 2-D and 3-D rigid objects.

12.1 1-D constrained motion and pulleys

The kinematic constraints we consider here are those imposed by connections with bars or ropes. Consider a car towing another with a strong light chain. We may not want to consider the elasticity of the chain but instead idealize the chain as an inextensible connection. This idealization of zero deformation is a simplification. But it is a simplification that requires special treatment. It is the simplest example of a kinematic constraint.

Figure 12.2 shows a schematic of one car pulling another. One-dimensional free body diagrams are also shown. The force F is the force transmitted from the road to the front car through the tires. The tension T is the tension in the connecting chain. From linear momentum balance for each of the objects (modeled as particles):

$$T = m_1\ddot{x}_1 \quad \text{and} \quad F - T = m_2\ddot{x}_2. \tag{12.1}$$

But these equations are exactly the same as we would have if the cars were connected by a spring, a dashpot, or any idealized-as-massless connector. And all these systems have different motions. We need our equations to somehow indicate that the two particles are not allowed to move independently. We need something to replace the constitutive law that we would have used for a spring or dashpot.

Kinematic constraint: two approaches

In the simplest example below we show two ways of dealing with kinematic constraints:

1. Use separate free body diagrams and equations of motion for each particle and then add extra kinematic constraint equations, or
2. do something clever to avoid having to find the constraint forces.

Finding the constraint force with the accelerations

The geometric (or kinematic) restriction that two masses must move in lock-step is

$$x_1 = x_2 + \text{Constant}.$$

We can differentiate the kinematic constraint twice to get

$$\ddot{x}_1 = \ddot{x}_2. \tag{12.2}$$

If we take F and the two masses as given, equations 12.1 and 12.2 are three equations for the unknowns $\ddot{x}_1$, $\ddot{x}_2$, and T. In matrix form,we have:

$$\begin{bmatrix} m_1 & 0 & -1 \\ 0 & m_2 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ T \end{bmatrix} = \begin{bmatrix} 0 \\ F \\ 0 \end{bmatrix}.$$

We can solve these equations to find $\ddot{x}_1$, $\ddot{x}_2$, and T in terms of F.

Finessing the finding of the constraint force

On the other hand, if all we are interested in are the accelerations of the cars it would be nice to avoid even having to think about the constraint force. One way to avoid dealing with the constraint force is to draw a free body diagram of the entire system as in figure 12.3. If we just call the acceleration of the system $\ddot{x}$ we have, from linear momentum balance, that

$$F = (m_1 + m_2)\ddot{x},$$

which is one equation in one unknown.

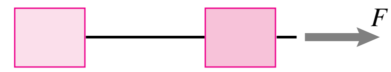

Figure 12.3: A free body diagram of the whole system. Note that the unknown tension (constraint) force does not show. As usual for 1D mechanics, vertical forces are left off for simplicity (although it would be more correct to include them).

Filename:tfigure-twocarstogether

Kinematic constraints

A generalization of the 1D inextensible-cable constraint example above is the rigid-object constraint where not just two, but many particles are assumed to keep constant distance from one another, and in two or three dimensions. Another important constraint is an ideal hinge connection between two objects. Much of the theory of mechanics after Newton has been motivated by a desire to deal easily with these and other kinematic constraints. In fact, one way of characterizing the primary difficulty of dynamics is as the difficulty of dealing with kinematic constraints.

Pulleys

Pulleys are used to redirect force to amplify or attenuate force and to amplify or attenuate motion. Like a lever, a pulley system is an example of a mechanical transmission. Objects connected by inextensible ropes around ideal pulleys are also examples of kinematic constraint.

Constant length and constant tension

Problems with pulleys are solved by using two facts about idealized strings. First, an ideal string is inextensible so the sum of the string lengths, over the different inter-pulley sections, adds to a constant (not varying in time).

$$\ell_1 + \ell_2 + \ell_3 + \ell_4 + \ldots = \text{constant} \tag{12.3}$$

Second, for round pulleys of negligible mass and no bearing friction, tension is constant along the length of the string*.The tension on one side of a pulley is the same as the tension on the other side. And this can carry on if a rope is wrapped around several pulleys.

$$T_1 = T_2 = T_3 \ldots \tag{12.4}$$

* See figure 4.24 on page 177 and the related text which shows why $T_1 = T_2$ for one round pulley idealized as frictionless and massless.

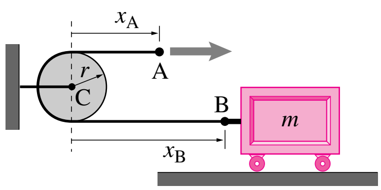

Figure 12.4: One mass, one pulley, and one string

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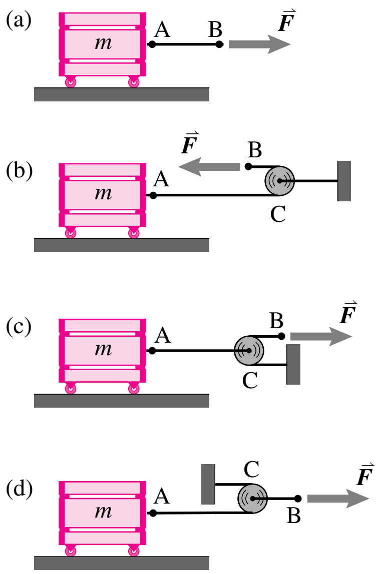

Figure 12.5: The four classic cases: (a) no pulley, (b) a pulley system with no mechanical advantage, (c) a pulley system that multiplies force and attenuates motion, and (d) a pulley system that attenuates force and amplifies motion.

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We use the trivial pulley example in figure 12.4 to show how to analyze the relative motion of various points in a pulley system.

Example: **Length of string calculation**

Starting from point A, we add up the lengths of string

$$\ell_{tot} = x_A + \pi r + x_B \equiv constant. \tag{12.5}$$

The portion of string wrapped around the pulley contacts half of the pulley so that it's length is half the pulley circumference, πr. Even if x_A and x_B change in time and different portions of string wrap around the pulley, the length of string touching the pulley is always πr.

We can now formally deduce the intuitively obvious relations between the velocities and accelerations of points A and B. Differentiating equation 12.5 with respect to time once and then again, we get

$$\begin{aligned} \dot{\ell}_{tot} = 0 &= \dot{x}_A + 0 + \dot{x}_B \\ \Rightarrow \dot{x}_A &= -\dot{x}_B \\ \Rightarrow \ddot{x}_A &= -\ddot{x}_B \end{aligned} \tag{12.6}$$

When point A is displaced to the right by an amount Δx_A, point B is displaced exactly the same amount but to the left; that is, $\Delta x_A = -\Delta x_B$. Note that in order to derive the kinematic relations 12.6 for the pulley system, we never need to know the total length of the string, only that it is constant in time. The constant-in-time quantities (the pulley half-circumference and the string length) get 'killed' in the process of differentiation.

Commonly we think of pulleys as small and thus never account for the pulley-contacting string length. Luckily this approximation generally leads to no error because we most often are interested in displacements, velocities, and accelerations in which cases the pulley contact length drops out of the equations anyway.

The classic simple uses of pulleys

First imagine trying to move a load with no pulley as in Fig. 12.5a. The force you apply goes right to the mass. This is like direct drive with no transmission.

Now you would like to use pulleys to help you move the mass. In the cases we consider here the mass is on a frictionless support and we are trying to accelerate it. But the concepts are the same if there are also resisting forces on the mass. What can we do with one pulley? Three possibilities are shown in Fig. 12.5b-d which might, at a blinking glance, look roughly the same. But they are quite different. Here we discuss each design qualitatively. The details of the calculations are a homework problem.

In Fig. 12.5b we pull one direction and the mass accelerates the other way. This illustrates one use of a pulley, to redirect an applied force. The force on the mass has magnitude $|\vec{F}|$ and there is no mechanical advantage.

Fig. 12.5c shows the most classic use of a pulley. A free body diagram of the pulley at C will show you that the tension in rope AC is $2|\vec{F}|$ and we have thus doubled the force acting on the mass. However, counting string length and displacement you will see that point A moves only half the distance that point B moves. Thus the force at B is multiplied by two to give the force at A and the displacement at B is divided by two to give the displacement at A. This result for Fig. 12.5c is most solidly understood using energy balance. The power of the force at B goes eventually entirely into the mass; the string and pulley do not absorb any energy. On the other hand if we cut the string AC, the same amount of power must be applied to the mass (it gains the same energy). Thus the product of the tension and velocity at A must equal the product of the tension and velocity at B,

$$T_A v_A = T_B v_B.$$

This is a general property of ideal transmissions, from levers to pulleys to gear boxes:

> If force is amplified then motion is equally attenuated.

Fig. 12.5d shows a use of a pulley opposite to the use in Fig. 12.5d. A free body diagram of the pulley shows that the tension in AC is $\frac{1}{2}|\vec{F}|$. Thus the force is attenuated by a factor of 2. A kinematic analysis reveals that the motion of A is twice that of B. Thus, as expected from energy considerations, the motion is amplified when the force is attenuated.

Summarizing,

> Relative to Fig. 12.5a the design Fig. 12.5b does nothing and the designs Fig. 12.5c and Fig. 12.5d are opposite in their effects.

12.1 *THEORY*

The 'effective mass' of a point of force application

The feel of the machine is of concern for machines that people handle. One aspect of feel is the effective mass. The *effective mass* is defined by the response of a point when a force is applied.

$$m_{\text{eff}} = \frac{|\vec{F}_{\text{B}}|}{|\vec{a}_{\text{B}}|}.$$

For the case of Fig. 12.5a and Fig. 12.5b the effective mass of point B is the mass of the block, m. For the case of Fig. 12.5c the block at A has $2|\vec{F}|$ acting on it and point B has twice the acceleration of point A. So the acceeration of poing B is $4F/m = F/(m/4)$ and the effective mass of point B is $m/4$. For the case of Fig. 12.5d, the mass only has $|\vec{F}|/2$ acting on it and point B only has half the acceleration of point A, so the effective mass is $4m$.

These special cases exemplify the general rule:

> The effective mass of one end of a transmission is the mass of the other end multiplied by the square of the motion amplification ratio.

In terms of the effective mass, the systems shown inFig. 12.5c and Fig. 12.5d which look so similar to a novice, actually differ by a factor of $2^2 \cdot 2^2 = 16$. With a given F and m point B in Fig. 12.5c has 16 times the acceleration of point B in Fig. 12.5d.

SAMPLE 12.1 Find the motion of two cars. One car is towing another of equal mass on level ground. The thrust of the wheels of the first car is F. The second car rolls frictionlessly. Find the acceleration of the system two ways:

1. using separate free body diagrams,
2. using a system free body diagram.

Solution

1. From linear momentum balance of the two cars, we get

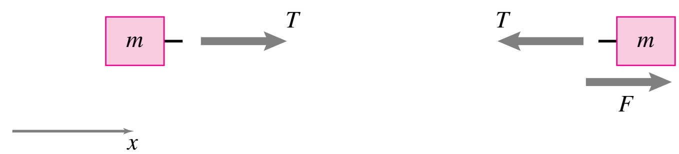

Figure 12.6:
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$$m\ddot{x}_1 = T \quad (12.7)$$
$$F - T = m\ddot{x}_2 \quad (12.8)$$

The kinematic constraint of towing (the cars move together, *i.e.*, no relative displacement between the cars) gives

$$\ddot{x}_1 - \ddot{x}_2 = 0 \quad (12.9)$$

Solving eqns. (12.7), (12.8), and (12.9) simultaneously, we get

$$\ddot{x}_1 = \ddot{x}_2 = \frac{F}{2m} \quad (T = \frac{F}{2})$$

2. From linear momentum balance of the two cars as one system, we get

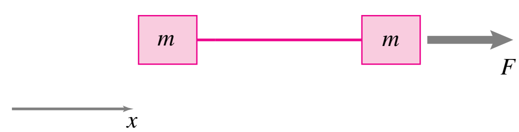

Figure 12.7:
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$$m\ddot{x} + m\ddot{x} = F$$
$$\ddot{x} = F/2m$$

$$\boxed{\ddot{x} = \ddot{x}_1 = \ddot{x}_2 = F/2m}$$

SAMPLE 12.2 Pulley kinematics. For the masses and ideal-massless pulleys shown in figure 12.8, find the acceleration of mass A in terms of the acceleration of mass B. Pulley C is fixed to the ceiling and pulley D is free to move vertically. All strings are inextensible.

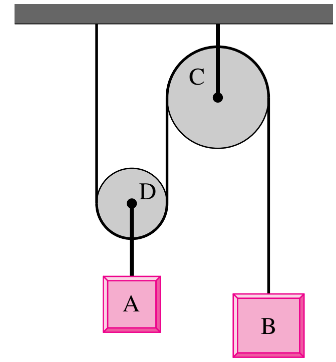

Figure 12.8:
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Solution Let us measure the position of the two masses from a fixed point, say the center of pulley C. (Since C is fixed, its center is fixed too.) Let y_A and y_B be the vertical distances of masses A and B, respectively, from the chosen reference (C). Then the position vectors of A and B are:

$$\vec{r}_A = y_A\hat{\jmath} \quad \text{and} \quad \vec{r}_B = y_B\hat{\jmath}.$$

Therefore, the velocities and accelerations of the two masses are

$$\begin{aligned} \vec{v}_A &= \dot{y}_A\hat{\jmath}, & \vec{v}_B = \dot{y}_B\hat{\jmath}, \\ \vec{a}_A &= \ddot{y}_A\hat{\jmath}, & \vec{a}_B = \ddot{y}_B\hat{\jmath}. \end{aligned}$$

Since all quantities are in the same direction ($\hat{\jmath}$), we can drop $\hat{\jmath}$ from our calculations and just do scalar calculations. We are asked to relate $\ddot{y}_A$ to $\ddot{y}_B$.

In all pulley problems, the trick in doing kinematic calculations is to relate the variable positions to the fixed length of the string. Here, the length of the string ℓ_{tot} is: *

$$\begin{aligned} \ell_{tot} &= \text{ab + bc + cd + de + ef = constant} \\ \text{where} \quad \text{ab} &= \underbrace{\text{aa}'}_{\text{constant}} + \underbrace{\text{a}'\text{b}}_{(=\text{cd}=y_D)} \\ \text{bc} &= \text{string over the pulley D = constant} \\ \text{de} &= \text{string over the pulley C = constant} \\ \text{ef} &= y_B \\ \text{thus} \quad \ell_{tot} &= 2y_D + y_B + \overbrace{\text{constant}}^{(\text{aa}'+\text{bc}+\text{de})}. \end{aligned}$$

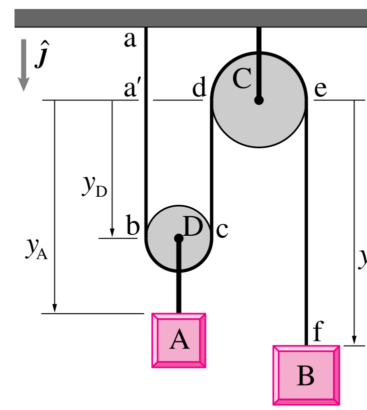

Figure 12.9:
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Taking the time derivative on both sides, we get

$$\underbrace{\frac{d}{dt}(\ell_{tot})}_{\text{0 because } \ell_{tot} \text{ does not change with time}} = 2\dot{y}_D + \dot{y}_B \quad \Rightarrow \quad \dot{y}_D = -\frac{1}{2}\dot{y}_B \tag{12.10}$$

$$\Rightarrow \quad \ddot{y}_D = -\frac{1}{2}\ddot{y}_B. \tag{12.11}$$

$$\begin{aligned} \text{But} \quad & y_D = y_A - \text{AD and AD = constant} \\ \Rightarrow \quad & \dot{y}_D = \dot{y}_A \quad \text{and} \quad \ddot{y}_D = \ddot{y}_A. \end{aligned}$$

Thus, substituting $\dot{y}_A$ and $\ddot{y}_A$ for $\dot{y}_D$ and $\ddot{y}_D$ in (12.10) and (12.11) we get

$$\dot{y}_A = -\frac{1}{2}\dot{y}_B \quad \text{and} \quad \ddot{y}_A = -\frac{1}{2}\ddot{y}_B$$

$$\boxed{\ddot{y}_A = -\tfrac{1}{2}\ddot{y}_B}$$

* We have done an elaborate calculation of ℓ_{tot} here. Usually, the constant lengths over the pulleys and some constant segments such as aa′ are ignored in calculating ℓ_{tot}. These constant length segments can be ignored because they drop out of the equation when we take time derivatives to relate velocities and accelerations of different points, such as B and D here.

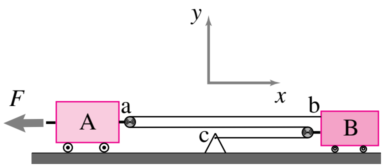

Figure 12.10: A two-mass pulley system.
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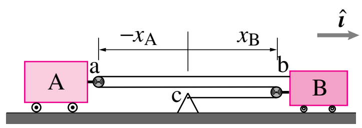

Figure 12.11: Pulley kinematics. Note that the distance from c to a is minus the x coordinate of a.
Filename:sfig3-3-1b

* You may be tempted to use angular momentum balance (AMB) to get an extra equation. In this case AMB could help determine the vertical reactions, but offers no help in finding the rope tension or the accelerations.

SAMPLE 12.3 **A two-mass pulley system.** The two masses shown in Fig. 12.10 have frictionless bases and round frictionless pulleys. The inextensible cord connecting them is always taut. Given that $F = 130\,\mathrm{N}$, $m_A = m_B = m = 40\,\mathrm{kg}$, find the acceleration of the two blocks using:

1. linear momentum balance and
2. energy balance.

Solution

1. **Using Linear Momentum Balance:**

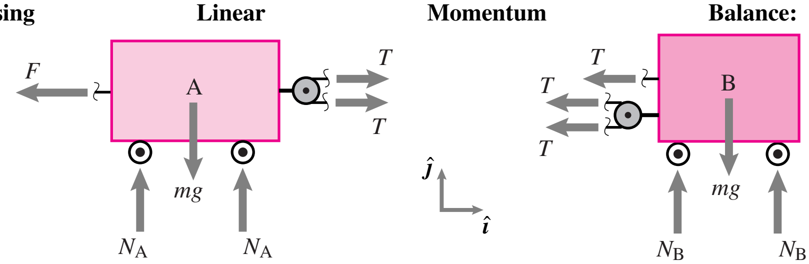

Figure 12.12:
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The free-body diagrams of the two masses A and B are shown in Fig. 12.12 above. Linear momentum balance for mass A gives (assuming $\vec{a}_A = a_A\hat{\imath}$ and $\vec{a}_B = a_B\hat{\imath}$):

$$\begin{aligned} (2T-F)\hat{\imath} + (2N_A - mg)\hat{\jmath} &= m\vec{a}_A = -ma_A\hat{\imath} \\ \text{(dotting with } \hat{\jmath}) \quad \Rightarrow \quad 2N_A &= mg \\ \text{(dotting with } \hat{\imath}) \quad \Rightarrow \quad 2T - F &= ma_A \end{aligned} \tag{12.12}$$

Similarly, linear momentum balance for mass B gives:

$$\begin{aligned} -3T\hat{\imath} + (2N_B - mg)\hat{\jmath} &= m\vec{a}_B = ma_B\hat{\imath} \\ \Rightarrow \quad 2N_B &= mg \\ \text{and} \quad -3T &= ma_B. \end{aligned} \tag{12.13}$$

From (12.12) and (12.13) we have three unknowns: T, a_A, a_B, but only 2 equations!. We need an extra equation to solve for the three unknowns.*

We can get the extra equation from kinematics. Since A and B are connected by a string of fixed length, their accelerations must be related. For simplicity, and since these terms drop out anyway, we neglect the radius of the pulleys and the lengths of the little connecting cords. Using the fixed point C as the origin of our xy coordinate system we can write

$$\begin{aligned} \ell_{tot} &\equiv \text{length of the string connecting A and B} \\ &= 3x_B + 2(-x_A) \\ \Rightarrow \quad \overbrace{\dot{\ell}_{tot}}^{0} &= 3\dot{x}_B + 2(-\dot{x}_A) \\ \Rightarrow \quad \dot{x}_B &= -\frac{2}{3}(-\dot{x}_A) \quad \Rightarrow \quad \ddot{x}_B = -\frac{2}{3}(-\ddot{x}_A) \end{aligned} \tag{12.14}$$

Since

$$\begin{aligned} \vec{v}_A &= v_A\hat{\imath} = -(-\dot{x}_A)\hat{\imath}, \\ \vec{a}_A &= a_A\hat{\imath} = \ddot{x}_A\hat{\imath}, \\ \vec{v}_B &= v_B\hat{\imath} = \dot{x}_B\hat{\imath}, \text{ and} \\ \vec{a}_B &= a_B\hat{\imath} = \ddot{x}_B\hat{\imath}, \end{aligned}$$

we get

$$a_B = \frac{2}{3} a_A. \tag{12.15}$$

Substituting (12.15) into (12.13), we get

$$9T = -2m_B a_A. \tag{12.16}$$

Now solving (12.12) and (12.16) for T, we get

$$T = \frac{2F}{13} = \frac{2 \cdot 130\,\text{N}}{13} = 20\,\text{N}.$$

Therefore,

$$\begin{aligned} a_A &= -\frac{9T}{2m} = -\frac{9 \cdot 20\,\text{N}}{2 \cdot 40\,\text{kg}} = -2.25\,\text{m/s}^2 \\ a_B &= \frac{2}{3} a_A = -1.5\,\text{m/s}^2 \end{aligned}$$

$$\boxed{\vec{\boldsymbol{a}}_A = -2.25\,\text{m/s}^2 \hat{\boldsymbol{\imath}},\ \vec{\boldsymbol{a}}_B = -1.5\,\text{m/s}^2 \hat{\boldsymbol{\imath}}.}$$

2. **Using Power Balance (III):** We have,

$$P = \dot{E}_\text{K}.$$

The power balance equation becomes

$$\sum \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}} = m\, a_A\, v_A + m_B\, a_B\, v_B.$$

Because the force at A is the only force that does work on the system, when we apply power balance to the whole system (see the FBD in Fig. 12.13), we get,

$$\begin{aligned} -Fv_A - T \overbrace{v_q}^{=v_c=0} &= m_A v_A a_A + m v_B a_B \\ \text{or} \qquad F &= -m a_A - m \frac{v_B}{v_A} a_B \\ &= -a_A (m + m \frac{v_B}{v_A} \frac{a_B}{a_A}). \end{aligned}$$

Substituting $a_B = 2/3 a_A$ and $v_B = 2/3 v_A$ from Eqn. (12.15),

$$a_A = \frac{-F}{m + \frac{4}{9} m} = \frac{-130\,\text{N}}{40\,\text{kg}(1 + \frac{4}{9})} = -2.25\,\text{m/s}^2,$$

and since $a_B = 2/3 a_A$,

$$a_B = -1.5\,\text{m/s}^2,$$

which are the same accelerations as found before.

$$\boxed{a_A = -2.25\,\text{m/s}^2 \hat{\boldsymbol{\imath}},\ a_B = -1.5\,\text{m/s}^2 \hat{\boldsymbol{\imath}}}$$

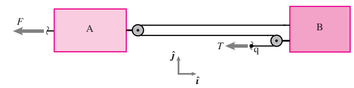

Figure 12.13: 1-D free body diagram of the whole system. Note that except F, no other forces do any work.

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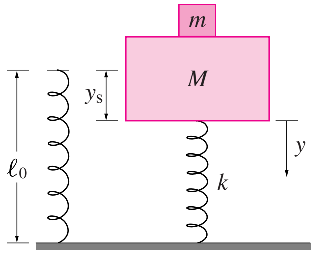

Figure 12.14:

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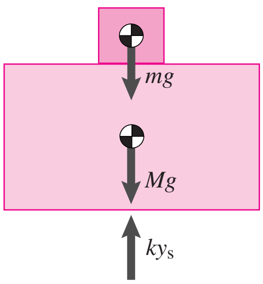

Figure 12.15: Free body diagram of the two masses as one system when in static equilibrium (this special case could be skipped as it follows from the free body diagram below).

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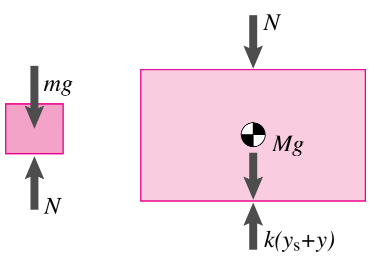

Figure 12.16: Free body diagrams of the individual masses.

Filename:sfig10-1-5b

SAMPLE 12.4 In static equilibrium the spring in Fig. 12.14 is compressed by y_s from its unstretched length ℓ_0. Now, the spring is compressed by an additional amount y_0 and released with no initial velocity.

1. Find the force on the top mass m exerted by the lower mass M.
2. When does this force become minimum? Can this force become zero?
3. Can the force on m due to M ever be negative?

Solution

1. The free body diagram of the two masses is shown in Figure 12.15 when the system is in static equilibrium. From linear momentum balance we have

$$\sum \vec{F} = \vec{0} \quad \Rightarrow \quad ky_s = (m+M)g. \tag{12.17}$$

The free body diagrams of the two masses at an arbitrary position y during motion are given in Figure 12.16. Since the two masses oscillate together, they have the same acceleration. From linear momentum balance for mass m we get

$$mg - N = m\ddot{y}. \tag{12.18}$$

We are interested in finding the normal force N. Clearly, we need to find $\ddot{y}$ to calculate N. Now, from linear momentum balance for mass M we get

$$Mg + N - k(y + y_s) = M\ddot{y}. \tag{12.19}$$

Adding eqn. (12.18) with eqn. (12.19) we get

$$(m+M)g - ky - ky_s = (m+M)\ddot{y}.$$

But $ky_s = (m+M)g$ from eqn. (12.17). Therefore, the equation of motion of the system is

$$\begin{aligned} -ky &= (m+M)\ddot{y} \\ \text{or} \quad \ddot{y} + \frac{k}{(m+M)}y &= 0. \end{aligned} \tag{12.20}$$

As you recall from your study of the harmonic oscillator, the general solution of this differential equation is

$$y(t) = A\sin\lambda t + B\cos\lambda t \tag{12.21}$$

where $\lambda = \sqrt{\frac{k}{m+M}}$ and the constants A and B are to be determined from the initial conditions. From eqn. (12.21) we obtain

$$\dot{y}(t) = A\lambda\cos\lambda t - B\lambda\sin\lambda t. \tag{12.22}$$

Substituting the given initial conditions $y(0) = y_0$ and $\dot{y}(0) = 0$ in eqns. (12.21) and (12.22), respectively, we get

$$\begin{aligned} y(0) &= y_0 = B \\ \dot{y}(0) &= 0 = A\lambda \quad \Rightarrow \quad A = 0. \end{aligned}$$

Thus,

$$y(t) = y_0\cos\lambda t. \tag{12.23}$$

Now we can find the acceleration by differentiating eqn. (12.23) twice :

$$\ddot{y} = -y_0\lambda^2\cos\lambda t.$$

Substituting this expression in eqn. (12.18) we get the force applied by mass M on the smaller mass m:

$$\begin{aligned} mg - N &= m\,\overbrace{(-y_0\lambda^2\cos\lambda t)}^{\ddot{y}} \\ \Rightarrow \quad N &= mg + my_0\lambda^2\cos\lambda t \\ &= m(g + y_0\lambda^2\cos\lambda t) \end{aligned} \tag{12.24}$$

$$\boxed{N = m(g + y_0\lambda^2 \cos\lambda t)}$$

2. Since $\cos\lambda t$ varies between ± 1, the value of the force N varies between $mg \pm y_0\lambda^2$. Clearly, N attains its minimum value when $\cos\lambda t = -1$, *i.e.*, when $\lambda t = \pi$. This condition is met when the spring is fully stretched and the mass is at its highest vertical position. At this point,

$$N \equiv N_{min} = m(g - y_0\lambda^2)$$

If y_0, the initial displacement from the static equilibrium position, is chosen such that $y_0 = \frac{g}{\lambda^2}$, then $N = 0$ when $\cos\lambda t = -1$, *i.e.*, at the topmost point in the vertical motion. This condition means that the two masses momentarily lose contact with each other when they are about to begin their downward motion. ◁

3. From eqn. (12.24) we can get a negative value of N when $\cos\lambda t = -1$ and $y_0\lambda^2 > g$. However, a negative value for N is nonsense unless the blocks are glued. Without glue the bigger mass M cannot apply a negative compression on m, *i.e.*, it cannot “suck” m. When $y_0\lambda^2 > g$ then N becomes zero before $\cos\lambda t$ decreases to -1. That is, assuming no bonding, the two masses lose contact on their way to the highest vertical position but before reaching the highest point. Beyond that point, the equations of motion derived above are no longer valid for unglued blocks because the equations assume contact between m and M. eqn. (12.24) is inapplicable when $N \leq 0$. ◁

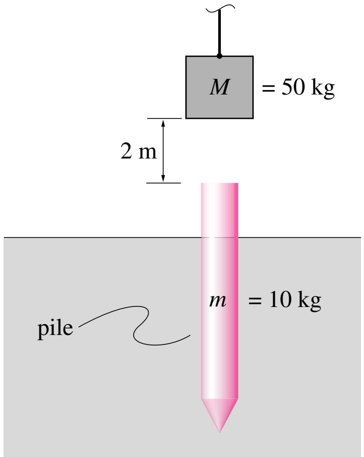

Figure 12.17:

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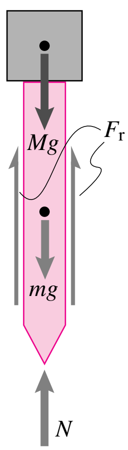

Figure 12.18: Free body diagram of the hammer and pile system. F_r is the total resistance of the ground.

Filename:sfig3-5-DH2

SAMPLE 12.5 Driving a pile into the ground. A cylindrical wooden pile of mass 10 kg and cross-sectional diameter 20 cm is driven into the ground with the blows of a hammer. The hammer is a block of steel with mass 50 kg which is dropped from a height of 2 m to deliver the blow. At the nth blow the pile is driven into the ground by an additional 5 cm. Assuming the impact between the hammer and the pile to be totally inelastic (*i.e.*, the two stick together), find the average resistance of the soil to penetration of the pile.

Solution Let F_r be the average (constant over the period of driving the pile by 5 cm) resistance of the soil. From the free body diagram of the pile and hammer system, we have

$$\sum \vec{F} = -mg\hat{\jmath} - Mg\hat{\jmath} + N\hat{\jmath} + F_r\hat{\jmath}.$$

But N is the normal reaction of the ground, which from static equilibrium, must be equal to $mg + Mg$. Thus,

$$\sum \vec{F} = Fr\hat{\jmath}.$$

Therefore, from linear momentum balance ($\sum \vec{F} = m\vec{a}$),

$$\vec{a} = \frac{F_r}{M+m}\hat{\jmath}.$$

Now we need to find the acceleration from given conditions. Let v be the speed of the hammer just before impact and V be the combined speed of the hammer and the pile immediately after impact. Then, treating the hammer and the pile as one system, we can ignore all other forces *during* the impact (none of the external forces: gravity, soil resistance, ground reaction, is comparable to the impulsive impact force, see page **??**). The impact force is internal to the system. Therefore, during impact, $\sum \vec{F} = \vec{0}$ which implies that linear momentum is conserved. Thus

$$\begin{aligned} -Mv\hat{\jmath} &= -(m+M)V\hat{\jmath} \\ \Rightarrow \quad V &= \left(\frac{M}{m+M}\right)v = \frac{50\,\text{kg}}{60\,\text{kg}}v = \frac{5}{6}v. \end{aligned}$$

The hammer speed v can be easily calculated, since it is the free fall speed from a height of 2 m:

$$v = \sqrt{2gh} = \sqrt{2\cdot(9.81\,\text{m/s}^2)\cdot(2\,\text{m})} \quad = 6.26\,\text{m/s} \quad \Rightarrow \quad V = \frac{5}{6}v = 5.22\,\text{m/s}.$$

The pile and the hammer travel a distance of $s = 5\,\text{cm}$ under the deceleration a. The initial speed $V = 5.22\,\text{m/s}$ and the final speed = 0. Plugging these quantities into the one-dimensional kinematic formula

$$v^2 = v_0^2 + 2as,$$

we get,

$$\begin{aligned} 0 &= V^2 - 2as \quad \text{(Note that } a \text{ is negative)} \\ \Rightarrow \quad a &= \frac{V^2}{2s} = \frac{(5.22\,\text{m/s})^2}{2\times 0.05\,\text{m}} = 272.48\,\text{m/s}^2. \end{aligned}$$

Thus $\vec{a} = 272.48\,\text{m/s}^2\hat{\jmath}$. Therefore,

$$F_r = (m+M)a = (60\,\text{kg})\cdot(272.48\,\text{m/s}^2) = 1.635\times 10^4\,\text{N}$$

$$\boxed{F_r \approx 16.35\,\text{kN}}$$

12.2 1D motion: 2D and 3D forces

Even if all the motion is in a single direction, an engineer may still have to consider two- or three-dimensional forces.

Example: **Piston in a cylinder.**
Consider a piston sliding vertically in a cylinder. For now neglect the spatial extent of the cylinder. Let's assume a coefficient of friction μ between the piston and the cylinder wall and that the connecting rod has negligible mass so it can be treated as a two-force member as discussed in section 4.2b.The free body diagram of the piston (with a bit of the connecting rod) is shown in figure 12.19. We have assumed that the piston is moving up so the friction force is directed down, resisting the motion. Linear momentum balance for this system is:

$$\sum \vec{F}_i = \dot{\vec{L}}$$
$$-N\hat{\imath} - \mu N\hat{\jmath} + T\hat{\lambda}_{rod} = m_{piston}\, a\hat{\jmath}.$$

If we assume that the acceleration $a\hat{\jmath}$ of the piston is known, as is its mass m_{piston}, the coefficient of friction μ, and the orientation of the connecting rod $\hat{\lambda}_{rod}$, then we can solve for the rod tension T and the normal reaction N.

Even though the piston moves in one direction, the momentum balance equation is a two-dimensional vector equation.

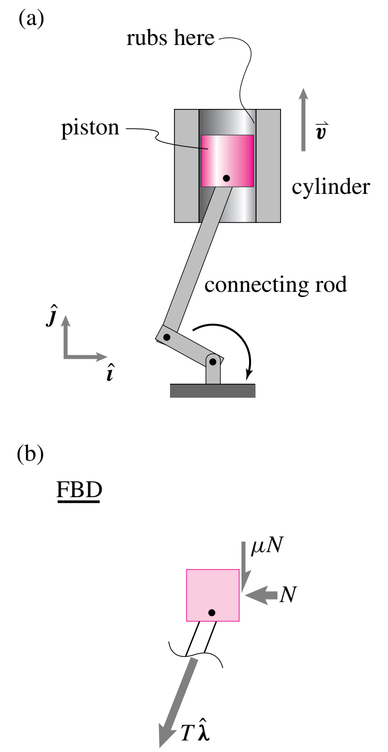

Figure 12.19: (a) shows a piston in a cylinder. (b) shows a free body diagram of the piston. To draw this FBD, we have assumed: (1) a coefficient of friction μ between the piston and cylinder wall, and (2) negligible mass for the connecting rod, and (3) ignored the spatial extent of the cylinder.

Filename:tfigure3-1

The kinematically simple 1-D motions we assume in this chapter simplify the evaluation of the right hand sides of the momentum balance equations. But the momentum balance equations are still vector equations.

Highly constrained bodies

This chapter is about rigid objects that do not rotate or deform. Most objects will not agree to be the topic of such discussion without being forced into doing so. In general, one expects bodies to rotate or move along a curved path. To keep an object that is subject to various forces from rotating or curving takes some constraint. The object needs to be rigid and held by wires, rods, rails, hinges, welds, etc. that keep it from spinning, keeping it in parallel motion. Of course the presence of constraint is not always associated with the disallowance of rotation — constraints could even cause rotation. But to keep a rigid object in straight-line motion usually does require some kind of constraint.

Of common interest for constrained structures is making sure that static and dynamic loads do not cause failure of the parts that enforce the constraints. For example, suppose a truck hauls a very heavy load that is held down by chains or straps. When the truck accelerates, what is the tension in the chains, and will it exceed the strength limit of the chains so that they might break?

In this chapter, we assume all points of a system or body are moving in a straight line with the same velocity and acceleration. Let's consider a set of points in the system of interest. Let's call them A to G, or generically, P. For convenience we distinguish a reference point O'. O' may be the center-of-mass, the origin of a local coordinate system, or a fleck of dirt that serves as a marker. By *parallel motion*, we mean that the system happens to move in such a way that $\vec{a}_P = \vec{a}_{O'}$, and $\vec{v}_P = \vec{v}_{O'}$ (Fig. 12.20). That is,

$$\vec{a}_A = \vec{a}_B = \vec{a}_C = \vec{a}_D = \vec{a}_E = \vec{a}_F = \vec{a}_G = \vec{a}_P = \vec{a}_{O'}$$

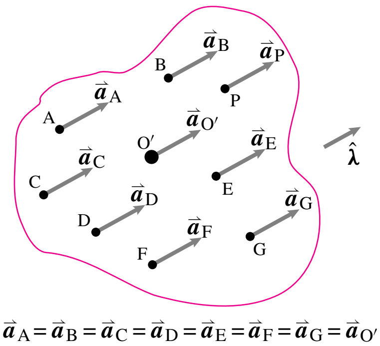

Figure 12.20: Parallel motion: all points on the body have the same acceleration $\vec{a} = a\hat{\boldsymbol{\lambda}}$. For straight-line motion: $\hat{\boldsymbol{\lambda}}(t)$=constant in time and $\vec{v} = v\hat{\boldsymbol{\lambda}}$.

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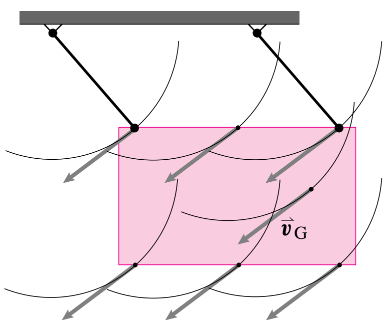

Figure 12.21: A swing showing instantaneous parallel motion which is *curvilinear*. At every instant, each point has the same velocity as the others, but the motion is not in a straight line.

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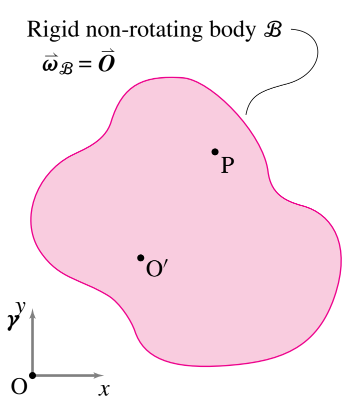

Figure 12.22: A non-rotating body $\mathcal{B}$ with points O' and P.

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at every instant in time. We also assume that $\vec{v}_A = \ldots = \vec{v}_P = \vec{v}_{O'}$.

A special case of parallel motion is straight-line motion.

a system moves with straight-line motion if it moves like a non-rotating rigid body, in a straight line.

For straight-line motion, the velocity of the body is in a fixed unchanging direction. If we call a unit vector in that direction $\hat{\boldsymbol{\lambda}}$, then we have

$$\vec{v}(t) = v(t)\hat{\boldsymbol{\lambda}}, \qquad \vec{a}(t) = a(t)\hat{\boldsymbol{\lambda}} \qquad \text{and} \qquad \vec{r}(t) = \vec{r}_0 + s(t)\hat{\boldsymbol{\lambda}}$$

for every point in the system. $\vec{r}_0$ is the position of a point at time 0 and s is the distance the point moves in the $\hat{\boldsymbol{\lambda}}$ direction. Every point in the system has the same s, v, a, and $\hat{\boldsymbol{\lambda}}$ as the other points. There are a variety of problems of practical interest that can be idealized as fitting into this class, notably, the motions of things constrained to move on belts, roads, and rails, like the train in figure **??**.

Example: **Parallel swing is not straight-line motion**

The swing shown does not rotate — all points on the swing have the same velocity. The velocity of all particles are parallel but, since paths are curved, this motion is not straight-line motion. Such curvilinear parallel motion will be discussed in Chapter 7.

A special way of analyzing straight-line motion is with one-dimensional mechanics as we did in the previous chapter. For one-dimensional mechanics, we assume that, in addition to the restricted kinematics, everything of interest mechanically happens in the $\hat{\boldsymbol{\lambda}}$ direction, often taken to be the x direction. That is, we ignore *all* torques and angular momenta, and only consider the $\hat{\boldsymbol{\lambda}}$ components of the forces (*i.e.*, $\vec{F}\cdot\hat{\boldsymbol{\lambda}}$) and linear momentum ($\vec{L}\cdot\hat{\boldsymbol{\lambda}}$). For example, if $\hat{\boldsymbol{\lambda}}$ is in the $\hat{\imath}$ direction, the components would be F_x and L_x.

Before we proceed with discussion of the details of the mechanics of straight-line motion we present some ideas that are also more generally applicable. That is, the concept of the center-of-mass allows some useful simplifications of the general expressions for $\vec{L}$, $\dot{\vec{L}}$, $\vec{H}_C$, $\dot{\vec{H}}_C$ and E_K.

Velocity of a point

The velocity of any point P on a non-rotating rigid body (such as for straight-line motion) is the same as that of any reference point on the body (see Fig. 12.22).

$$\vec{v}_P = \vec{v}_{O'}$$

A more general case, which you will learn in later chapters, is shown as 5b in Table II at the back of the book. This formula concerns rotational rate which we will measure with the vector $\vec{\omega}$. For now all you need to know is that $\vec{\omega} = \vec{0}$ when something is not rotating. In 5b in Table II, if you set $\vec{\omega}_{\mathcal{B}} = 0$ and $\vec{v}_{P/\mathcal{B}} = \vec{0}$ it says that $\vec{v}_P = \dot{\vec{r}}_{O'/O}$ or in shorthand, $\vec{v}_P = \vec{v}_{O'}$, as we have written above.

Acceleration of a point

Similarly, the acceleration of every point on a non-rotating rigid body is the same as every other point. The more general case, not needed in this chapter, is shown as entry 5c in Table II at the back of the book.

Angular momentum and its rate of change, $\vec{H}_C$ and $\dot{\vec{H}}_C$ for straight-line motion

For the motions in this chapter, where $\vec{a}_i = \vec{a}_{cm}$ and thus $\vec{a}_{i/cm} = \vec{0}$, angular momentum considerations are simplified, as explained in Box 12.2 on page 559*. But for straight-line motion (and for parallel motion), the calculations turn out to be the same as we would get if we put a single point mass at the center-of-mass:*

$$\begin{aligned}\vec{H}_C &\equiv \sum(\vec{r}_{i/C} \times m_i\vec{v}_i) &= \vec{r}_{cm/C} \times (m_{\text{total}}\vec{v}_{\text{cm}}),\\ \dot{\vec{H}}_C &\equiv \sum(\vec{r}_{i/C} \times m_i\vec{a}_i) &= \vec{r}_{cm/C} \times (m_{\text{total}}\vec{a}_{cm}).\end{aligned}$$

*Calculating rate of change of angular momentum will get more difficult as the book progresses. For a rigid body $\mathcal{B}$ in more general motion, the calculation of rate of change of angular momentum involves the angular velocity $\vec{\omega}_{\mathcal{B}}$, its rate of change $\dot{\vec{\omega}}_{\mathcal{B}}$, and the moment of inertia matrix $[\boldsymbol{I}^{\text{cm}}]$. If you look in the back of the book at Table I, entries 6c and 6d, you will see formulas that reduce to the formulas below if you assume no rotation and thus use $\vec{\omega} = \vec{0}$ and $\dot{\vec{\omega}} = \vec{0}$.
But rate of change of linear momentum is simple, at least in concept, in this chapter, as well as in the rest of this book, where

$$\dot{\vec{L}} = m_{tot}\vec{a}_{cm}$$

always applies.

***Caution:** Unfortunately, the special motions in this chapter are almost the only cases where the angular momentum and its rate of change are so easy to calculate.

Approach

To study systems in straight-line motion (as always) we:

- draw a free body diagram, showing the appropriate forces and couples at places where connections are 'cut',
- state reasonable kinematic assumptions based on the motions that the constraints allow,
- write linear and/or angular momentum balance equations and/or energy balance, and
- solve for quantities of interest.

12.2 *THEORY*

Calculation of $\vec{H}_C$ and $\dot{\vec{H}}_C$ for straight-line motion

For straight-line motion, and parallel motion in general, we can derive the simplification in the calculation of $\vec{H}_C$ as follows:

$$\begin{aligned}\vec{H}_C &\equiv \sum \vec{r}_{i/C} \times \text{m}_i\,\vec{v}_i \text{(definition)}\\ &= \sum \vec{r}_{i/C} \times \text{m}_i\,\vec{v}_{\text{cm}} \quad (\text{since, } \vec{v}_i = \vec{v}_{\text{cm}})\\ &= \left(\sum \vec{r}_{i/C}\,\text{m}_i\right) \times \vec{v}_{\text{cm}},\\ &= \vec{r}_{\text{cm}/C} \times (m_{\text{tot}}\vec{v}_{\text{cm}}),\end{aligned}$$

$$(\text{ since, } \sum \vec{r}_{i/C}\,\text{m}_i \equiv m_{\text{tot}}\vec{r}_{\text{cm}/C}).$$

The derivation that $\dot{\vec{H}}_C = \vec{r}_{\text{cm}/C} \times (m\vec{a}_{\text{cm}})$ follows from $\dot{\vec{H}}_C \equiv \sum \vec{r}_{i/C} \times \text{m}_i\,\vec{a}_i$ by the same reasoning.

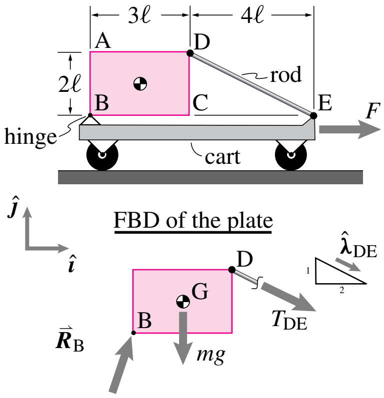

Figure 12.23: Uniform plate supported by a hinge and a rod on an accelerating cart.

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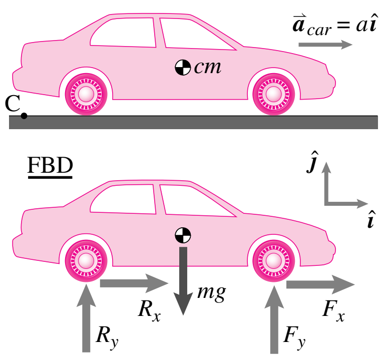


Figure 12.24: A four-wheel drive car accelerating but not tipping. See fig. 3.32 on page 148 for more about FBDs involving wheel contact.

Filename:tfigure3-4wd-car

Angular momentum balance about a judiciously chosen axis is a particularly useful tool for reducing the number of equations that need to be solved.

Example: **Plate on a cart**

A uniform rectangular plate $ABCD$ of mass m is supported by a light rigid rod DE and a hinge joint at point B. The dimensions are as shown. The cart has acceleration $a_x\hat{\imath}$ due to a force $F\hat{\imath}$ and the constraints of the wheels. Referring to the free body diagram in figure 12.23 and writing angular momentum balance for the plate about point B, we can get an equation for the tension in the rod T_{DE} in terms of m and a_x:

$$\sum \vec{M}_{/B} = \dot{\vec{H}}_{/B}$$

$$\left\{\vec{r}_{D/B}\times(T_{DE}\hat{\lambda}_{DE}) + \vec{r}_{G/B}\times(-mg\hat{\jmath}) = \vec{r}_{G/B}\times(ma_x\hat{\imath})\right\}$$

$$\{\}\cdot\hat{k}\Rightarrow \quad T_{DE} = \frac{\sqrt{5}}{7}m(a_x-\frac{3}{2}g).$$

Summarizing note:

> angular momentum balance is important even when there is no rotation.

Sliding and pseudo-sliding objects

A car coming to a stop can be roughly modeled as a rigid body that translates and does not rotate. That is, at least for a first approximation, the rotation of the car due to the suspension and tire deformation, can be neglected. The free body diagram will show various forces with lines of action that do not all act through a single point so that angular momentum balance must be used to analyze the system. Similarly, a bicycle which is braking or a box that is skidding (if not tipping) may be analyzed by assuming straight-line motion.

Example: **Car skidding**

Consider the accelerating four-wheel drive car in figure 12.24. The motion quantities for the car are $\dot{\vec{L}} = m_{car}\vec{a}_{car}$ and $\dot{\vec{H}}_{C} = \vec{r}_{cm/C}\times\vec{a}_{car}m_{car}$. We could calculate angular momentum balance relative to the car's center of mass in which case $\sum\vec{M}_{cm} = \dot{\vec{H}}_{cm} = \vec{0}$ (because the position of the center-of-mass relative to the center-of-mass is $\vec{0}$).

As mentioned, it is often useful to calculate angular momentum balance of sliding objects about points of contact (such as where tires contact the road) or about points that lie on lines of action of applied forces when writing angular momentum balance to solve for forces or accelerations. To do so usually eliminates some unknown reactions from the equations to be solved. For example, the angular momentum balance equation about the rear-wheel contact of a car does not contain the rear-wheel contact forces.

Wheels

The function of wheels is to allow easy sliding-like (pseudo-sliding) motion between objects, at least in the direction they are pointed. On the other hand, wheels do sometimes slip due to:

- being overpowered (as in a screeching accelerating car),
- being braked hard, or
- having very bad bearings (like a rusty toy car).

How wheels are treated when analyzing cars, bikes, and the like depends on both the application and on the level of detail one requires. In *this chapter*, we will always assume that wheels have negligible mass. Thus, when we treat the special case of un-driven and un-braked wheels our free body diagrams will be as in figure 3.33 on page 148 and *not* like the one in figure **??** on page **??**. With the ideal wheel approximation, all of the various cases for a car traveling to the right are shown with partial free body diagrams of a wheel in figure 3.32. For the purposes of actually solving problems, we have accepted Coulomb's law of friction as a model for contacting interaction (see pages **??**-146).

3-D forces in straight-line motion

The ideas we have discussed apply as well in three dimensions as in two. As you learned from doing statics problems, working out the details in 3D, where vector methods must be used carefully, is more involved than in 2D. As for statics, three dimensional problems often yield simple results and simple intuitions by considering angular momentum balance about an axis.

Angular momentum balance about an axis

The simplest way to think of angular momentum balance about an axis is to look at angular momentum balance about a point and then take a dot product with a unit vector along an axis:

$$\hat{\boldsymbol{\lambda}} \cdot \left\{ \sum \vec{\boldsymbol{M}}_{/\mathrm{C}} = \dot{\vec{\boldsymbol{H}}}_{/\mathrm{C}} \right\}.$$

Note that the axis need not correspond to any mechanical device in any way resembling an axle. The equation above applies for any point C and any vector $\hat{\boldsymbol{\lambda}}$. If you choose C and $\hat{\boldsymbol{\lambda}}$ judiciously many terms in your equations may drop out.

SAMPLE 12.6 Force in braking. A front-wheel-drive car of mass $m = 1200\,\mathrm{kg}$ is cruising at $v = 60\,\mathrm{mph}$ on a straight road when the driver slams on the brake. The car slows down to 20 mph in 4 s while maintaining its straight path. What is the average force (average in time) applied on the car during braking?

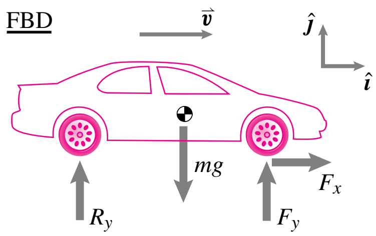


Figure 12.25: Free body diagram of a front-wheel-drive car during braking. Note that we have (arbitrarily) pointed F_x to the right. The algebra in this problem will tell us that $F_x < 0$.

Filename:sfig2-6-3a

Solution Let us assume that we have an xy coordinate system in which the car is traveling along the x-axis during the entire time under consideration. Then, the velocity of the car before braking, $\vec{v}_1$, and after braking, $\vec{v}_2$, are

$$\begin{aligned} \vec{v}_1 = v_1\hat{\imath} &= 60\,\mathrm{mph}\,\hat{\imath} \\ \vec{v}_2 = v_2\hat{\imath} &= 20\,\mathrm{mph}\,\hat{\imath}. \end{aligned}$$

The linear impulse during braking is $\vec{F}_{ave}\Delta t$ where $\vec{F} \equiv F_x\hat{\imath}$ (see free body diagram of the car). Now, from the impulse-momentum relationship,

$$\vec{F}\Delta t = \vec{L}_2 - \vec{L}_1,$$

where $\vec{L}_1$ and $\vec{L}_2$ are linear momenta of the car before and after braking, respectively, and $\vec{F}$ is the average applied force. Therefore,

$$\begin{aligned} \vec{F} &= \frac{1}{\Delta t}(\vec{L}_2 - \vec{L}_1) \\ &= \frac{m}{\Delta t}(\vec{v}_2 - \vec{v}_1) \\ &= \frac{1200\,\mathrm{kg}}{4\,\mathrm{s}}(20-60)\,\mathrm{mph}\hat{\imath} \\ &= -12000\frac{\mathrm{kg}}{\mathrm{s}}\cdot\frac{\not{\mathrm{m}}\mathrm{i}}{\not{\mathrm{h}}\mathrm{r}}\cdot\frac{1600\ \mathrm{m}}{1\ \not{\mathrm{m}}\mathrm{i}}\cdot\frac{1\ \not{\mathrm{h}}\mathrm{r}}{3600\,\mathrm{s}}\hat{\imath} \\ &= -\frac{16,000}{3}\,\mathrm{kg}\cdot\mathrm{m/s^2}\hat{\imath} \\ &= -5.33\,\mathrm{kN}\hat{\imath}. \end{aligned}$$

Thus

$$\begin{aligned} F_x\hat{\imath} &= -5.33\,\mathrm{kN}\hat{\imath} \\ \Rightarrow\quad F_x &= -5.33\,\mathrm{kN}. \end{aligned}$$

$$\boxed{F_x = -5.33\,\mathrm{kN}}$$

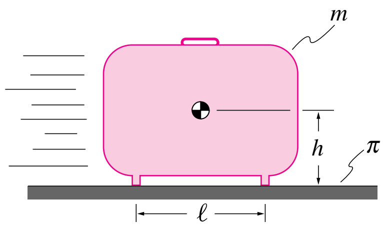


Figure 12.26: A suitcase in motion.

Filename:sfig3-5-1

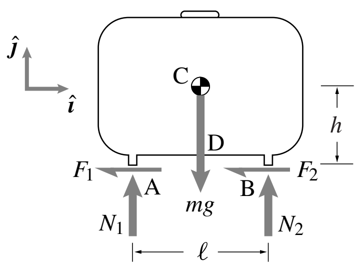


Figure 12.27: FBD of the suitcase.

Filename:sfig3-5-1a

SAMPLE 12.7 A suitcase skidding on frictional ground. A suitcase of mass m is pushed and sent sliding on a horizontal surface. The suitcase slides without any rotation. A and B are the only contact points of the suitcase with the ground. If the coefficient of friction between the suitcase and the ground is μ, find all the forces applied by the ground on the suitcase. Discuss the results obtained for normal forces.

Solution As usual, we first draw a free body diagram of the suitcase. The FBD is shown in Fig. 12.27. Assuming Coulomb's law of friction holds, we can write

$$\vec{F}_1 = -\mu N_1 \hat{\imath} \quad \text{and} \quad \vec{F}_2 = -\mu N_2 \hat{\imath}. \tag{12.25}$$

Now we write the balance of linear momentum for the suitcase:

$$\sum \vec{F} = m\vec{a}_{\text{cm}}$$

$$\Rightarrow \qquad -(F_1 + F_2)\hat{\imath} + (N_1 + N_2 - mg)\hat{\jmath} = ma_C\hat{\imath} \tag{12.26}$$

where $\vec{a}_C = a_C\hat{\imath}$ is the unknown acceleration. Dotting eqn. (12.26) with $\hat{\imath}$ and $\hat{\jmath}$ and substituting for F_1 and F_2 from eqn. (12.25) we get

$$-\mu(N_1 + N_2) = ma_C \tag{12.27}$$

$$N_1 + N_2 = mg. \tag{12.28}$$

Equations (12.27) and (12.28) represent 2 scalar equations in three unknowns N_1, N_2 and a. Obviously, we need another equation to solve for these unknowns.

We can write the balance of angular momentum about any point. Points A or B are good choices because they each eliminate some reaction components. Let us write the balance of angular momentum about point A:

$$\sum \vec{M}_{\text{A}} = \dot{\vec{H}}_{\text{A}}$$

$$\begin{aligned}
\sum \vec{M}_{\text{A}} &= \vec{r}_{B/A} \times N_2\hat{\jmath} + \vec{r}_{D/A} \times (-mg)\hat{\jmath} \\
&= \ell\hat{\imath} \times N_2\hat{\jmath} + \frac{\ell}{2}\hat{\imath} \times (-mg)\hat{\jmath} \\
&= (\ell N_2 - mg\frac{\ell}{2})\hat{k}
\end{aligned} \tag{12.29}$$

and

$$\begin{aligned}
\dot{\vec{H}}_{\text{A}} &= \vec{r}_{C/A} \times m\vec{a}_C \\
&= (\frac{\ell}{2}\hat{\imath} + h\hat{\jmath}) \times ma_C\hat{\imath} \\
&= -ma_C h\hat{k}.
\end{aligned} \tag{12.30}$$

Equating (12.29) and (12.30) and dotting both sides with $\hat{k}$ we get the following third scalar equation:

$$\ell N_2 - mg\frac{\ell}{2} = -ma_C h. \tag{12.31}$$

Solving eqns. (12.27) and (12.28) for a we get

$$a_C = -\mu g$$

and substituting this value of a_C in eqn. (12.31) we get

$$\begin{aligned}
N_2 &= \frac{m\mu gh + mg\ell/2}{\ell} \\
&= mg\left(\frac{1}{2} + \frac{h}{\ell}\mu\right).
\end{aligned}$$

Substituting the value of N_2 in either of the equations (12.27) or (12.28) we get

$$N_1 = mg\left(\frac{1}{2} - \frac{h}{\ell}\mu\right).$$

$$\boxed{N_1 = mg(\tfrac{1}{2} - \tfrac{h}{\ell}\mu),\ N_2 = mg(\tfrac{1}{2} + \tfrac{h}{\ell}\mu),\ f_1 = \mu N_1,\ f_2 = \mu N_2.}$$

Discussion: From the expressions for N_1 and N_2 we see that

1. $N_1 = N_2 = \frac{1}{2}mg$ if $\mu = 0$ because without friction there is no deceleration. The problem becomes equivalent to a statics problem.
2. $N_1 = N_2 \approx \frac{1}{2}mg$ if $\ell >> h$. In this case, the moment produced by the friction forces is too small to cause a significant difference in the magnitudes of the normal forces. For example, take $\ell = 20h$ and calculate moment about the center-of-mass to convince yourself.

Graphically, N_1, N_2 and their difference $N_1 - N_2$ are shown in the plot below as a function of h/ℓ for a particular value of μ and mg. As the equations indicate, $N_1 - N_2$ increases steadily as h/ℓ increases, showing how the moment produced by the friction forces makes a bigger and bigger difference between N_1 and N_2 as this moment gets bigger.

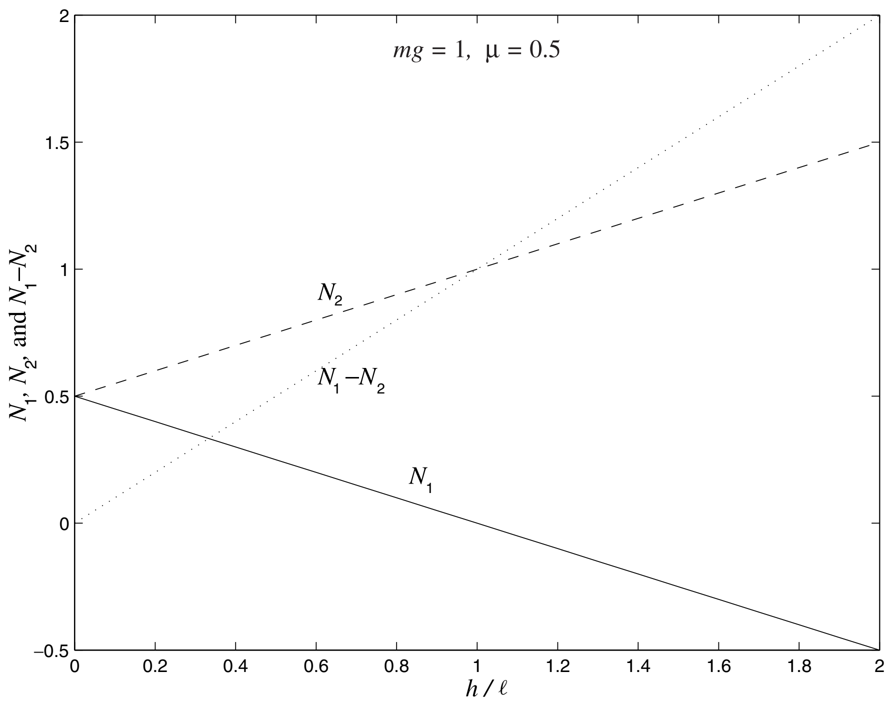


Figure 12.28: The normal forces N_1 and N_2 differ from each other more and more as h/ℓ increases.
Filename:sample6p8graph

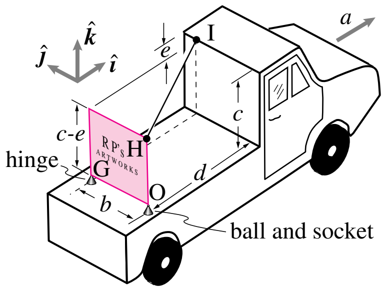


Figure 12.29: An accelerating board in 3-D
Filename:sfig3-5-2

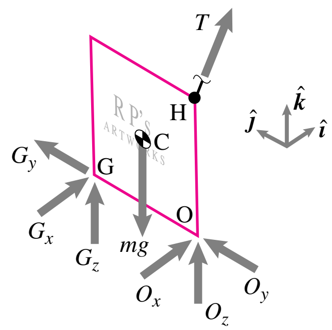


Figure 12.30: FBD of the board
Filename:sfig3-5-2a

SAMPLE 12.8 Uniform acceleration of a board in 3-D. A uniform sign-board of mass $m = 20\,\text{kg}$ sits in the back of an accelerating flatbed truck. The board is supported with a ball-and-socket joint at O and a hinge at G. A light rod from H to I keeps the board from falling over. The truck is on level ground and has forward acceleration $\vec{\boldsymbol{a}} = 0.6\,\text{m/s}^2\hat{\boldsymbol{\imath}}$. The relevant dimensions are $b = 1.5\,\text{m}$, $c = 1.5\,\text{m}$, $d = 3\,\text{m}$, $e = 0.5\,\text{m}$. There is gravity ($g = 10\,\text{m/s}^2$).

1. Draw a free body diagram of the board.
2. Set up equations to solve for all the unknown forces shown on the FBD.
3. Use the balance of angular momentum about an axis to find the tension in the rod.

Solution

1. The free body diagram of the board is shown in Fig. 12.30.
2. Linear momentum balance for the board:

$$\sum \vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}, \quad \text{or}$$

$$(G_x + O_x)\hat{\boldsymbol{\imath}} + (G_y + O_y)\hat{\boldsymbol{\jmath}} + (G_z + O_z - mg)\hat{\boldsymbol{k}} + T\hat{\boldsymbol{\lambda}}_{HI} = ma\hat{\boldsymbol{\imath}} \tag{12.32}$$

where

$$\hat{\boldsymbol{\lambda}}_{HI} = \frac{d\hat{\boldsymbol{\imath}} + b\hat{\boldsymbol{\jmath}} + e\hat{\boldsymbol{k}}}{\sqrt{d^2 + b^2 + e^2}} = \frac{d\hat{\boldsymbol{\imath}} + b\hat{\boldsymbol{\jmath}} + e\hat{\boldsymbol{k}}}{\ell},$$

and ℓ is the length of the rod HI.

Dotting eqn. (12.32) with $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ we get the following three scalar equations:

$$G_x + O_x + T\frac{d}{\ell} = ma \tag{12.33}$$

$$G_y + O_y + T\frac{b}{\ell} = 0 \tag{12.34}$$

$$G_z + O_z + T\frac{e}{\ell} = mg \tag{12.35}$$

Angular momentum balance about point G:

$$\sum \vec{\boldsymbol{M}}_G = \dot{\vec{\boldsymbol{H}}}_G$$

$$\begin{aligned}
\sum \vec{\boldsymbol{M}}_G &= \vec{\boldsymbol{r}}_{C/G} \times (-mg\hat{\boldsymbol{k}}) + \vec{\boldsymbol{r}}_{O/G} \times (O_x\hat{\boldsymbol{\imath}} + O_z\hat{\boldsymbol{k}}) + \vec{\boldsymbol{r}}_{H/G} \times T\hat{\boldsymbol{\lambda}}_{HI} \\
&= (-\frac{b}{2}\hat{\boldsymbol{\jmath}} + \frac{c-e}{2}\hat{\boldsymbol{k}}) \times (-mg\hat{\boldsymbol{k}}) - b\hat{\boldsymbol{\jmath}} \times (O_x\hat{\boldsymbol{\imath}} + O_z\hat{\boldsymbol{k}}) \\
&\quad + [-b\hat{\boldsymbol{\jmath}} + (c-e)\hat{\boldsymbol{k}}] \times \frac{T}{\ell}(d\hat{\boldsymbol{\imath}} + b\hat{\boldsymbol{\jmath}} + e\hat{\boldsymbol{k}}) \\
&= \left(\frac{b}{2}mg - bO_z - be\frac{T}{\ell} - (c-e)b\frac{T}{\ell}\right)\hat{\boldsymbol{\imath}} \\
&\quad + (c-e)d\frac{T}{\ell}\hat{\boldsymbol{\jmath}} + \left(bO_x + bd\frac{T}{\ell}\right)\hat{\boldsymbol{k}}
\end{aligned} \tag{12.36}$$

and

$$\begin{aligned}
\dot{\vec{\boldsymbol{H}}}_G &= \vec{\boldsymbol{r}}_{C/G} \times ma\hat{\boldsymbol{\imath}} \\
&= (-\frac{b}{2}\hat{\boldsymbol{\jmath}} + \frac{c-e}{2}\hat{\boldsymbol{k}}) \times ma\hat{\boldsymbol{\imath}} \\
&= \frac{b}{2}ma\hat{\boldsymbol{k}} + \frac{c-e}{2}ma\hat{\boldsymbol{\jmath}}.
\end{aligned} \tag{12.37}$$

Equating (12.36) and (12.37) and dotting both sides with $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ we get the following three additional scalar equations:

$$O_z + \frac{c}{\ell}T = \frac{1}{2}mg \quad (12.38)$$

$$\frac{d}{\ell}T = \frac{1}{2}ma \quad (12.39)$$

$$O_x + \frac{d}{\ell}T = \frac{1}{2}ma \quad (12.40)$$

Now we have six scalar equations in seven unknowns — O_x, O_y, O_z, G_x, G_y, G_z, and T. From basic linear algebra, we know that we cannot find unique solutions for all these unknowns from the given equations. A closer inspection of eqns. (12.33–12.35) and (12.38–12.40) shows that we can easily solve for O_x, O_z, G_x, G_z, and T, but O_y and G_y cannot be determined uniquely because they appear together as the sum $G_y + O_y$. * Fortunately, we can find the tension in the wire HI without worrying about the values of O_y and G_y as we show below.

* Note that G_y and O_y will always appear together as the sum $G_y + O_y$ even if you took the angular momentum balance about some other point. This is because they have the same line of action. Thus, they cannot be found independently. This mathematical problem corresponds to the physical reality that the supports at points O and G could be squeezing the plate along the line OG with, say, $O_y = 1000\,\mathrm{N}$ and $G_y = -1000\,\mathrm{N}$ even if there were no gravity, and the truck was not accelerating. To make prestress problems like this tractable, people often make assumptions like, 'Assume $G_y = 0$', that is, they try to get rid of the redundancy in supports to make the problem statically determinate.

3. Balance of angular momentum about axis OG gives:

$$\begin{aligned} \hat{\boldsymbol{\lambda}}_{OG} \cdot \sum \vec{\boldsymbol{M}}_G &= \hat{\boldsymbol{\lambda}}_{OG} \cdot \dot{\vec{\boldsymbol{H}}}_G \\ &= \hat{\boldsymbol{\lambda}}_{OG} \cdot (\vec{\boldsymbol{r}}_{C/G} \times ma\hat{\boldsymbol{\imath}}). \end{aligned} \quad (12.41)$$

Since all reaction forces and the weight go through axis OG, they do not produce any moment about this axis (convince yourself that the forces from the reactions have no torque about the axis by calculation or geometry). Therefore,

$$\begin{aligned} \hat{\boldsymbol{\lambda}}_{OG} \cdot \sum \vec{\boldsymbol{M}}_G &= \hat{\boldsymbol{\jmath}} \cdot (\vec{\boldsymbol{r}}_{H/G} \times T\hat{\boldsymbol{\lambda}}_{HI}) \\ &= T\frac{d(c-e)}{\ell}. \end{aligned} \quad (12.42)$$

$$\begin{aligned} \hat{\boldsymbol{\lambda}}_{OG} \cdot (\vec{\boldsymbol{r}}_{C/G} \times ma\hat{\boldsymbol{\imath}}) &= \hat{\boldsymbol{\jmath}} \cdot \left[\left(\frac{b}{2}\hat{\boldsymbol{\jmath}} + \frac{c-e}{2}\hat{\boldsymbol{k}} \right) \times ma\hat{\boldsymbol{\imath}} \right] \\ &= ma\frac{(c-e)}{2}. \end{aligned} \quad (12.43)$$

Equating (12.42) and (12.43), as required by eqn. (12.41), we get

$$\begin{aligned} T &= \frac{ma\ell}{2d} \\ &= \frac{20\,\mathrm{kg} \cdot 0.6\,\mathrm{m/s^2} \cdot 3.39\,\mathrm{m}}{2 \cdot 3\,\mathrm{m}} \\ &= 6.78\,\mathrm{N}. \end{aligned}$$

$$\boxed{T_{HI} = 6.78\,\mathrm{N}}$$

* Be careful with units. Most computer programs will not take care of your units. They only deal with numerical input and output. You should, therefore, make sure that your variables have proper units for the required calculations. Either do dimensionless calculations or use consistent units for all quantities.

SAMPLE 12.9 Computer solution of algebraic equations. In the previous sample problem (Sample 12.8), six equations were obtained to solve for the six unknown forces (assuming $G_y = 0$). (i) Set up the six equations in matrix form and (ii) solve the matrix equation on a computer. Check the solution by substituting the values obtained in one or two equations.

Solution

1. The six scalar equations — (12.33), (12.34), (12.35), (12.38), (12.39), and (12.40) are amenable to hand calculations. We, however, set up these equations in matrix form and solve the matrix equation on the computer. The matrix form of the equations is:

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & \frac{d}{\ell} \\ 0 & 1 & 0 & 0 & 0 & \frac{b}{\ell} \\ 0 & 0 & 1 & 0 & 1 & \frac{e}{\ell} \\ 0 & 0 & 1 & 0 & 0 & \frac{c}{\ell} \\ 0 & 0 & 0 & 0 & 0 & \frac{d}{\ell} \\ 1 & 0 & 0 & 0 & 0 & \frac{d}{\ell} \end{bmatrix} \begin{Bmatrix} O_x \\ O_y \\ O_z \\ G_x \\ G_z \\ T \end{Bmatrix} = \begin{Bmatrix} ma \\ 0 \\ mg \\ mg/2 \\ ma/2 \\ ma/2 \end{Bmatrix}. \quad (12.44)$$

The above equation can be written, in matrix notation, as

$$\mathbf{A}\,\mathbf{x} = \mathbf{b}$$

where **A** is the coefficient matrix, **x** is the vector of the unknown forces, and **b** is the vector on the right hand side of the equation. Now we are ready to solve the system of equations on the computer.

2. We use the following pseudo-code to solve the above matrix equation. *

```
m = 20, a = 0.6,
b = 1.5, c = 1.5, d = 3, e = 0.5, g = 10,
l = sqrt(b^2 + d^2 + e^2),

A = [1 0 0 1 0 d/l
     0 1 0 0 0 b/l
     0 0 1 0 1 e/l
     0 0 1 0 0 c/l
     0 0 0 0 0 d/l
     1 0 0 0 0 d/l]
b = [m*a, 0, m*g, m*g/2, m*a/2, m*a/2]'

{Solve A x = b for x}
x =                % this is the computer output
         0
   -3.0000
   97.0000
    6.0000
  102.0000
    6.7823
```

The solution obtained from the computer means:

$$\boxed{O_x = 0,\ O_y = -3\,\text{N},\ O_z = 97\,\text{N},\ G_x = 6\,\text{N},\ G_z = 102\,\text{N},\ T = 6.78\,\text{N}.}$$

We now hand-check the solution by substituting the values obtained in, say, Eqns. (12.34) and (12.39). Before we substitute the values of forces, we need to calculate the length ℓ.

$$\begin{aligned} \ell &= \sqrt{d^2 + b^2 + e^2} \\ &= 3.3912\,\text{m}. \end{aligned}$$

Therefore,

$$\begin{aligned}
\text{Eqn. (12.34):} \quad O_y + T\frac{b}{\ell} &= -3\,\mathrm{N} + 6.78\,\mathrm{N}\cdot\frac{1.5\,\mathrm{m}}{3.3912\,\mathrm{m}} \\
&\stackrel{\checkmark}{=} 0, \\
\text{Eqn. (12.39):} \quad \frac{d}{\ell}T - \frac{1}{2}ma &= \frac{3\,\mathrm{m}}{3.3912\,\mathrm{m}}6.78\,\mathrm{N} - \frac{1}{2}20\,\mathrm{kg}\,0.6\,\mathrm{m/s^2} \\
&\stackrel{\checkmark}{=} 0.
\end{aligned}$$

Thus, the computer solution agrees with our equations.

Comments: We could have solved the six equations for seven unknowns without assuming $G_y = 0$ if our computer program or package allows us to do so. We will, of course, not get a unique solution. For example, by taking the following **A**, a 6×7 matrix, and solving **A x = b** for $\mathbf{x} = [O_x\ O_y\ O_z; G_x\ G_y\ G_z\ T]^T$ with the same **b** as input above, we get the solution as shown below.

```
A = [1  0  0  1  0  0  d/l
     0  1  0  0  1  0  b/l
     0  0  1  0  0  1  e/l
     0  0  1  0  0  0  c/l
     0  0  0  0  0  0  d/l
     1  0  0  0  0  0  d/l]
b = [m*a, 0, m*g, m*g/2, m*a/2, m*a/2]'

{Solve A x = b for x}
x =                % this is the computer output
          0
   -3.0000
   97.0000
    6.0000
          0
  102.0000
    6.7823
```

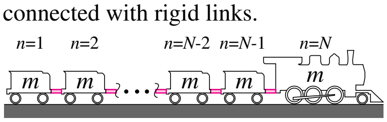
This is the same solution as we got before except that it includes $G_y = 0$ in the solution. Now, if we add a vector $\Delta\,\mathbf{x} = [0\ \alpha\ 0\ 0\ -\alpha\ 0\ 0]^T$ to $\mathbf{x}$ where α is any number, and compute $\mathbf{A}\ (\mathbf{x}+\Delta\mathbf{x})$, we get back **b**. That is, the six equilibrium conditions are satisfied irrespective of the actual values of O_y and G_y as long as the value of $O_y + G_y$ remains the same.

Problems for Chapter 12

Constrained straight line motion

12.1 1-D constrained motion and pulleys

12.1 A train engine of mass m pulls and accelerates N cars each of mass m. The power of the engine is P_t and its speed is v_t. Find the tension T_n between car n and car n+1. Assume there is no resistance and the ground is level. Assume the cars are connected with rigid links.

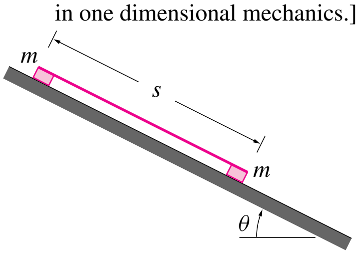


problem 12.1:

Filename:pfigure-newtrain

12.2 Two blocks, each of mass m, are connected together across their tops by a massless string of length S; the blocks' dimensions are small compared to S. They slide down a slope of angle θ. Do not neglect gravity but do neglect friction.

a) Draw separate free body diagrams of each block, the string, and the system of the two blocks and string.
b) Write separate equations for linear momentum balance for each block, the string, and the system of blocks and string.
c) What is the acceleration of the center of mass of the two blocks?
d) What is the force in the string?
e) What is the speed of the center of mass for the two blocks after they have traveled a distance d down the slope, having started from rest. [Hint: You need to dot your momentum balance equations with a unit vector along the ramp in order to reduce this problem to a problem in one dimensional mechanics.]

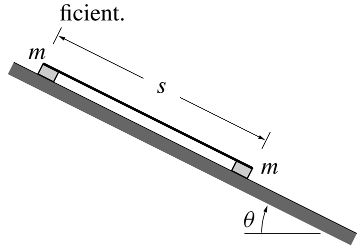


problem 12.2:

Filename:pfigure-blue-27-1

12.3 Two blocks, each of mass m, are connected together across their tops by a massless string of length S; the blocks' dimensions are small compared to S. They slide down a slope of angle θ. The materials are such that the coefficient of dynamic friction on the top block is μ and on the bottom block is $\mu/2$.

a) Draw separate free body diagrams of each block, the string, and the system of the two blocks and string.
b) Write separate equations for linear momentum balance for each block, the string, and the system of blocks and string.
c) What is the acceleration of the center of mass of the two blocks?
d) What is the force in the string?
e) What is the speed of the center of mass for the two blocks after they have traveled a distance d down the slope, having started from rest.
f) How would your solutions to parts (a) and (c) differ in the following two variations: i.) If the two blocks were interchanged with the slippery one on top or ii.) if the string were replaced by a massless rod? Qualitative responses to this part are sufficient.

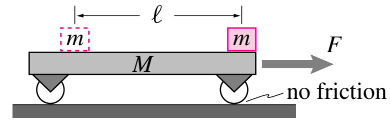


problem 12.3:

Filename:pfigure-blue-27-1a

12.4 A cart of mass M, initially at rest, can move horizontally along a frictionless track. When $t = 0$, a force F is applied as shown to the cart. During the acceleration of M by the force F, a small box of mass m slides along the cart from the front to the rear. The coefficient of friction between the cart and box is μ, and it is assumed that the acceleration of the cart is sufficient to cause sliding.

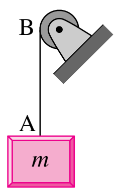
a) Draw free body diagrams of the cart, the box, and the cart and box together.
b) Write the equation of linear momentum balance for the cart, the box, and the system of cart and box.
c) Show that the equations of motion for the cart and box can be combined to give the equation of motion of the mass center of the system of two bodies.
d) Find the displacement of the cart at the time when the box has moved a distance ℓ along the cart.



problem 12.4:

Filename:pfigure-blue-28-1

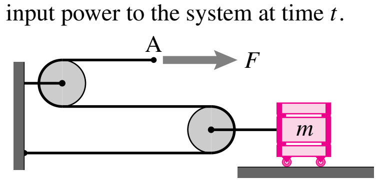
12.5 A motor at B allows the block of mass $m = 3\,\mathrm{kg}$ shown in the figure to accelerate downwards at $2\,\mathrm{m/s^2}$. There is gravity. What is the tension in the string AB?



problem 12.5:

Filename:pfigure-blue-12-2

12.6 For the mass and pulley system shown in the figure, the point of application A of the force moves twice as fast as the mass. At some instant in time t, the speed of the mass is $\dot{x}$ to the left. Find the input power to the system at time t.

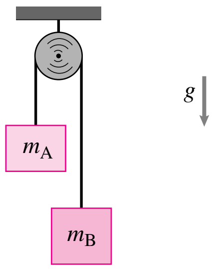


problem 12.6:

Filename:pfig2-3-rp8

12.7 Pulley and masses. Two masses connected by an inextensible string hang from an ideal pulley.

a) Find the downward acceleration of mass B. Answer in terms of any or all of m_A, m_B, g, and the present velocities of the blocks. As a check, your answer should give $a_B = g$ when $m_A = 0$ and $a_B = 0$ when $m_A = m_B$. .
b) Find the tension in the string. As a check, your answer should give $T = m_B g = m_A g$ when $m_A = m_B$ and $T = 0$ when $m_A = 0$.



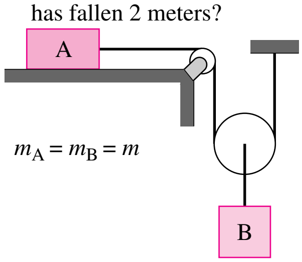
problem 12.7:

Filename:pfigure3-f95p1p2

12.8 The blocks shown are released from rest. Make reasonable assumptions about strings, pulleys, string lengths, and gravity.

a) What is the acceleration of block A at $t = 0^+$ (just after release)?

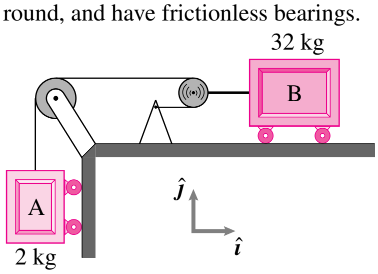
b) What is the speed of block B after it has fallen 2 meters?



problem 12.8:

Filename:pfigure-blue-29-2

12.9 What is the acceleration of block A? Use $g = 10\,\mathrm{m/s^2}$. Assume the string is massless and that the pulleys are massless, round, and have frictionless bearings.

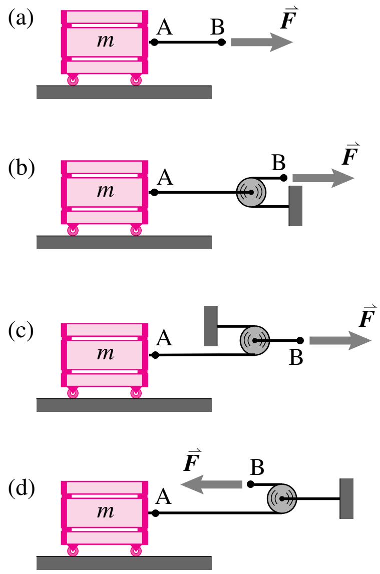


problem 12.9:

Filename:pfigure-f93q4

12.10 For the system shown in problem 12.7, find the acceleration of mass B using energy balance ($P = \dot{E}_K$).

12.11 For the various situations pictured, find the acceleration of mass A and point B shown using balance of linear momentum. Define any variables, coordinates or sign conventions that you need to do your calculations and to define your solution.

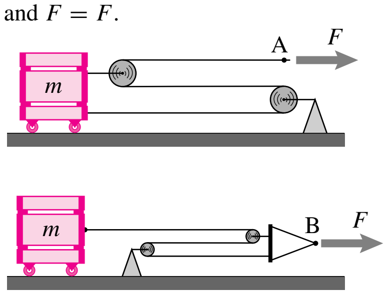


problem 12.11:

Filename:pulley1

12.12 For each of the various situations pictured in problem 12.11 find the acceleration of the mass using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

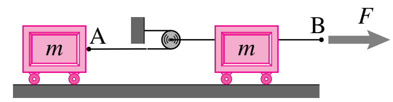
12.13 What is the ratio of the acceleration of point A to that of point B in each configuration? In both cases, the strings are inextensible, the pulleys massless, $m = m$ and $F = F$.



problem 12.13:

Filename:sum95-p1-p3

12.14 Find the acceleration of points A and B in terms of F and m. Assume that the carts stay on the ground, have good (frictionless) bearings, and have wheels of negligible mass.



problem 12.14:

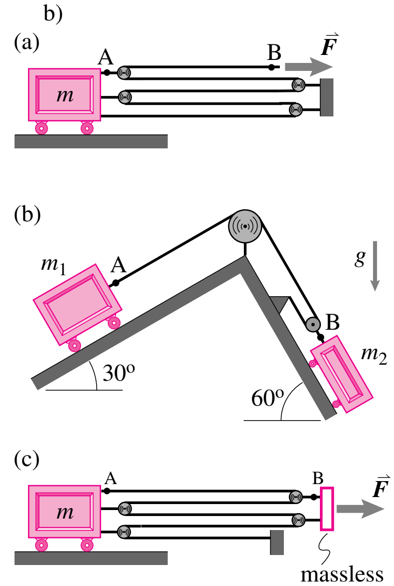
Filename:pfigure-s94q5p1

12.15 For the situation pictured in problem 12.14 find the accelerations of the two masses using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

12.16 For the various situations pictured, find the acceleration of mass A and point B shown using balance of linear momentum ($\sum \vec{F} = m\vec{a}$). Define any variables, coordinates or sign conventions that you need to do your calculations and to define your solution.

a)

b)



problem 12.16:

Filename:pulley4

12.17 For the various situations pictured in problem 12.16 find the acceleration of the mass using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

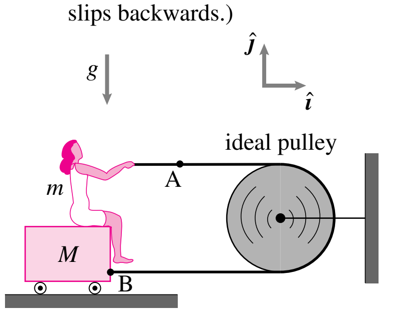
12.18 A person of mass m, modeled as a rigid body is sitting on a cart of mass

$M > m$ and pulling the massless inextensible string towards herself. The coefficient of friction between her seat and the cart is μ. All wheels and pulleys are massless and frictionless. Point B is attached to the cart and point A is attached to the rope.

a) If you are given that she is pulling rope in with acceleration a_0 relative to herself (that is, $\vec{\boldsymbol{a}}_{A/B} \equiv \vec{\boldsymbol{a}}_A - \vec{\boldsymbol{a}}_B = -a_0\hat{\boldsymbol{\imath}}$) and that she is not slipping relative to the cart, find $\vec{\boldsymbol{a}}_A$. (Answer in terms of some or all of m, M, g, μ, $\hat{\boldsymbol{\imath}}$ and a_0.)

b) Find the largest possible value of a_0 without the person slipping off the cart? (Answer in terms of some or all of m, M, g and μ. You may assume her legs get out of the way if she slips backwards.)

c) If instead, $m < M$, what is the largest possible value of a_0 without the person slipping off the cart? (Answer in terms of some or all of m, M, g and μ. You may assume her legs get out of the way if she slips backwards.)



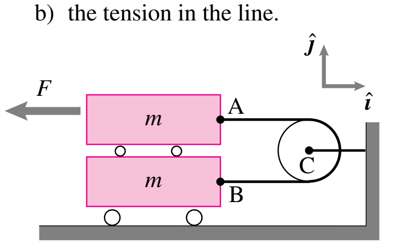
problem 12.18: Pulley.

Filename:s97p2-2

12.19 Two blocks and a pulley. Two identical blocks are stacked and tied together by the pulley as shown. All bearings are frictionless. All rotating parts have negligible mass. Find

a) the acceleration of point A, and

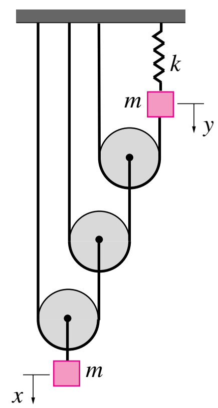
b) the tension in the line.



problem 12.19:

Filename:p-s96-p1-1

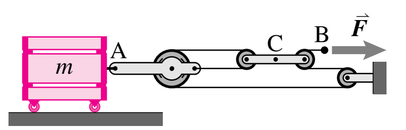
12.20 The pulleys are massless and frictionless. Include gravity. x measures the vertical position of the lower mass from equilibrium. y measures the vertical position of the upper mass from equilibrium. What is the natural frequency of vibration of this system?



problem 12.20:

Filename:pfigure-s95q4

12.21 For the situation pictured, find the acceleration of mass A and points B and C shown. [Hint: the situation with point C is tricky and the answer is genuinely subtle.]



problem 12.21:

Filename:pulley2

12.22 For the situation pictured in problem 12.21, find the acceleration of point A using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

12.23 Design a pulley system. You are to design a pulley system to move a mass. There is no gravity. Point A has a force $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{\imath}}$ pulling it to the right. Mass B has mass m_B. You can connect point A to the mass with any number of ideal strings and ideal pulleys. You can make use of rigid walls or supports anywhere you like (say, to the right or left of the mass). You must design the system so that mass B accelerates to the left with $\frac{F}{2m_B}$ (i.e., $\vec{\boldsymbol{a}}_B = -\frac{F}{2m_B}\hat{\boldsymbol{\imath}}$).

a) Draw the system clearly. Justify your answer with enough words or equations so that a reasonable person, say a grader, can tell that you understand your solution.

b) Find the acceleration of point A.

12.24 Design a pulley system. You are to design a pulley system to move a mass. There is no gravity. Point A has a force $\vec{\boldsymbol{F}} = F\hat{\boldsymbol{\imath}}$ pulling it to the right. Mass B has mass m_B. You can connect the point A to the mass with any number of ideal strings and ideal pulleys. You can make use of rigid walls or supports anywhere you like (say, to the right or left of the mass). Draw the system clearly. Justify your answer with enough words or equations to convince a skeptical person that your solution is correct. You must design the system so that the mass B accelerates .

a) to the left with $\frac{F}{m_B}$ (i.e., $\vec{\boldsymbol{a}}_B = -\frac{F}{m_B}\hat{\boldsymbol{\imath}}$)

b) to the left with $\frac{2F}{m_B}$

c) to the left with $\frac{F}{2m_B}$

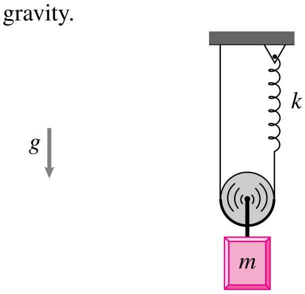
d) to the right with $\frac{2F}{m_B}$

e) to the right with $\frac{F}{2m_B}$

f) to the left with $\frac{8F}{m_B}$

g) to the right with $\frac{F}{5m_B}$

12.25 Pulley and spring. For the hanging mass find the period of oscillation. Assume a massless pulley with good bearings. The massless string is inextensible. Only vertical motion is of interest. There is gravity.

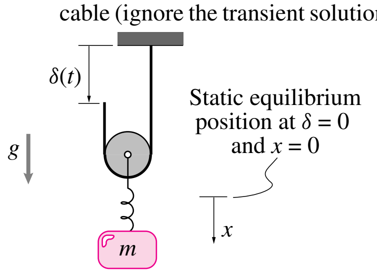


problem 12.25:

Filename:pfigure-s94h4p4

12.26 The spring-mass system shown (m = 10 slugs ($\equiv$ lb $\cdot$ sec^2/ft), $k = 10\,\text{lb}/ft$) is excited by moving the free end of the cable vertically according to $\delta(t) = 4\sin(\omega t)$ in, as shown in the figure. Assuming that the cable is inextensible and massless and that the pulley is massless, do the following.

a) Derive the equation of motion for the block in terms of the displacement x from the static equilibrium position, as shown in the figure.
b) If $\omega = 0.9\,\mathrm{rad/s}$, check to see if the pulley is always in contact with the cable (ignore the transient solution).

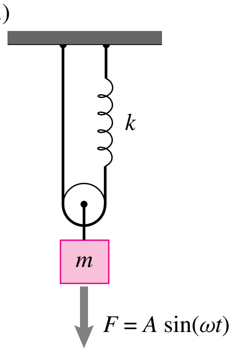


problem 12.26:

Filename:pfigure-blue-151-1

12.27 The block of mass m hanging on the spring with constant k and a string shown in the figure is forced by $F = A\sin(\omega t)$. Do not neglect gravity. The pulley is massless.

a) What is the differential equation governing the motion of the block? You may assume that the only motion is vertical motion.
b) Given A, m and k, for what values of ω would the string go slack at some point in the cyclical motion? (The common assumption in such problems, which you can use, is to neglect the homogeneous solution to the differential equation. It is assumed that the damping, small enough to be neglected in the governing equations is large enough so that the particular solution will have damped out at the time of observation.)



problem 12.27:

Filename:pfigure-blue-155-1

12.28 Block A, with mass m_A, is pulled to the right a distance d from the position it would have if the spring were relaxed. It is then released from rest. Assume ideal string, pulleys and wheels. The spring has constant k.

a) What is the acceleration of block A just after it is released (in terms of k, m_A, and d)?
b) What is the speed of the mass when the mass passes through the position where the spring is relaxed?

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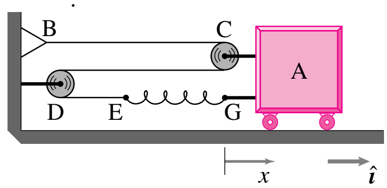
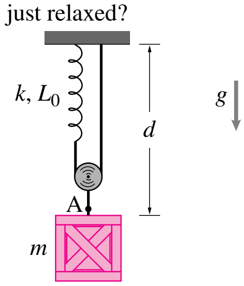


problem 12.28:

Filename:pfigure-f93q5

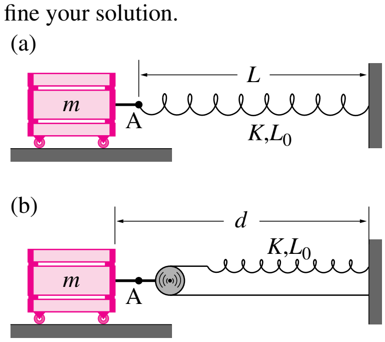
12.29 What is the static displacement of the mass from the position where the spring is just relaxed?



problem 12.29:

Filename:pulley3-a

12.30 For the two situations pictured, find the acceleration of point A shown using balance of linear momentum ($\sum \vec{F} = m\vec{a}$). Assuming both masses are deflected an equal distance from the position where the spring is just relaxed, how much smaller or bigger is the acceleration of block (b) than that of block (a). Define any variables, coordinate system origins, coordinates or sign conventions that you need to do your calculations and to define your solution.



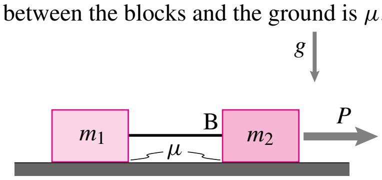
problem 12.30:

Filename:pulley3

12.31 For each of the situations pictured in problem 12.30, find the acceleration of the mass using energy balance ($P = \dot{E}_K$). Define any variables, coordinates, or sign conventions that you need to do your calculations and to define your solution.

12.2 2D and 3D forces even though the motion is straight

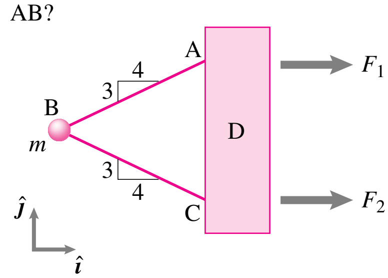
12.32 The two blocks, $m_1 = m_2 = m$, are connected by an inextensible string AB. The string can only withstand a tension T_{cr}. Find the maximum value of the applied force P so that the string does not break. The sliding coefficient of friction between the blocks and the ground is μ.



problem 12.32:

Filename:Danef94s3q5

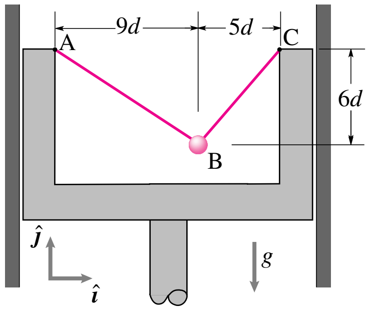
12.33 Mass pulled by two strings. F_1 and F_2 are applied so that the system shown accelerates to the right at $5\,\mathrm{m/s^2}$ (*i.e.*, $\boldsymbol{a} = 5\,\mathrm{m/s^2}\hat{\imath} + 0\hat{\jmath}$) and has no rotation. The mass of D and forces F_1 and F_2 are unknown. What is the tension in string AB?



problem 12.33:

Filename:pg9-1

12.34 A point mass m is attached to a piston by two inextensible cables. The piston has upwards acceleration of $a_y\hat{\jmath}$. There is gravity. In terms of some or all of m, g, d, and a_y find the tension in cable AB.

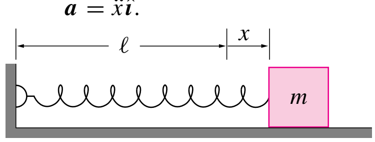


problem 12.34:

Filename:ch3-13

12.35 A point mass of mass m moves on a frictional surface with coefficient of friction μ and is connected to a spring with constant k and unstretched length ℓ. There is gravity. At the instant of interest, the mass is at a distance x to the right from its position where the spring is unstretched and is moving with $\dot{x} > 0$ to the right.

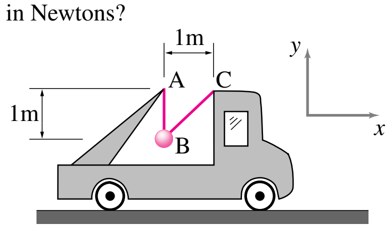
a) Draw a free body diagram of the mass at the instant of interest.
b) At the instant of interest, write the equation of linear momentum balance for the block evaluating the left hand side as explicitly as possible. Let the acceleration of the block be $\vec{a} = \ddot{x}\hat{\imath}$.



problem 12.35:

Filename:ch2-10a

12.36 Find the tension in two strings. Consider the mass at B (2 kg) supported by two strings in the back of a truck which has acceleration of $3\,\mathrm{m/s^2}$. Use $g = 10\,\mathrm{m/s^2}$. What is the tension T_{AB} in the string AB in Newtons?



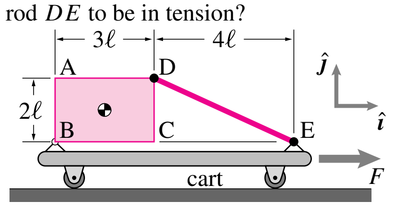
problem 12.36:

Filename:pfigure-s94h2p8

12.37 Coin on a car on a ramp. A student engineering design course asked students to build a cart (mass $= m_c$) that rolls down a ramp with angle θ. A small weight (mass $m_w \ll m_c$) is placed on top of the cart on a surface tipped with respect to the cart (angle ϕ). Assume the small mass does not slide. Assume massless wheels with frictionless bearings. $\hat{\imath}$ is horizontal and $\hat{\jmath}$ is vertical up.

a) Find the acceleration of the cart. Answer in terms of some or all of $m_c, g, \hat{\imath}, \theta$ and $\hat{\jmath}$.
b) What coefficient of friction μ is required (the smallest that will work) to keep the small mass from sliding as the cart rolls down the slope? Answer in terms of some or all of m_c, m_w, g, θ, and ϕ.
c) What angle ϕ will allow a small mass to ride on the cart with the smallest coefficient of friction? Answer in terms of some or all of m_c, m_w, g, and θ.

12.38 Guyed plate on a cart A uniform rectangular plate $ABCD$ of mass m is supported by a rod DE and a hinge joint at point B. The dimensions are as shown. There is gravity. What must the acceleration of the cart be in order for massless rod DE to be in tension?

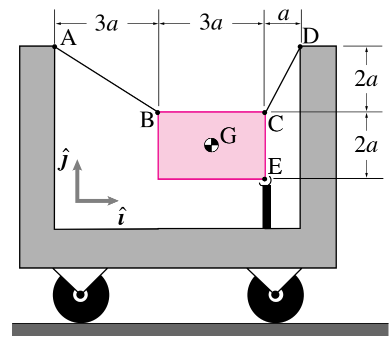


problem 12.38: Uniform plate supported by a hinge and a cable on an accelerating cart.

Filename:tfigure3-2D-a-guyed

12.39 A uniform rectangular plate of mass m is supported by two inextensible cables AB and CD and by a hinge at point E on the cart as shown. The cart has acceleration $a_x\hat{\imath}$ due to a force not shown. There is gravity.

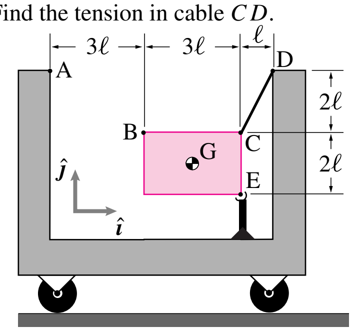
a) Draw a free body diagram of the plate.
b) Write the equation of linear momentum balance for the plate and evaluate the left hand side as explicitly as possible.
c) Write the equation for angular momentum balance about point E and evaluate the left hand side as explicitly as possible.



problem 12.39:

Filename:ch2-12

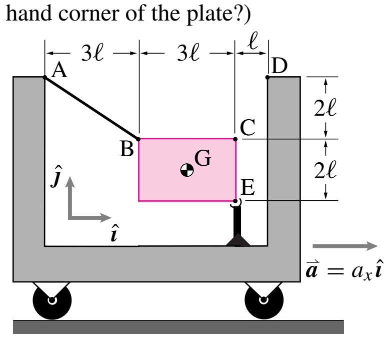
12.40 A uniform rectangular plate of mass m is supported by an inextensible cable CD and a hinge joint at point E on the cart as shown. The hinge joint is attached to a rigid column welded to the floor of the cart. The cart is at rest. There is gravity. Find the tension in cable CD.



problem 12.40:

Filename:ch3-11-b

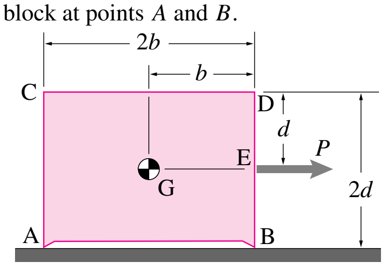
12.41 A uniform rectangular plate of mass m is supported by an inextensible cable AB and a hinge joint at point E on the cart as shown. The hinge joint is attached to a rigid column welded to the floor of the cart. The cart has acceleration $a_x\hat{\imath}$. There is gravity. Find the tension in cable AB. (What's 'wrong' with this problem? What if instead point B were at the bottom left hand corner of the plate?)



problem 12.41:

Filename:ch3-11a

12.42 A block of mass m is sitting on a frictionless surface and acted upon at point E by the horizontal force P through the center of mass. Draw a free body diagram of the block. There is gravity. Find the acceleration of the block and reactions on the block at points A and B.

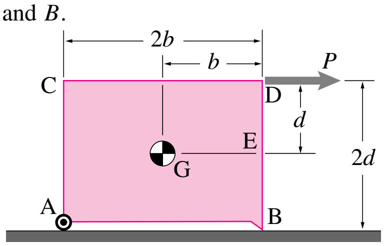


problem 12.42:

Filename:ch3-9

12.43 Reconsider the block in problem 12.42. This time, find the acceleration of the block and the reactions at A and B if the force P is applied instead at point D. Are the acceleration and the reactions on the block different from those found when P is applied at point E?

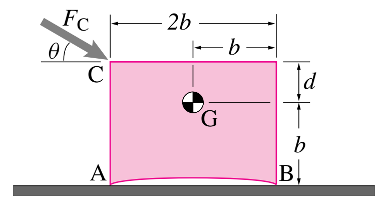
12.44 A block of mass m is sitting on a frictional surface and acted upon at point D by the horizontal force P. The block is resting on a sharp edge at point B and is supported by an ideal wheel at point A. There is gravity. Assuming the block is sliding with coefficient of friction μ at point B, find the acceleration of the block and the reactions on the block at points A and B.



problem 12.44:

Filename:ch3-12

12.45 A force F_C is applied to the corner C of a box of weight W with dimensions and center of gravity at G as shown in the figure. The coefficient of sliding friction between the floor and the points of contact A and B is μ. Assuming that the box slides when F_C is applied, find the acceleration of the box and the reactions at A and B in terms of W, F_C, θ, b, and d.

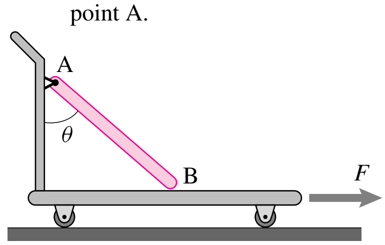


problem 12.45:

Filename:Mikes92p3

12.46 Forces of rod on a cart. A uniform rod with mass m_r rests on a cart (mass m_c) which is being pulled to the right. The rod is hinged at one end (with a frictionless hinge) and has no friction at the contact with the cart. The cart is rolling on wheels that are modeled as having no mass and no bearing friction (ideal massless wheels). Answer in terms of g, m_r, m_c, θ and F. Find:

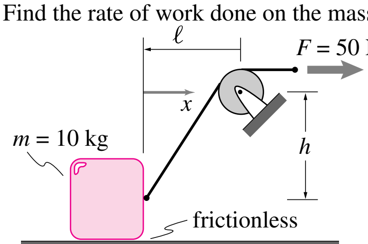
a) The force on the rod from the cart at point B.
b) The force on the rod from the cart at point A.



problem 12.46:

Filename:pfigure-s94h3p3

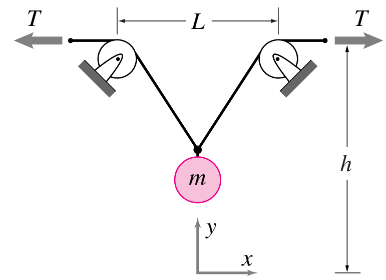
12.47 At the instant shown, the mass is moving to the right at speed $v = 3\,\text{m/s}$. Find the rate of work done on the mass.



problem 12.47:

Filename:pfig2-3-rp9

12.48 A point mass 'm' is pulled straight up by two strings. The two strings pull the mass symmetrically about the vertical axis with constant and equal force T. At an instant in time t, the position and the velocity of the mass are $y(t)\hat{\boldsymbol{\jmath}}$ and $\dot{y}(t)\hat{\boldsymbol{\jmath}}$, respectively. Find the power input to the moving mass.

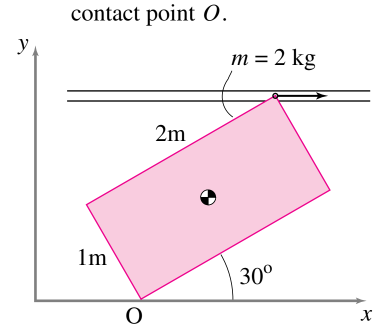


problem 12.48:

Filename:pfig2-3-rp2

12.49 The box shown in the figure is dragged in the x-direction with a constant acceleration $\vec{\boldsymbol{a}} = 0.5\,\text{m/s}^2\hat{\boldsymbol{\imath}}$. At the instant shown, the velocity of (every point on) the box is $\vec{\boldsymbol{v}} = 0.8\,\text{m/s}\hat{\boldsymbol{\imath}}$.

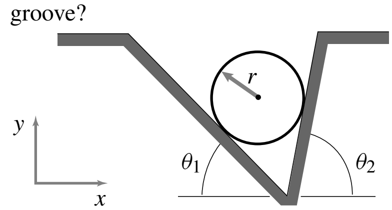
a) Find the linear momentum of the box.
b) Find the rate of change of linear momentum of the box.
c) Find the angular momentum of the box about the contact point O.
d) Find the rate of change of angular momentum of the box about the contact point O.



problem 12.49:

Filename:pfigure3-mom-rp1

12.50 The groove and disk accelerate upwards, $\vec{\boldsymbol{a}} = a\hat{\boldsymbol{\jmath}}$. Neglecting gravity, what are the forces on the disk due to the groove?



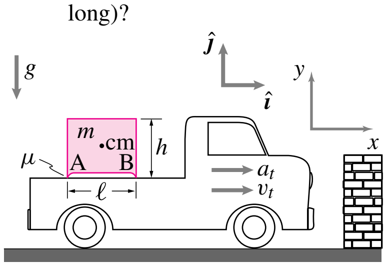
problem 12.50:

Filename:ch2-5-ba

12.51 The following problems concern a box that is in the back of a pickup truck. The pickup truck is moving forward with acceleration of a_t. The truck's speed is v_t. The box has sharp feet at the front and back ends so the only place it contacts the truck is at the feet. The center of mass of the box is at the geometric center of the box. The box has height h, length ℓ and depth w (into the paper.) Its mass is m. There is gravity. The friction coefficient between the truck and the box edges is μ.

In the problems below you should express your solutions in terms of the variables given in the figure, ℓ, h, μ, m, g, a_t, and v_t. If any variables do not enter the expressions comment on why they do not. In all cases you may assume that the box does not rotate (though it might be on the verge of doing so).

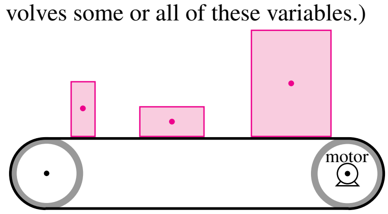
a) Assuming the box does not slide, what is the total force that the truck exerts on the box (i.e. the sum of the reactions at A and B)?
b) Assuming the box does not slide what are the reactions at A and B? [Note: You cannot find both of them without additional assumptions.]
c) Assuming the box does slide, what is the total force that the truck exerts on the box?
d) Assuming the box does slide, what are the reactions at A and B?
e) Assuming the box does not slide, what is the maximum acceleration of the truck for which the box will not tip over (hint: just at that critical acceleration what is the vertical reaction at B?)?
f) What is the maximum acceleration of the truck for which the block will not slide?
g) The truck hits a brick wall and stops instantly. Does the block tip over? Assuming the block does not tip over, how far does it slide on the truck before stopping (assume the bed of the truck is sufficiently long)?



problem 12.51:

Filename:pfigure-blue-22-1

12.52 A collection of uniform boxes with various heights h and widths w and masses m sit on a horizontal conveyer belt. The acceleration $a(t)$ of the conveyer belt gets extremely large sometimes due to an erratic over-powered motor. Assume the boxes touch the belt at their left and right edges only and that the coefficient of friction there is μ. It is observed that some boxes never tip over. What is true about μ, g, w, h, and m for the boxes that always maintain contact at both the right and left bottom edges? (Write an inequality that involves some or all of these variables.)

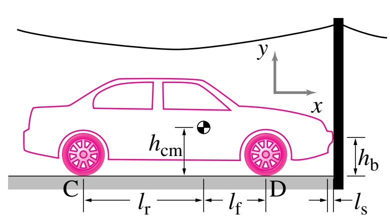


problem 12.52:

Filename:pfigure-f93q3

12.53 After failure of her normal brakes, a driver pulls the emergency brake of her old car. This action locks the rear wheels (friction coefficient $= \mu$) but leaves the well lubricated and light front wheels spinning freely. The car, braking inadequately as is the case for rear wheel braking, hits a stiff and slippery phone pole which compresses the car bumper. The car bumper is modeled here as a linear spring (constant $= k$, rest length $= l_0$, present length $= l_s$). The car is still traveling forward at the moment of interest. The bumper is at a height h_b above the ground. Assume that the car, excepting the bumper, is a non-rotating rigid body and that the wheels remain on the ground (that is, the bumper is compliant but the suspension is stiff).

- What is the acceleration of the car in terms of g, m, μ, l_f, l_r, k, h_b, h_{cm}, l_0, and l_s (and any other parameters if needed)?



problem 12.53:

Filename:pfigure-s94q4p1

12.54 Car braking: front brakes versus rear brakes versus all four brakes. There are a few puzzles in dynamics concerning the differences between front and rear braking of a car. Here is one you can deal with now. What is the peak deceleration of a car when you apply: the front brakes till they skid, the rear brakes till they skid, and all four brakes till they skid? Assume that the coefficient of friction between rubber and road is $\mu = 1$ (about right, the coefficient of friction between rubber and road varies between about .7 and 1.3) and that $g = 10\,\text{m/s}^2$ (2% error). Pick the dimensions and mass of the car, but assume the center of mass height h is above the ground. The height h, should be less than half the wheel base w, the distance between the front and rear wheel. Further assume that the CM is halfway between the front and back wheels ($i.e.$, $l_f = l_r = w/2$). Assume also that the car has a stiff suspension so the car does not move up or down or tip during braking; i. e., the car does not rotate in the xy-plane. Neglect the mass of the rotating wheels in the linear and angular momentum balance equations. Treat this problem as two-dimensional problem; i. e., the car is symmetric left to right, does not turn left or right, and that the left and right wheels carry the same loads. To organize your work, here are some steps to follow.

a) Draw a FBD of the car assuming rear wheel is skidding. The FBD should show the dimensions, the gravity force, what you know *a priori* about the forces on the wheels from the ground (i.e., that the friction force $F_r = \mu N_r$, and that there is no friction at the front wheels), and the coordinate directions. Label points of interest that you will use in your momentum balance equations. (Hint: also draw a free body diagram of the rear wheel.)
b) Write the equation of linear momentum balance.
c) Write the equation of angular momentum balance relative to a point of your choosing. Some particularly useful points to use are:
   - the point above the front wheel and at the height of the center of mass;
   - the point at the height of the center of mass, behind the rear wheel that makes a 45 degree angle line down to the rear wheel ground contact point; and
   - the point on the ground straight under the front wheel

that is as far below ground as the wheel base is long.

d) Solve the momentum balance equations for the wheel contact forces and the deceleration of the car. If you have used any or all of the recommendations from part (c) you will have the pleasure of only solving one equation in one unknown at a time.

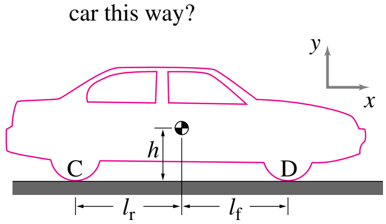
e) Repeat steps (a) to (d) for front-wheel skidding. Note that the advantageous points to use for angular momentum balance are now different. Does a car stop faster or slower or the same by skidding the front instead of the rear wheels? Would your solution to (e) be different if the center of mass of the car were at ground level(h=0)?

f) Repeat steps (a) to (d) for all-wheel skidding. There are some shortcuts here. You determine the car deceleration without ever knowing the wheel reactions (or using angular momentum balance) if you look at the linear momentum balance equations carefully.

g) Does the deceleration in (f) equal the sum of the decelerations in (d) and (e)? Why or why not?

h) What peculiarity occurs in the solution for front-wheel skidding if the wheel base is twice the height of the CM above ground and $\mu = 1$?

i) What impossibility does the solution predict if the wheel base is shorter than twice the CM height? What wrong assumption gives rise to this impossibility? What would really happen if one tried to skid a car this way?



problem 12.54:

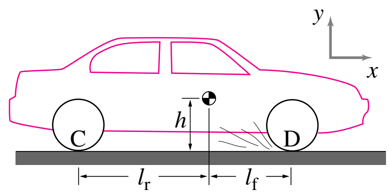
Filename:pfigure-s94h3p6

12.55 Assuming massless wheels, an infinitely powerful engine, a stiff suspension (*i.e.*, no rotation of the car) and a coefficient of friction μ between tires and road,

a) what is the maximum forward acceleration of this front wheel drive car?

b) what is the force of the ground on the rear wheels during this acceleration?

c) what is the force of the ground on the front wheels?



problem 12.55:

Filename:pfigure-f00p1-3

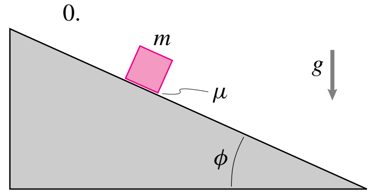
12.56 At time $t = 0$, the block of mass m is released from rest on the slope of angle ϕ. The coefficient of friction between the block and slope is μ.

a) What is the acceleration of the block for $\mu > 0$?

b) What is the acceleration of the block for $\mu = 0$?

c) Find the position and velocity of the block as a function of time for $\mu > 0$.

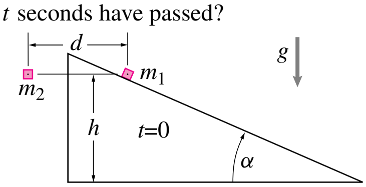
d) Find the position and velocity of the block as a function of time for $\mu = 0$.



problem 12.56:

Filename:Danef94s1q5

12.57 A small block of mass m_1 is released from rest at altitude h on a frictionless slope of angle α. At the instant of release, another small block of mass m_2 is dropped vertically from rest at the same altitude. The second block does not interact with the ramp. What is the velocity of the first block relative to the second block after t seconds have passed?



problem 12.57:

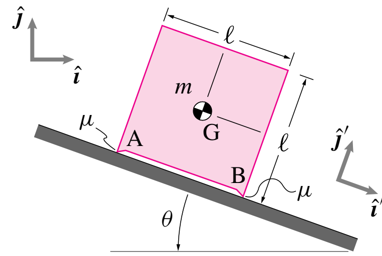
Filename:ch8-7

12.58 Block sliding on a ramp with friction. A square box is sliding down a ramp of angle θ with instantaneous velocity $v\hat{\imath}'$. Assume it does not tip over.

a) What is the force on the block from the ramp at point A? Answer in terms of any or all of θ, ℓ, m, g, μ, v, $\hat{\imath}'$, and $\hat{\jmath}'$. As a check, your answer should reduce to $\frac{mg}{2}\hat{\jmath}'$ when $\theta = \mu = 0$.

b) In addition to solving the problem by hand, see if you can write a set of computer commands that, if θ, μ, ℓ, m, v and g were specified, would give the correct answer.

c) Assuming $\theta = 80°$ and $\mu = 0.9$, can the box slide this way or would it tip over? Why?

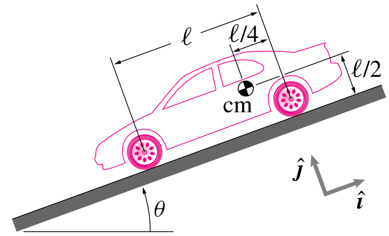


problem 12.58:

Filename:pfigure3-f95p1p1

12.59 A coin is given a sliding shove up a ramp with angle ϕ with the horizontal. It takes twice as long to slide down as it does to slide up. What is the coefficient of friction μ between the coin and the ramp. Answer in terms of some or all of m, g, ϕ and the initial sliding velocity v.

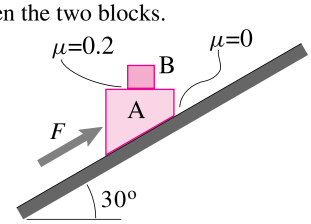
12.60 A skidding car. What is the braking acceleration of the front-wheel braked car as it slides down hill. Express your answer as a function of any or all of the following variables: the slope θ of the hill, the mass of the car m, the wheel base ℓ, and the gravitational constant g. Use $\mu = 1$.



problem 12.60: A car skidding downhill on a slope of angle θ

Filename:pfigure3-car

12.61 Two blocks A and B are pushed up a frictionless inclined plane by an external force F as shown in the figure. The coefficient of friction between the two blocks is $\mu = 0.2$. The masses of the two blocks are $m_A = 5\,\mathrm{kg}$ and $m_B = 2\,\mathrm{kg}$. Find the magnitude of the maximum allowable force such that no relative slip occurs between the two blocks.

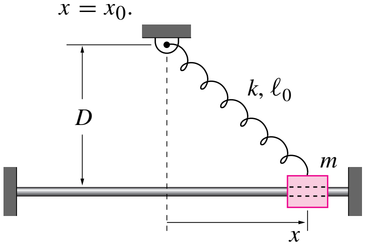


problem 12.61:

Filename:summer95f-1

12.62 A bead slides on a frictionless rod. The spring has constant k and rest length ℓ_0. The bead has mass m.

a) Given x and $\dot{x}$ find the acceleration of the bead (in terms of some or all of D, ℓ_0, x, $\dot{x}$, m, k and any base vectors that you define).
b) If the bead is allowed to move, as constrained by the slippery rod and the spring, find a differential equation that must be satisfied by the variable x. (Do not try to solve this somewhat ugly non-linear equation.)
c) In the special case that $\ell_0 = 0^*$ find how long it takes for the block to return to its starting position after release with no initial velocity at $x = x_0$.



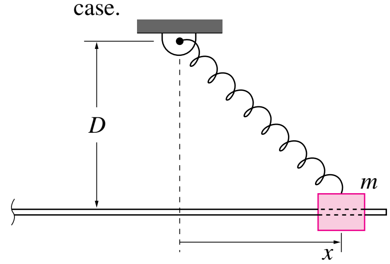
problem 12.62:

Filename:p-s96-p1-2

12.63 A bead oscillates on a straight frictionless wire. The spring obeys the equation $F = k\,(\ell - \ell_o)$, where ℓ = length of the spring and ℓ_0 is the 'rest' length. Assume

$$x(t = 0) = x_0,\ \dot{x}(t = 0) = 0.$$

a) Write a differential equation satisfied by $x(t)$.
b) What is $\dot{x}$ when $x = 0$? [hint: Don't try to solve the equation in (a)!]
c) Note the simplification in (a) if $\ell_o = 0$ (spring is then a so-called "zero–length" spring).
d) For this special case ($\ell_o = 0$) solve the equation in (a) and show the result agrees with (b) in this special case.



problem 12.63:

Filename:pfigure-blue-60-1

12.64 A cart on an elastic leash. A cart B (mass m) rolls on a frictionless level floor. One end of an inextensible string is attached to the the cart. The string wraps around a pulley at point A and the other end is attached to a spring with constant k. When the cart is at point O, it is in static equilibrium. The spring relaxed length, rope length, and room height h are such that the spring would be relaxed if the end of rope at B were diconnected from the cart and brought up to point A. The gravitational constant is g. The cart is pulled a horizontal distance d from the center of the room (at O) and released.

a) Assuming that the cart never leaves the floor, what is the speed of the cart when it passes through the center of the room, in terms of m, h, g and d.
b) Does the cart undergo simple harmonic motion for small or large oscillations (specify which if either)? (Simple harmonic motion occurs when position varies inusoidally with time.)

problem 12.64:

Filename:cartosc

12.65 The cart moves to the right with constant acceleration a. The ball has mass m. The spring has unstretched length ℓ_0 and spring constant k. Assuming the ball is stationary with respect to the cart find the distance from O to A in terms of k, ℓ_0, and a . [Hint: find θ first.]

problem 12.65:

Filename:Danef94s3q6

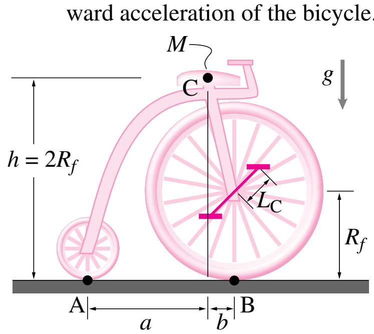
12.66 Consider a person, modeled as a rigid body, riding an accelerating motorcycle (2-D). The person is sitting on the seat and cannot slide fore or aft, but is free to rock in the plane of the motorcycle (as if there were a hinge connecting the motorcycle to the rider at the seat). The person's feet are off the pegs and the legs are sticking down and not touching anything.

The person's arms are like cables (they are massless and only carry tension). Assume all dimensions and masses are known (you have to define them carefully with a sketch and words). Assume the forward acceleration of the motorcycle is known. You may use numbers and/or variables to describe the quantities of interest.

a) Draw a clear sketch of the problem showing needed dimensional information and the coordinate system you will use.
b) Draw a Free Body Diagram of the rider.
c) Write the equations of linear and angular momentum balance for the rider.
d) Find all forces on the rider from the motorcycle (i.e., at the hands and the seat).
e) What are the forces on the motorcycle from the rider?

12.67 Acceleration of a bicycle on level ground. 2-D . A very compact bicycler (modeled as a point mass M at the bicycle seat C with height h, and distance b behind the front wheel contact), rides a very light old-fashioned bicycle (all components have negligible mass) that is well maintained (all bearings have no frictional torque) and streamlined (neglect air resistance). The rider applies a force F_p to the pedal perpendicular to the pedal crank (with length L_c). No force is applied to the other pedal. The radius of the front wheel is R_f.

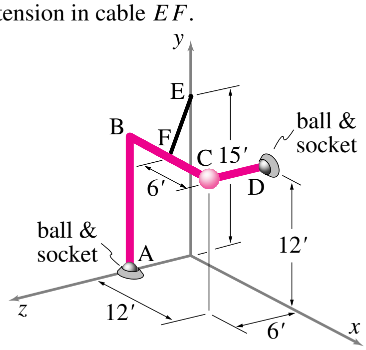
a) Assuming no slip, what is the forward acceleration of the bicycle? [Hint: draw a FBD of the front wheel and crank, and another FBD of the whole bicycle-rider system.]
b) (Harder) Assuming the rider can push arbitrarily hard but that $\mu = 1$, what is the maximum possible forward acceleration of the bicycle.



problem 12.67:

Filename:f-3

12.68 A 320 lbm mass is attached at the corner C of a light rigid piece of pipe bent as shown. The pipe is supported by ball-and-socket joints at A and D and by cable EF. The points A, D, and E are fastened to the floor and vertical sidewall of a pick-up truck which is accelerating in the z-direction. The acceleration of the truck is $\vec{a} = 5\,\text{ft/s}^2\hat{\boldsymbol{k}}$. There is gravity. Find the tension in cable EF.



problem 12.68:

Filename:mikef91p1

12.69 A 5 ft by 8 ft rectangular plate of uniform density has mass $m = 10$ lbm and is supported by a ball-and-socket joint at point A and the light rods CE, BD, and GH. The entire system is attached to a truck which is moving with acceleration $\vec{a}_T$. The plate is moving without rotation or angular acceleration relative to the truck. Thus, the center of mass acceleration of the plate is the same as the truck's. Dimensions are as shown. Points A, C, and D are fixed to the truck but the truck is not touching the plate at any other points. Find the tension in rod BD.

a) If the truck's acceleration is $\boldsymbol{a}_{cm} = (5\,\text{ft/s}^2)\boldsymbol{k}$, what is the tension or compression in rod BD?
b) If the truck's acceleration is $\vec{\boldsymbol{a}}_{\text{cm}} = (5\,\text{ft/s}^2)\hat{\boldsymbol{\jmath}} + (6\,\text{ft/s}^2)\hat{\boldsymbol{k}}$, what is the tension or compression in rod GH?

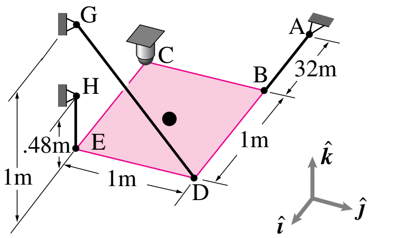
problem 12.69:

Filename:Mikesp92p1

12.70 Hanging a shelf. A shelf with negligible mass supports a 0.5 kg mass at its center. The shelf is supported at one corner with a ball and socket joint and at the other three corners with strings. At the moment of interest the shelf is in a rocket in outer space and accelerating at $10\,\text{m/s}^2$ in the $\boldsymbol{k}$ direction. The shelf is in the xy plane.

a) Draw a FBD of the shelf.
b) Challenge: without doing any calculations on paper can you find one of the reaction force components or the tension in any of the cables? Give yourself a few minutes of staring to try this approach. If you can't, then come back to this question after you have done all the calculations.
c) Write down the linear momentum balance equation (a vector equation).
d) Write down the angular momentum balance equation using the center of mass as a reference point.
e) By taking components, turn (b) and (c) into six scalar equations in six unknowns.
f) Solve these equations by hand or on the computer.
g) Instead of using a system of equations try to find a single equation which can be solved for T_{EH}. Solve it and compare to your result from before.
h) Challenge: For how many of the reactions can you find one equation which will tell you that particular reaction without knowing any of the

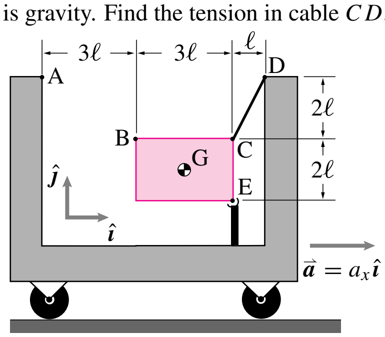
other reactions? [Hint, try angular momentum balance about various axes as well as linear momentum balance in an appropriate direction. It is possible to find five of the six unknown reaction components this way.] Must these solutions agree with (d)? Do they?



problem 12.70:

Filename:pfigure-s94h2p10

12.71 A uniform rectangular plate of mass m is supported by an inextensible cable CD and a hinge joint at point E on the cart as shown. The hinge joint is attached to a rigid column welded to the floor of the cart. The cart has acceleration $a_x\hat{\imath}$. There is gravity. Find the tension in cable CD.

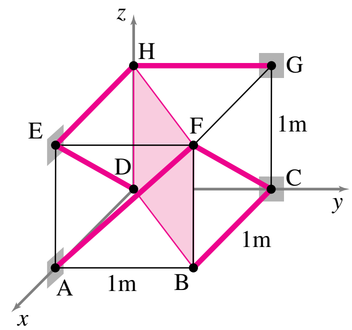


problem 12.71:

Filename:ch3-11

12.72 The uniform 2 kg plate DBFH is held by six massless rods (AF, CB, CF, GH, ED, and EH) which are hinged at their ends. The support points A, C, G, and E are all accelerating in the x-direction with acceleration $\boldsymbol{a} = 3\,\mathrm{m/s^2}\boldsymbol{\imath}$. There is no gravity.

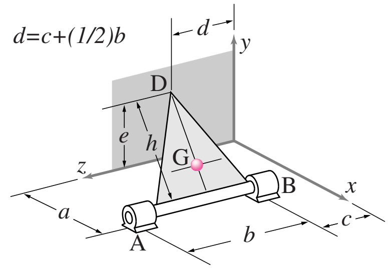
a) What is $\{\sum \vec{\boldsymbol{F}}\}\cdot\hat{\boldsymbol{\imath}}$ for the forces acting on the plate?
b) What is the tension in bar CB?



problem 12.72:

Filename:pfigure-s94q3p1

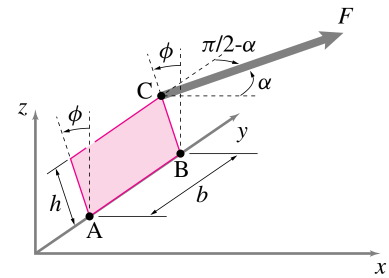
12.73 A massless triangular plate rests against a frictionless wall of a pick-up truck at point D and is rigidly attached to a massless rod supported by two ideal bearings fixed to the floor of the pick-up truck. A ball of mass m is fixed to the centroid of the plate. There is gravity. The pick-up truck skids across a road with acceleration $\vec{\boldsymbol{a}} = a_x\hat{\boldsymbol{\imath}} + a_z\hat{\boldsymbol{k}}$. What is the reaction at point D on the plate?



problem 12.73:

Filename:ch3-1a

12.74 Towing a bicycle. A bicycle on the level xy plane is steered straight ahead and is being towed by a rope. The bicycle and rider are modeled as a uniform plate with mass m (for the convenience of the artist). The tow force F applied at C has no z component and makes an angle α with the x axis. The rolling wheel contacts are at A and B. The bike is tipped an angle ϕ from the vertical. The towing force F is the magnitude needed to keep the bike accelerating in a straight line (along the y axis) without tipping any more or less than the angle ϕ. What is the acceleration of the bicycle? Answer in terms of some or all of b, h, α, ϕ, m, g and $\hat{\boldsymbol{\jmath}}$ (Note: F should *not* appear in your final answer.)



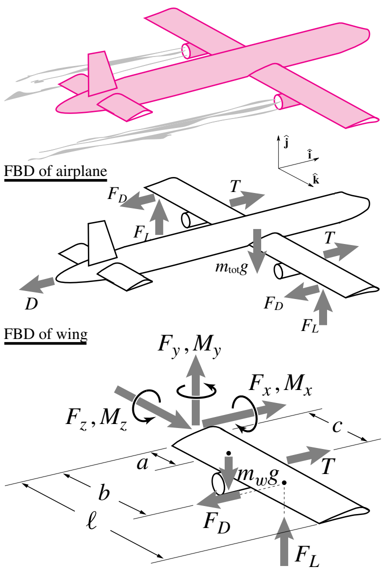
problem 12.74:

Filename:s97f3

12.75 An airplane is in straight level flight but is accelerating in the forward direction. In terms of some or all of the following parameters,

- $m_{tot} \equiv$ the total mass of the plane (including the wings),
- D= the drag force on the fuselage,
- F_D= the drag force on each wing,
- g= gravitational constant, and,
- T = the thrust of one engine.

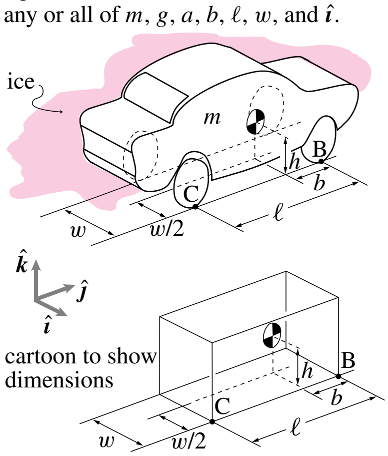
a) What is the lift on each wing F_L?
b) What is the acceleration of the plane $\vec{\boldsymbol{a}}_P$?
c) A free body diagram of one wing is shown. The mass of one wing is m_w. What, in terms of m_{tot}, m_w, F_L, F_D, g, a,b, c, and ℓ are the reactions at the base of the wing (where it is attached to the plane), $\vec{\boldsymbol{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}} + F_z\hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{M}} = M_x\hat{\boldsymbol{\imath}} + M_y\hat{\boldsymbol{\jmath}} + M_z\hat{\boldsymbol{k}}$?



problem 12.75:

Filename:pfigure3-airplane

12.76 A rear-wheel drive car on level ground. The two left wheels are on perfectly slippery ice. The right wheels are on dry pavement. The negligible-mass front right wheel at B is steered straight ahead and rolls without slip. The right rear wheel at C also rolls without slip and drives the car forward with velocity $\vec{v} = v\hat{\jmath}$ and acceleration $\vec{a} = a\hat{\jmath}$. Dimensions are as shown and the car has mass m . What is the sideways force from the ground on the right front wheel at B? Answer in terms of any or all of m, g, a, b, ℓ, w, and $\hat{\imath}$.

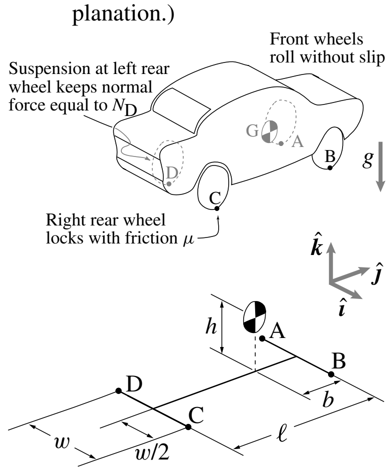


problem 12.76:

Filename:pfigure3-f95p1p3

12.77 A somewhat crippled car slams on the brakes. The suspension springs at A, B, and C are frozen and keep the car level and at constant height. The normal force at D is kept equal to N_D by the only working suspension spring which is on the left rear wheel at D. The only brake which is working is that of the right rear wheel at C which slides on the ground with friction coefficient μ. Wheels A, B, and D roll freely without slip. Dimensions are as shown.

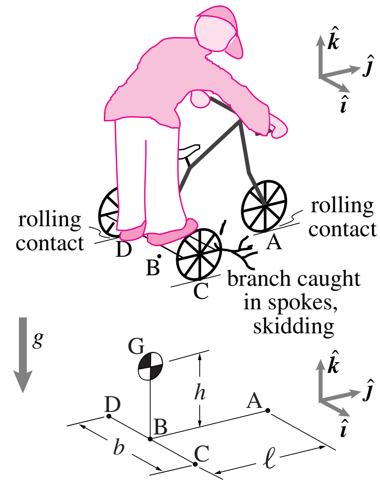
a) Find the acceleration of the car in terms of some or all of m, w, ℓ, b, h, g, μ, $\hat{\jmath}$, and N_D.
b) From the information given could you also find all of the reaction forces at all of the wheels? If so, why? If not, what can't you find and why? (No credit for correct answer. Credit depends on clear explanation.)



problem 12.77:

Filename:s97p2-3

12.78 Speeding tricycle gets a branch caught in the right rear wheel. A scared-stiff tricyclist riding on level ground gets a branch stuck in the right rear wheel so the wheel skids with friction coefficient μ. Assume that the center of mass of the tricycle-person system is directly above the rear axle. Assume that the left rear wheel and the front wheel have negligible mass, good bearings, and have sufficient friction that they roll in the $\hat{\jmath}$ direction without slip, thus constraining the overall motion of the tricycle. Dimensions are shown in the lower sketch. *Find the acceleration of the tricycle* (in terms of some or all of ℓ, h, b, m, $[I^{cm}]$, μ, g, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$). [Hint: check your answer against special cases for which you might guess the answer, such as when $\mu = 0$ or when $h = 0$.]

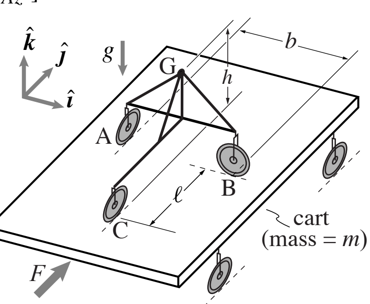


problem 12.78:

Filename:p-f96-f-1

12.79 -3-wheeled robot. A 3-wheeled robot with mass m is being transported on a level flatbed trailer also with mass m. The trailer is being pushed with a force $F\hat{\jmath}$. The ideal massless trailer wheels roll without slip. The ideal massless robot wheels also roll without slip. The robot steering mechanism has turned the wheels so that wheels at A and C are free to roll in the $\hat{\jmath}$ direction and the wheel at B is free to roll in the $\hat{\imath}$ direction. The center of mass of the robot at G is h above the trailer bed and symmetrically above the axle connecting wheels A and B. The wheels A and B are a distance b apart. The length of the robot is ℓ.

Find the force vector $\vec{F}_A$ of the trailer on the robot at A in terms of some or all of m, g, ℓ, F, b, h, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$. [Hints: Use a free body diagram of the cart with robot to find their acceleration. With reference to a free body diagram of the robot, use angular momentum balance about axis BC to find F_{Az}.]



problem 12.79:

Filename:pfigure-threewheelrobot

9.87) $h_{max} = e^2 h$.

10.4a) $\vec{\boldsymbol{v}}(5\,\mathrm{s}) = (30\hat{\boldsymbol{\imath}} + 300\hat{\boldsymbol{\jmath}})\,\mathrm{m/s}$.

b) $\vec{\boldsymbol{a}}(5\,\mathrm{s}) = (6\hat{\boldsymbol{\imath}} + 120\hat{\boldsymbol{\jmath}})\,\mathrm{m/s^2}$.

10.5) $\vec{\boldsymbol{r}}(t) = \left(x_0 + \frac{u_0}{\Omega} - \frac{u_0}{\Omega}\cos(\Omega t)\right)\hat{\boldsymbol{\imath}} + (y_0 + v_0 t)\,\hat{\boldsymbol{\jmath}}$.

10.13) $\vec{\boldsymbol{v}} = 2t\,\mathrm{m/s^2}\hat{\boldsymbol{\imath}} + e^{\frac{t}{s}}\,\mathrm{m/s}\hat{\boldsymbol{\jmath}}$, $\vec{\boldsymbol{a}} = 2\,\mathrm{m/s^2}\hat{\boldsymbol{\imath}} + e^{\frac{t}{s}}\,\mathrm{m/s^2}\hat{\boldsymbol{\jmath}}$.

10.48) $T_3 = 13\,\mathrm{N}$

10.61) Equation of motion: $-mg\hat{\boldsymbol{\jmath}} - b(\dot{x}^2 + \dot{y}^2)\left(\frac{\dot{x}\hat{\boldsymbol{\imath}} + \dot{y}\hat{\boldsymbol{\jmath}}}{\sqrt{\dot{x}^2+\dot{y}^2}}\right) = m(\ddot{x}\hat{\boldsymbol{\imath}} + \ddot{y}\hat{\boldsymbol{\jmath}})$.

10.62a) System of equations:

$$\begin{aligned}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{v}_x &= -\frac{b}{m} v_x \sqrt{v_x^2 + v_y^2} \\
\dot{v}_y &= -g - \frac{b}{m} v_y \sqrt{v_x^2 + v_y^2}
\end{aligned}$$

11.6) No. You need to know the angular momenta of the particles relative to the center of mass to complete the calculation, information which is not given.

11.17a) $v_0 = \frac{1}{m}(m v_B + m_B v_B + m_A v_A)$.

b) $v_1 = \frac{(m+m_B)}{m} v_B$.

c) (1) $E_{loss} = \frac{1}{2}m\left[v_0^2 - \left(\frac{m+m_B}{m}\right)v_B^2\right] - \frac{1}{2}m_A v_A^2$. $E_{loss} = \frac{1}{2}m\left[\frac{(m+m_B)^2}{m}v_B^2 - (m+m_B)v_B^2\right]$.

11.18) $v_A = \sqrt{\frac{m_B k \delta^2}{m_A^2 + m_B m_A}}$.

11.19) The trajectories should all be the same figure 8.

12.1) $T_n = \frac{P_t}{v_t}\frac{n}{N}$.

12.7a) $a_B = \left(\frac{m_B - m_A}{m_A + m_B}\right) g$

b) $T = 2\frac{m_A m_B}{m_A + m_B} g$.

12.11) (a) $\vec{\boldsymbol{a}}_A = \vec{\boldsymbol{a}}_B = \frac{F}{m}\hat{\boldsymbol{\imath}}$, where $\hat{\boldsymbol{\imath}}$ is parallel to the ground and pointing to the right., (b) $\vec{\boldsymbol{a}}_A = \frac{2F}{m}\hat{\boldsymbol{\imath}}$, $\vec{\boldsymbol{a}}_B = \frac{4F}{m}\hat{\boldsymbol{\imath}}$, (c) $\vec{\boldsymbol{a}}_A = \frac{F}{2m}\hat{\boldsymbol{\imath}}$, $\vec{\boldsymbol{a}}_B = \frac{F}{4m}\hat{\boldsymbol{\imath}}$, (d) $\vec{\boldsymbol{a}}_A = \frac{F}{m}\hat{\boldsymbol{\imath}}$, $\vec{\boldsymbol{a}}_B = -\frac{F}{m}\hat{\boldsymbol{\imath}}$.

12.13) $\frac{a_A}{a_B} = 81$.

12.16a) $\vec{\boldsymbol{a}}_A = \frac{5F}{m}\hat{\boldsymbol{\imath}}$, $\vec{\boldsymbol{a}}_B = \frac{25F}{m}\hat{\boldsymbol{\imath}}$, where $\hat{\boldsymbol{\imath}}$ is parallel to the ground and points to the right.

b) $\vec{\boldsymbol{a}}_A = \frac{g}{(4m_1+m_2)}(2m_2 - \sqrt{3}m_2)\hat{\boldsymbol{\lambda}}_1$, $\vec{\boldsymbol{a}}_B = -\frac{g}{2(4m_1+m_2)}(2m_2 - \sqrt{3}m_2)\hat{\boldsymbol{\lambda}}_2$, where $\hat{\boldsymbol{\lambda}}_1$ is parallel to the slope that mass m_1 travels along, pointing down and to the left, and $\hat{\boldsymbol{\lambda}}_2$ is parallel to the slope that mass m_2 travels along, pointing down and to the right.

12.20) angular frequency of vibration $\equiv \lambda = \sqrt{\frac{64k}{65m}}$.

12.27a) $m\ddot{x} + 4kx = A\sin\omega t + mg$, where x is the distance measured from the unstretched position of the center of the pulley.

b) The string will go slack if $\omega > \sqrt{\frac{4k}{m}\left(1 - \frac{A}{mg}\right)}$.

12.28a) $\vec{\boldsymbol{a}}_A = -\frac{9kd}{m_A}\hat{\boldsymbol{\imath}}$.

b) $v = 3d\sqrt{\frac{k}{m_A}}$

12.34) $T_{AB} = \frac{5\sqrt{39}}{28}m(a_y + g)$

12.38) $a_x > \frac{3}{2}g$

12.41) Can't solve for T_{AB}.

12.54d) Normal reaction at rear wheel: $N_r = \frac{mgw}{2(h\mu+w)}$, normal reaction at front wheel: $N_f = mg - \frac{mgw}{2(h\mu+w)}$, deceleration of car: $a_{car} = -\frac{\mu g w}{2(h\mu+w)}$.

e) Normal reaction at rear wheel: $N_r = mg - \frac{mgw}{2(w-\mu h)}$, normal reaction at front wheel: $N_f = \frac{mgw}{2(w-\mu h)}$, deceleration of car: $a_{car} = -\frac{\mu g w}{2(w-\mu h)}$. Car stops more quickly for front wheel skidding. Car stops at same rate for front or rear wheel skidding if $h = 0$.

f) Normal reaction at rear wheel: $N_r = \frac{mg(w/2-\mu h)}{w}$, normal reaction at front wheel: $N_f = \frac{mg(w/2+\mu h)}{w}$, deceleration of car: $a_{car} = -\mu g$.

g) No. Simple superposition just doesn't work.

h) No reaction at rear wheel.

i) Reaction at rear wheel is negative. Not allowing for rotation of the car in the xy-plane gives rise to this impossibility. In actuality, the rear of the car would flip over the front.

12.55a) Hint: the answer reduces to $a = \ell_r g/h$ in the limit $\mu \to \infty$.]

12.56a) $\vec{\boldsymbol{a}} = g(\sin\phi - \mu\cos\phi)\hat{\boldsymbol{\imath}}$, where $\hat{\boldsymbol{\imath}}$ is parallel to the slope and pointing downwards

b) $\vec{\boldsymbol{a}} = g\sin\phi$

c) $\vec{\boldsymbol{v}} = g(\sin\phi - \mu\cos\phi)t\hat{\boldsymbol{\imath}}$, $\vec{\boldsymbol{r}} = g(\sin\phi - \mu\cos\phi)\frac{t^2}{2}$

d) $\vec{\boldsymbol{v}} = g\sin\phi t\hat{\boldsymbol{\imath}}$, $\vec{\boldsymbol{r}} = g\sin\phi\frac{t^2}{2}\hat{\boldsymbol{\imath}}$

12.58a) $\vec{\boldsymbol{R}}_A = \frac{(1-\mu)mg\cos\theta}{2}(\hat{\boldsymbol{\jmath}}' - \mu\hat{\boldsymbol{\imath}}')$.

c) No tipping if $N_A = \frac{(1-\mu)mg\cos\theta}{2} > 0$; i.e., no tipping if $\mu < 1$ since $\cos\theta > 0$ for $0 < \theta < \frac{\pi}{2}$.(Here $\mu = 0.9$)

12.60) braking acceleration= $g(\frac{1}{2}\cos\theta - \sin\theta)$.

12.64a) $v = d\sqrt{\frac{k}{m}}$.

b) The cart undergoes simple harmonic motion for any size oscillation.

12.67a) $\vec{\boldsymbol{a}}_{bike} = \frac{F_p L_c}{MR_f}$.

b) max($\vec{\boldsymbol{a}}_{bike}$)=$\frac{ga}{a+b+2R_f}$.

12.68) $T_{EF} = 640\sqrt{2}$ lbf.

12.69a) $T_{BD} = 92.6\,\text{lbm} \cdot \text{ft/s}^2$.

b) $T_{GH} = 5\sqrt{61}\,\text{lbm} \cdot \text{ft/s}^2$.

12.70b) $T_{EH} = 0$

c) $(R_{C_x} - T_{AB})\hat{\boldsymbol{\imath}} + (R_{C_y} - \frac{T_{GD}}{\sqrt{2}})\hat{\boldsymbol{\jmath}} + (T_{HE} + R_{C_z} + \frac{T_{GD}}{\sqrt{2}})\hat{\boldsymbol{k}} = m\vec{\boldsymbol{a}} = 10\,\text{N}\hat{\boldsymbol{k}}$.

d) $\sum \vec{\boldsymbol{M}}_{\text{cm}} = (\frac{T_{GD}}{\sqrt{2}} - T_{HE} - R_{C_z})\hat{\boldsymbol{\imath}} + (R_{C_z} - \frac{T_{GD}}{\sqrt{2}} - T_{HE})\hat{\boldsymbol{\jmath}} + (T_{AB} + R_{C_x} - R_{C_y} - \frac{T_{GD}}{\sqrt{2}})\hat{\boldsymbol{k}} = \vec{\boldsymbol{0}}$

e)

$$
\begin{aligned}
R_{C_x} - T_{AB} &= 0 \\
R_{C_y} - \frac{T_{GD}}{\sqrt{2}} &= 0 \\
R_{C_z} + \frac{T_{GD}}{\sqrt{2}} + T_{EH} &= 5\,\text{N} \\
-T_{EH} + \frac{T_{GD}}{\sqrt{2}} - R_{C_z} &= 0 \\
-T_{EH} - \frac{T_{GD}}{\sqrt{2}} + R_{C_z} &= 0 \\
T_{AB} - \frac{T_{GD}}{\sqrt{2}} + R_{C_x} - R_{C_y} &= 0
\end{aligned}
$$

f) $R_{C_x} = 5\,\text{N}$, $R_{C_y} = 5\,\text{N}$, $R_{C_z} = 5\,\text{N}$, $T_{GD} = \frac{10}{\sqrt{2}}\,\text{N}$, $T_{EH} = 0\,\text{N}$, $T_{AB} = 5\,\text{N}$.

g) Find moment about CD axis; e.g., $(\sum \vec{\boldsymbol{M}}_{\text{C}} = \vec{\boldsymbol{r}}_{cm/C} \times \text{m}\vec{\boldsymbol{a}}_{cm}) \cdot \hat{\boldsymbol{\lambda}}_{CD}$, where $\hat{\boldsymbol{\lambda}}_{CD}$ is a unit vector in the direction of axis CD.

12.75a) $F_L = \frac{1}{2} m_{\text{tot}} g$.

b) $\vec{\boldsymbol{a}}_P = \frac{1}{m_{\text{tot}}}\left[2(T - F_D) - D\right]\hat{\boldsymbol{\imath}}$.

c) $\vec{\boldsymbol{F}} = \left[\frac{m_w}{m_{\text{tot}}}(2T - D - 2F_D) - T + F_D\right]\hat{\boldsymbol{\imath}} + (m_w g - F_L)\hat{\boldsymbol{\jmath}}$ and $\vec{\boldsymbol{M}} = (bF_L - am_w g)\hat{\boldsymbol{\imath}} + \left[(bF_D - cT) + a\frac{m_w}{m_{\text{tot}}}(2T - D - 2F_D)\right]\hat{\boldsymbol{\jmath}}$.

12.76) sideways force = $F_B\hat{\boldsymbol{\imath}} = \frac{wma}{2\ell}\hat{\boldsymbol{\imath}}$.